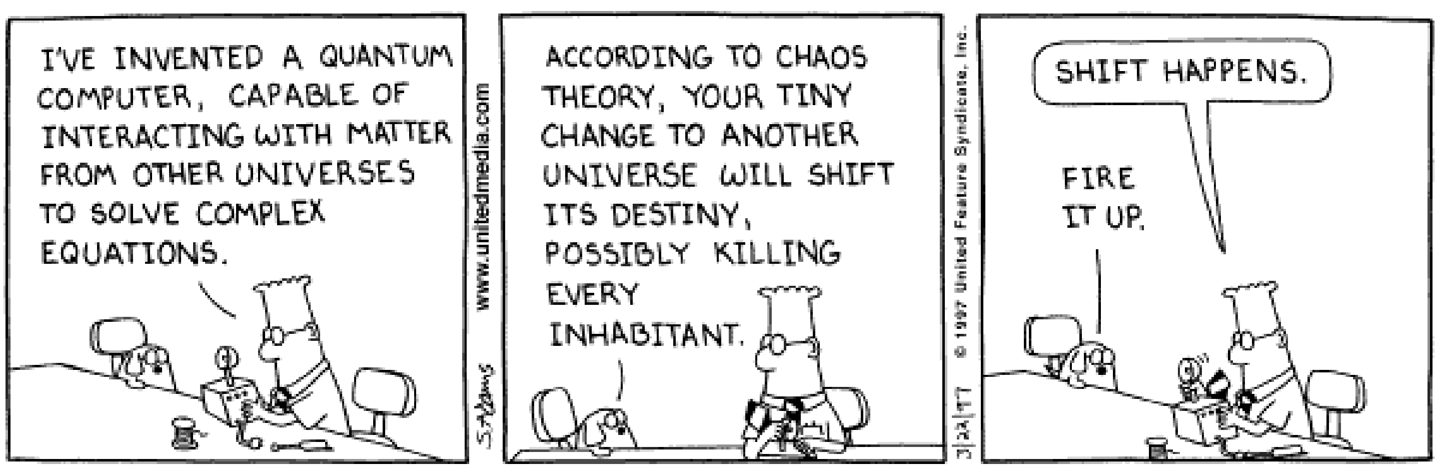
# Quantum Computation

Michael A. Nielsen

University of Queensland



# What does it mean to compute?

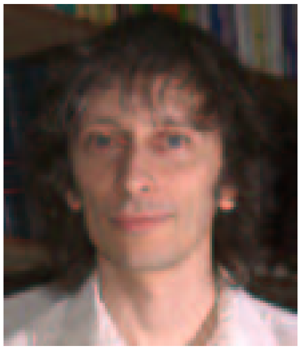
**Q:** Is there a general algorithm to determine whether a mathematical conjecture is true or false?


Church-Turing:
**NO!**

Church-Turing thesis: Any algorithmic process can be simulated on a Turing machine.

*Ad hoc* empirical justification.

Strong Church-Turing thesis: Any algorithmic process can be efficiently simulated on a probabilistic Turing machine.



# Deutsch:

## Can we justify C-T thesis using laws of physics?

Quantum mechanics seems to be very hard to simulate on a classical computer.

Might it be that computers exploiting quantum mechanics are not efficiently simulatable on a probabilistic Turing machine?

(Violation of strong C-T thesis!)

Might it be that such a computer can solve some computational problems faster than a probabilistic Turing machine?

