Quantum Mechanics, Quantum Information, and all that

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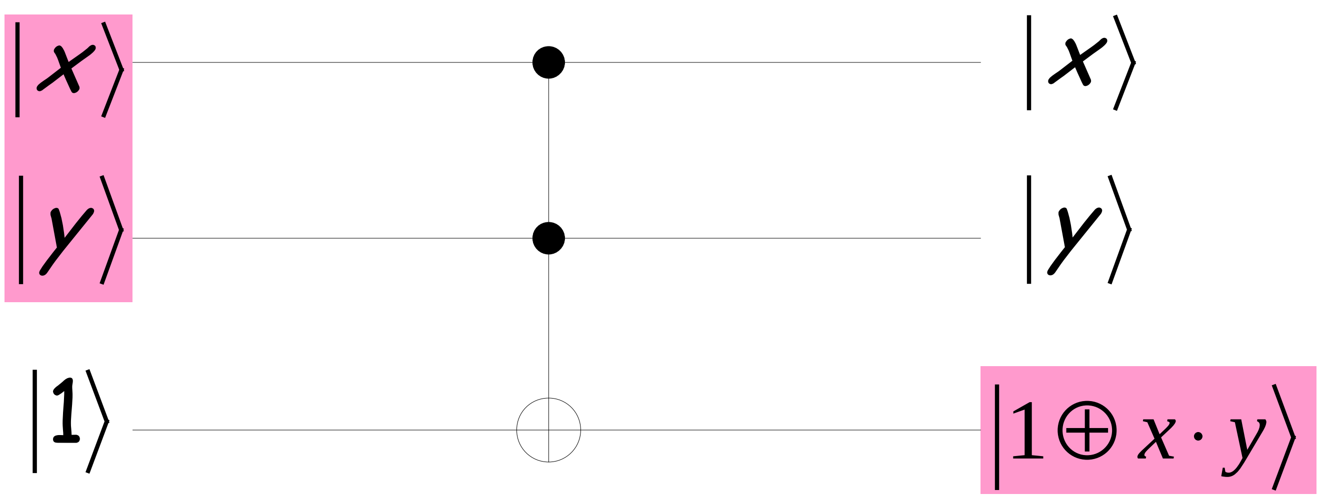
How to compute classical functions on quantum computers?

Problem: reversible nature of quantum gates.

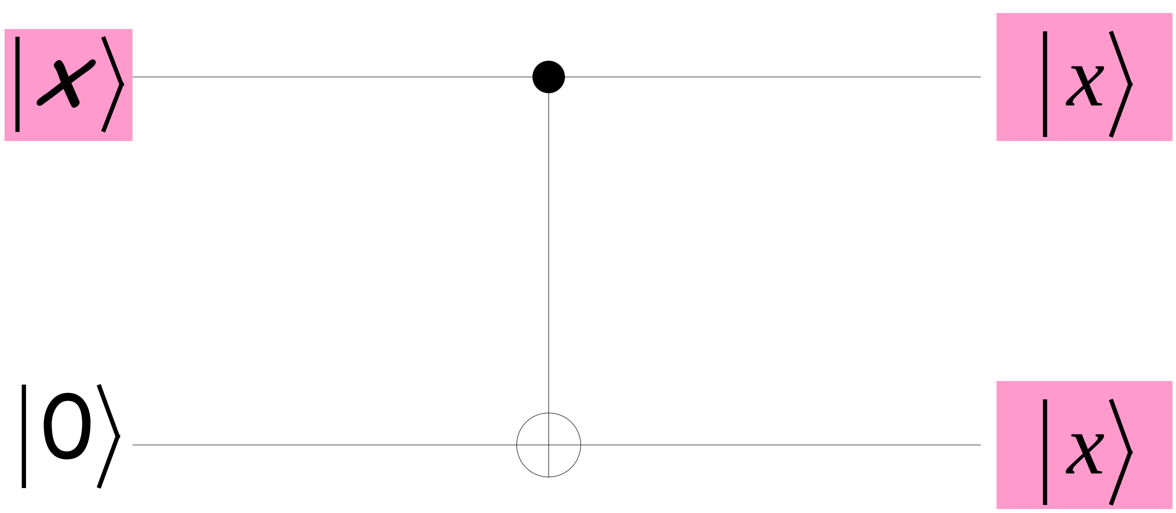
Solution: embed irreversible classical gates in reversible quantum gates.

Example: The quantum NAND gate.



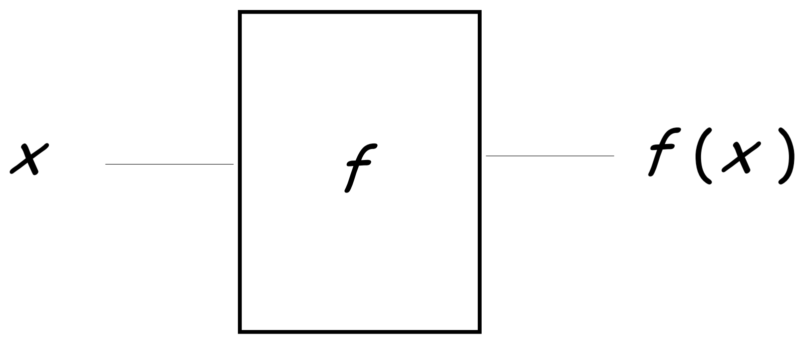
How to compute classical functions on quantum computers?

Any classical circuit can always be written in terms of NAND and FANOUT gates.

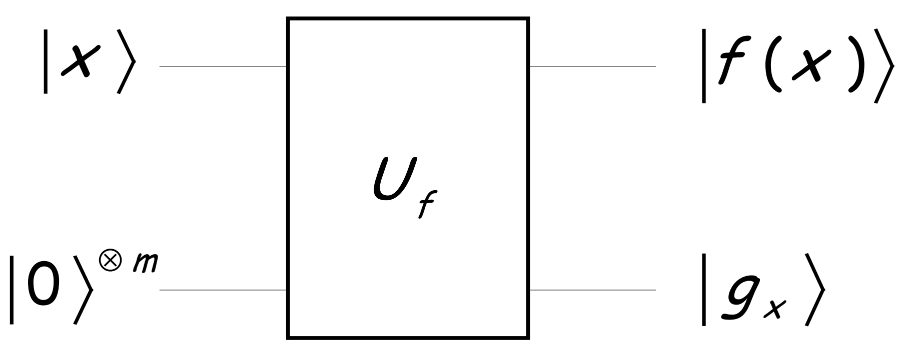


How to compute classical functions on quantum computers?

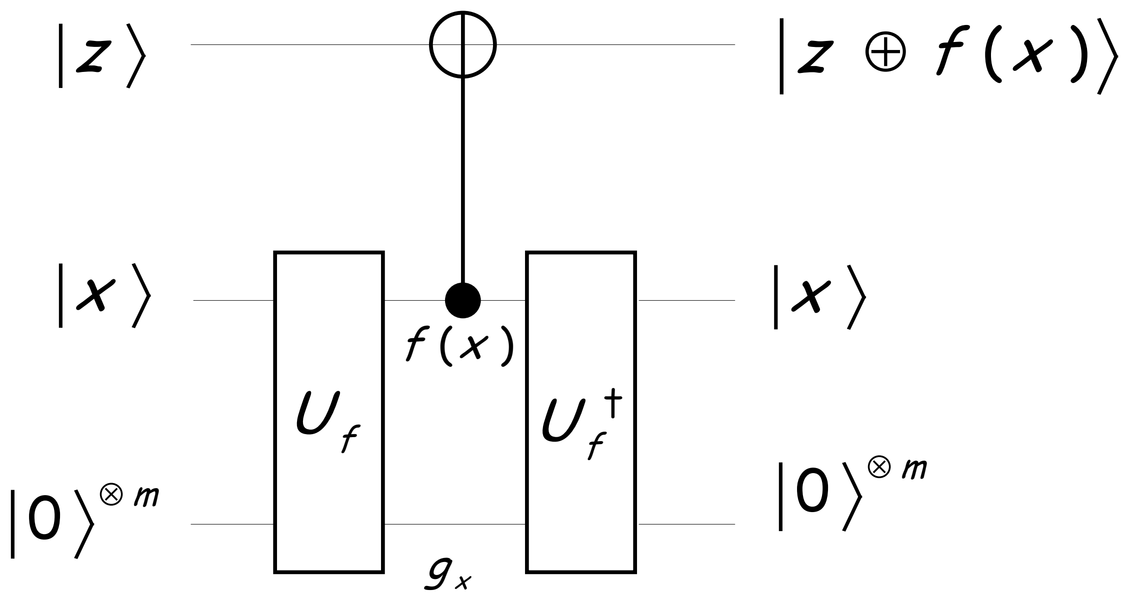
Classical circuit



Quantum circuit



How to compute classical functions on quantum computers?