Candidate universal computer:

quantum computer

# Quantum circuit model

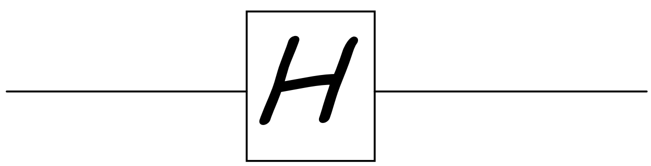
| Classical | Quantum |
|---|---|
| Unit: bit | Unit: qubit |
| 1. Prepare n bit input | 1. Prepare n qubit input |
| 2. Logic gates | 2. Reversible quantum logic gates |
| 3. Readout value of bits | 3. Readout partial information about qubits |

# Single qubit quantum logic gates

## Pauli gates:

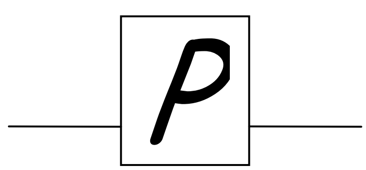
$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \quad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}; \quad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

## Hadamard gate:



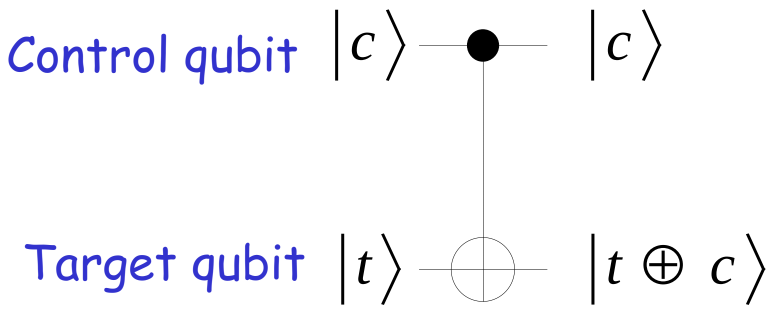

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}; \quad H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}; \quad H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

## Phase gate:



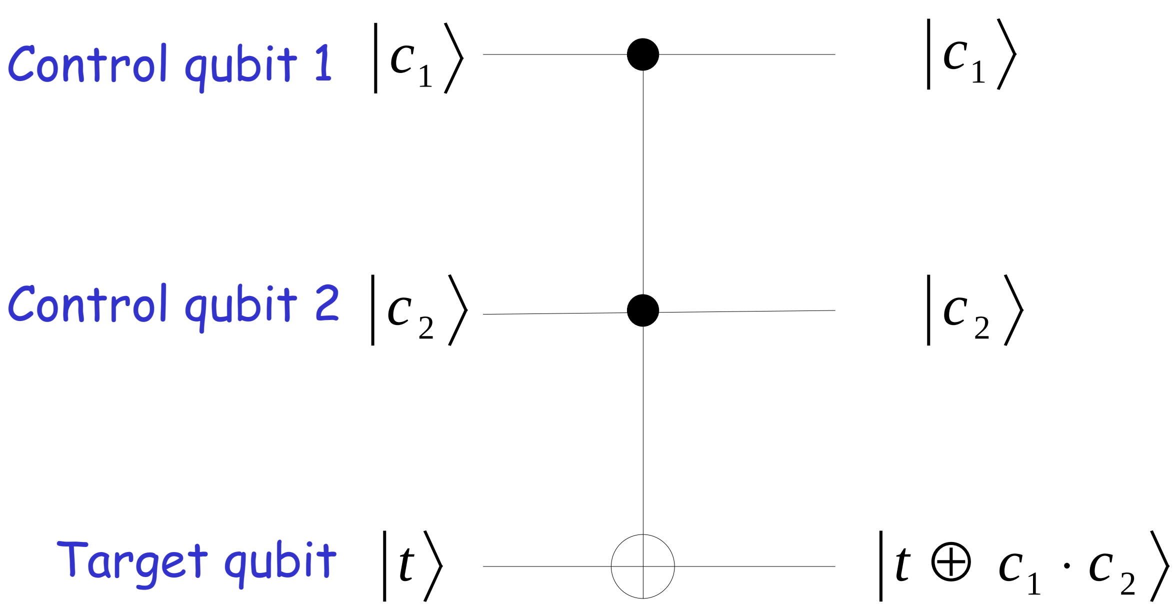

$$P|0\rangle = |0\rangle; \quad P|1\rangle = i|1\rangle$$

# Controlled not gate:




$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

# Toffoli gate



# Universal Logic Gates

Suppose U is an arbitrary unitary transformation on n qubits.

U can be approximated arbitrarily well by a sequence of Hadamard gates, phase gates, controlled not gates, and Toffoli gates.

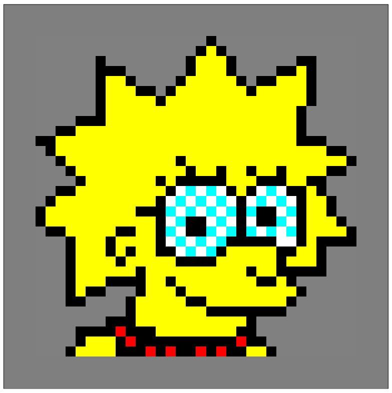
 These gates may all be performed fault-tolerantly, so in principle noise is not a problem.

# Quantum circuit model

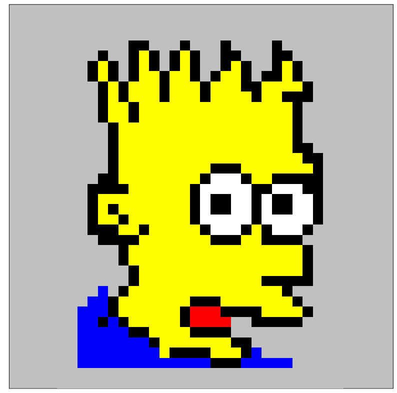
1. Prepare state $|0,0,\ldots,0\rangle$
2. Apply circuit of Hadamard, phase, controlled not and Toffoli gates.
3. Measure in the computational basis.