Net effect:

$$|x\rangle|z\rangle \rightarrow |x\rangle|z \oplus f(x)\rangle$$

What is quantum information theory?

1. Identify classes of static resources in Q.M.

Bits, qubits, shared states ("entangled states")

2. Identify classes of dynamical processes in Q.M.

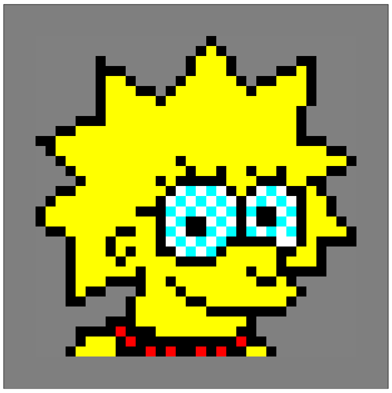
Memory, quantum information transmission, copying, compression

3. Quantify resource tradeoffs incurred performing elementarydynamical processes.

Minimal resources to communicate quantum states, in the presence of noise.

Quantum teleportation

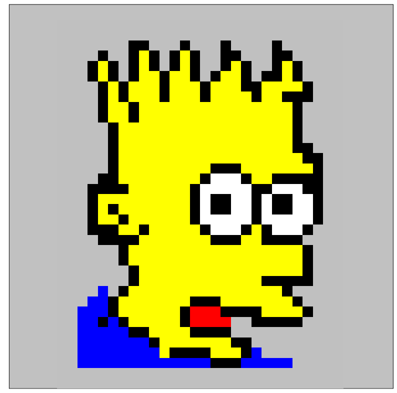
Alice



Preshare:

$$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

Bob



$|\psi\rangle$ → 

After some algebra:

$$\left(|00\rangle + |11\rangle\right)|\psi\rangle + \left(|00\rangle - |11\rangle\right) Z|\psi\rangle +$$
$$\left(|01\rangle + |10\rangle\right) X|\psi\rangle + \left(|01\rangle - |10\rangle\right) XZ|\psi\rangle$$

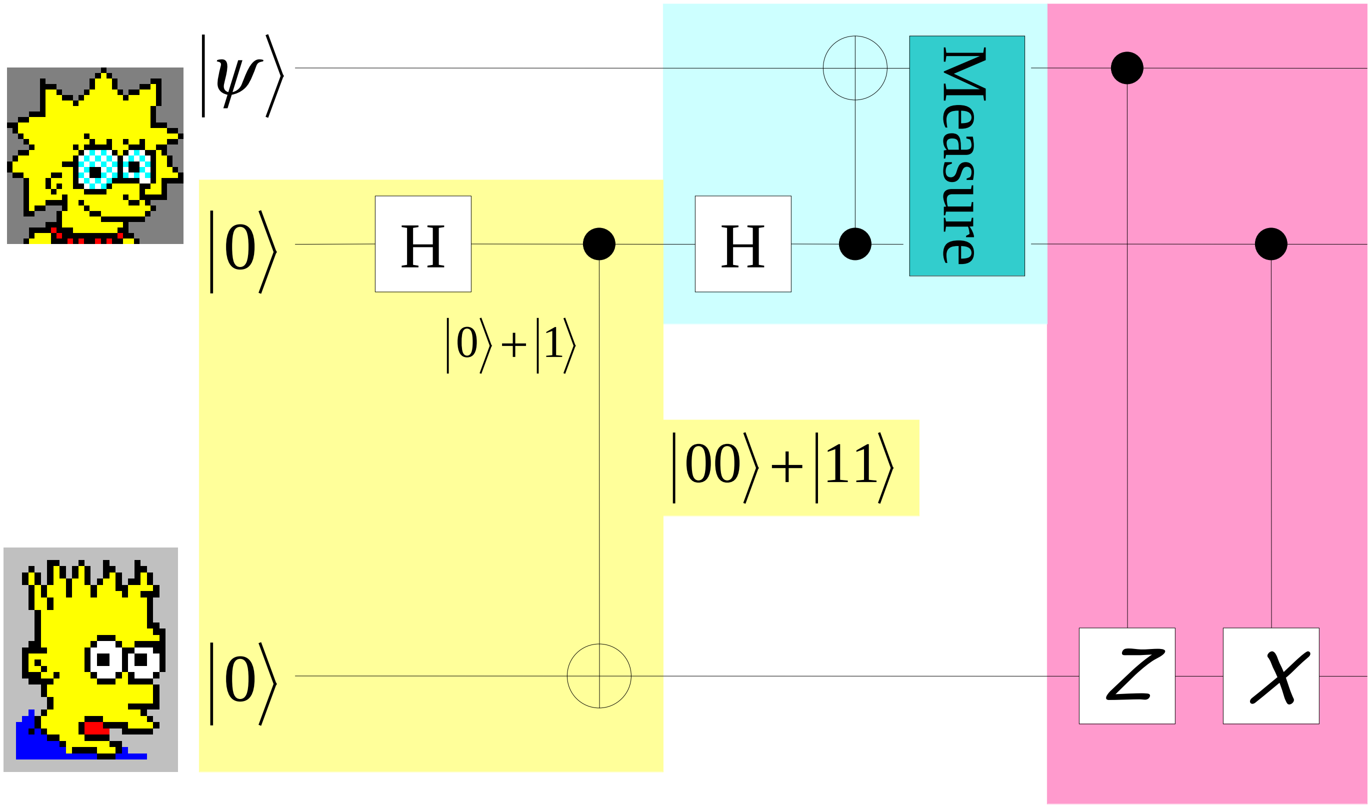
Alice measures:

$$|00\rangle + |11\rangle;\ |00\rangle - |11\rangle;\ |01\rangle + |10\rangle;\ |01\rangle - |10\rangle$$

Bob sees:

$$|\psi\rangle; \qquad Z|\psi\rangle; \qquad X|\psi\rangle; \qquad XZ|\psi\rangle$$

Teleportation as a quantum circuit



Legend:

Initial entanglement

Alice measures in the Bell basis

Conditional operations

1 ebit + 2 classical bits of communication

$$\geq$$

1 qubit of communication

Outer products

Let $|\psi\rangle$ and $|\phi\rangle$ be vectors.

Define an operator (matrix) $|\psi\rangle\langle\phi|$ by

$$|\psi\rangle\langle\phi|\big(|\gamma\rangle\big) \equiv |\psi\rangle\langle\phi|\gamma\rangle$$

Example:

$$|1\rangle\langle 0|\big(\alpha|0\rangle + \beta|1\rangle\big) \equiv |1\rangle\alpha = \alpha|1\rangle$$

Example: $X|0\rangle = |1\rangle$; $X|1\rangle = |0\rangle$

Easy to verify that: $X = |1\rangle\langle 0| + |0\rangle\langle 1|$

Revised form of postulate 2

The evolution of a closed quantum system is described by a unitary transformation.

$$|\psi'\rangle = U|\psi\rangle$$

But quantum dynamics occurs in continuous time.

Hermitian matrices

Hermitian matrix H is one such that $H^{\dagger}=H$.

Example:

$$X^{\dagger} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}^{\dagger} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = X$$

Hermitian matrices

Hermitian matrices satisfy the spectral theorem: there exists an orthonormal basis of eigenvectors for any Hermitian matrix, with real eigenvalues.

Example:

$$X\frac{|0\rangle+|1\rangle}{\sqrt{2}}=\frac{|0\rangle+|1\rangle}{\sqrt{2}}$$

$$X\frac{|0\rangle-|1\rangle}{\sqrt{2}}=-\frac{|0\rangle-|1\rangle}{\sqrt{2}}$$

Revised form of postulate 2

The evolution of a closed quantum system is described by Schroedinger's equation:

$$i\frac{d|\psi\rangle}{dt} = H|\psi\rangle$$

where H is a constant Hermitian matrix known as the Hamiltonian of the system.

The eigenvectors of H are known as the energy eigenstates of the system, and the corresponding eigenvalues are known as the energies.

Example: $H = \omega X$ has energy eigenstates $(|0\rangle + |1\rangle)/\sqrt{2}$ and $(|0\rangle - |1\rangle)/\sqrt{2}$, with corresponding energies $\pm\omega$

Connection to old form of postulate 2

The solution of Schroedinger's equation is

$$\left|\psi(t)\right\rangle = \exp(-iHt)\left|\psi(0)\right\rangle$$

$$U \equiv \exp(-iHt)\left|\psi(0)\right\rangle$$

$$\left|\psi'\right\rangle = U\left|\psi\right\rangle$$

[redacted]