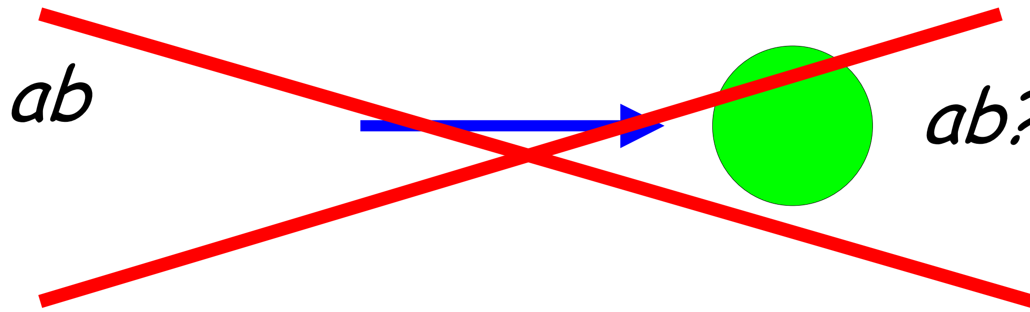
# Superdense coding

Alice



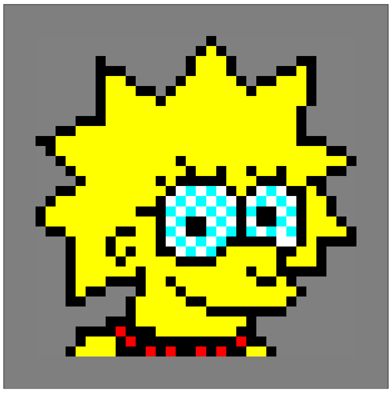
Bob



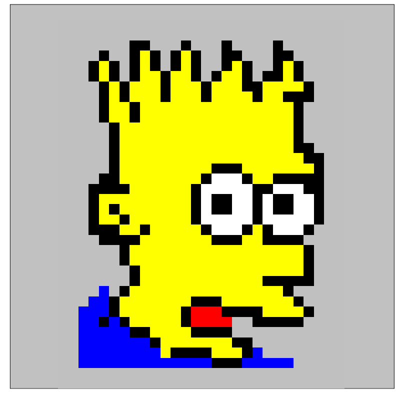


# Superdense coding

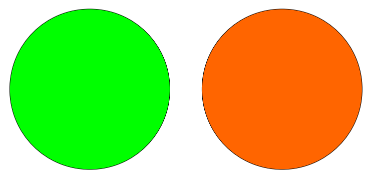
Alice



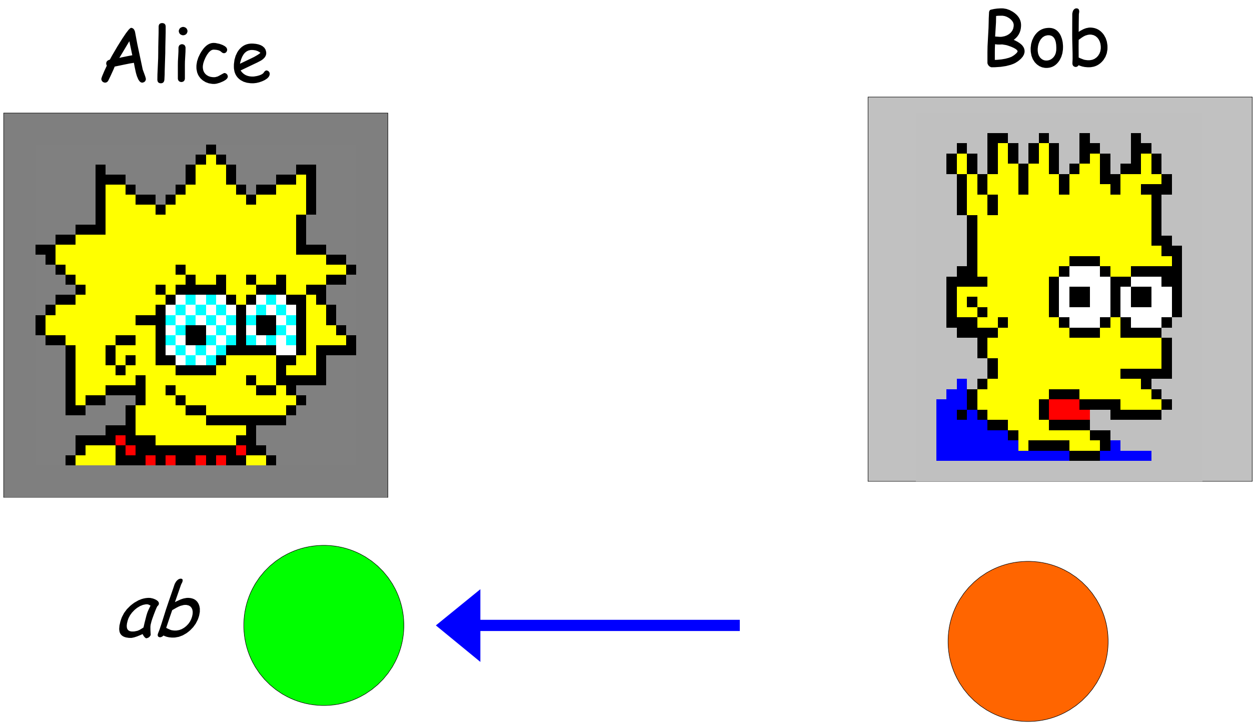
Bob



*ab*

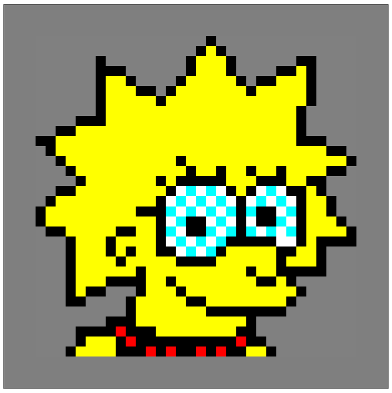


# Superdense coding

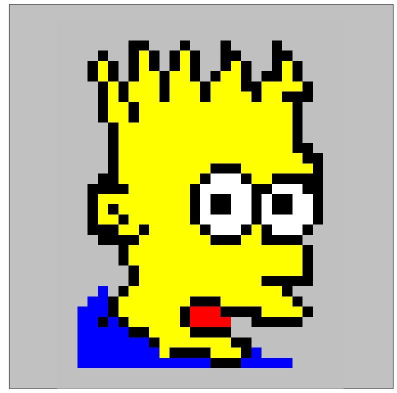


# Superdense coding

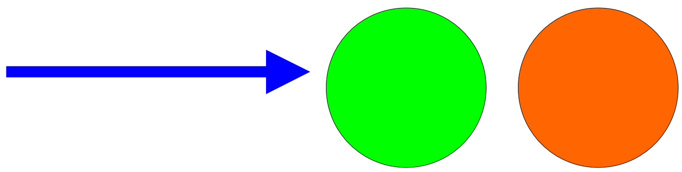
Alice



Bob

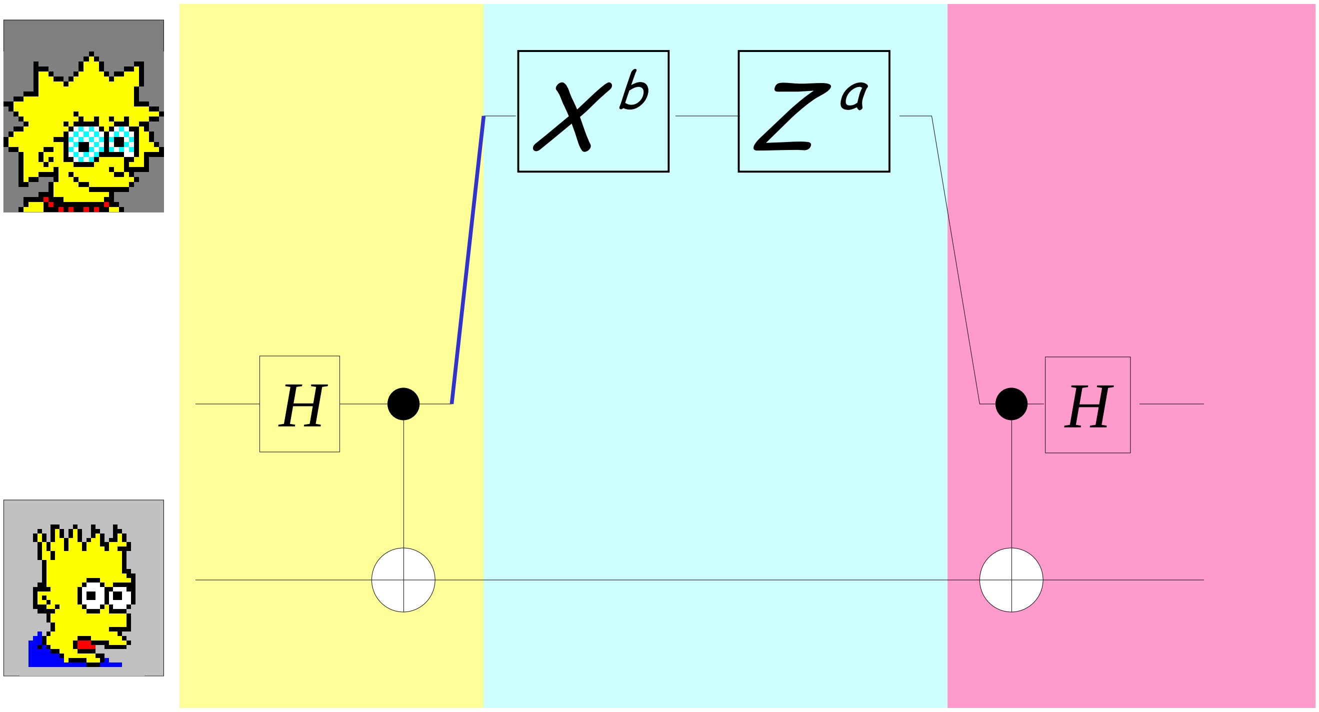


*ab*



*ab?*

# Superdense coding




$$00 \to 00 + 10 \to 00 + 11$$

$$00 + 11$$

$b = 0$ $\qquad$ $b = 1$

$$00 + 11 \qquad\qquad 10 + 01$$

$a = 0$ $\quad$ $a = 1$ $\quad$ $a = 0$ $\quad$ $a = 1$

$$00 + 11 \qquad 00 - 11 \qquad 10 + 01 \qquad -10 + 01$$

$$ab = 01: \;\; 10 + 01 \to 11 + 01 \to 01$$

# Example: Deutsch's problem

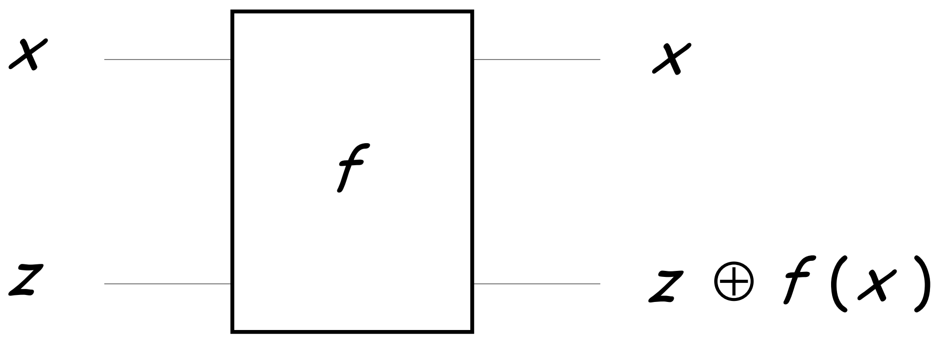
Given a black box computing a function $f:\{0,1\}\rightarrow\{0,1\}$

Our task is to determine whether $f$ is constant or balanced?

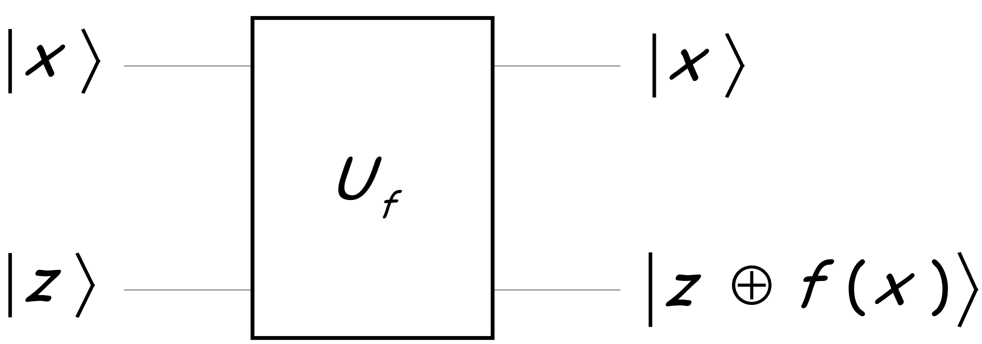
Classically we need to evaluate both $f(0)$ and $f(1)$.

Quantumly we need only use the black box for $f(\bullet)$ once!

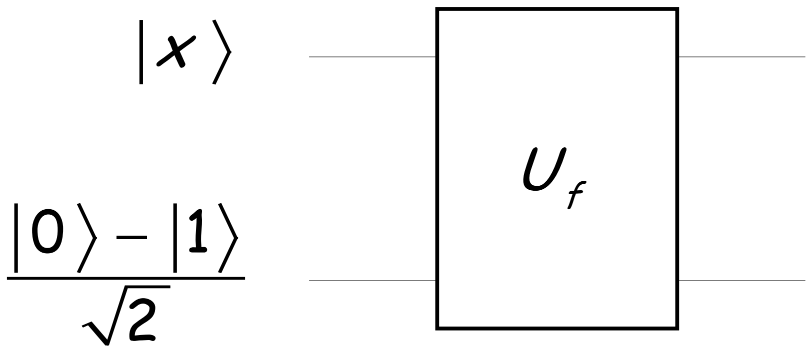
# Classical black box




# Quantum black box



# Putting information in the phase




$f(x) = 0:$

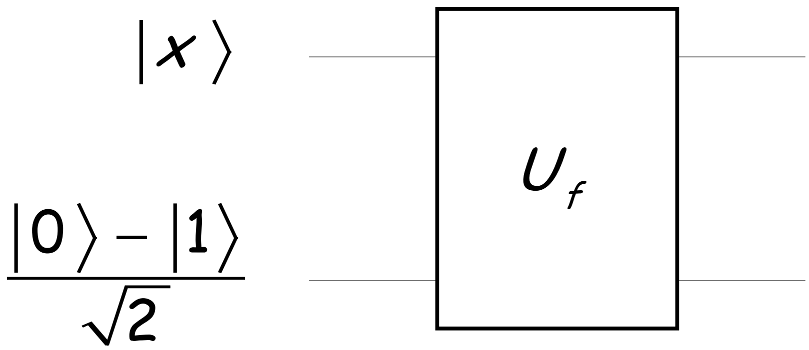
$$|x\rangle(|0\rangle - |1\rangle) \rightarrow |x\rangle(|0\rangle - |1\rangle)$$

$f(x) = 1:$

$$\begin{aligned} |x\rangle(|0\rangle - |1\rangle) &\rightarrow |x\rangle(|1\rangle - |0\rangle) \\ &= -|x\rangle(|0\rangle - |1\rangle) \end{aligned}$$

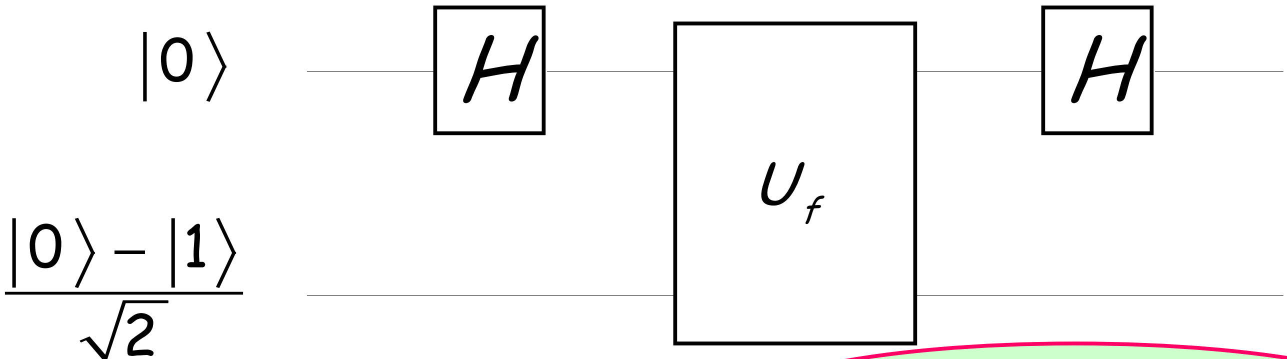
$$|x\rangle(|0\rangle - |1\rangle) \rightarrow (-1)^{f(x)}|x\rangle(|0\rangle - |1\rangle)$$

# Putting information in the phase




$$|x\rangle \rightarrow (-1)^{f(x)}|x\rangle$$

# Quantum algorithm for Deutsch's problem




Quantum parallelism

$$|0\rangle \rightarrow |0\rangle + |1\rangle$$

$$\rightarrow (-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle$$

$$\rightarrow (-1)^{f(0)}\left(|0\rangle + |1\rangle\right) + (-1)^{f(1)}\left(|0\rangle - |1\rangle\right)$$

$$= \left[(-1)^{f(0)} + (-1)^{f(1)}\right]|0\rangle + \left[(-1)^{f(0)} - (-1)^{f(1)}\right]|1\rangle$$

$f$ const. $\Rightarrow$ all amplitude in $|0\rangle$.

$f$ balan. $\Rightarrow$ all amplitude in $|1\rangle$.