Collisionless Damping in Plasmas and Neutral Gases.

Fluid-like closures for the long mean-free-path regimes

A. Smolyakov
University of Saskatchewan, Canada

Kinetic Theory Workshop, Fields Institute

Motivation

- Discuss fluid like closures for the long mean-free-path regimes

- Develop the closure model in terms of the source profile functions

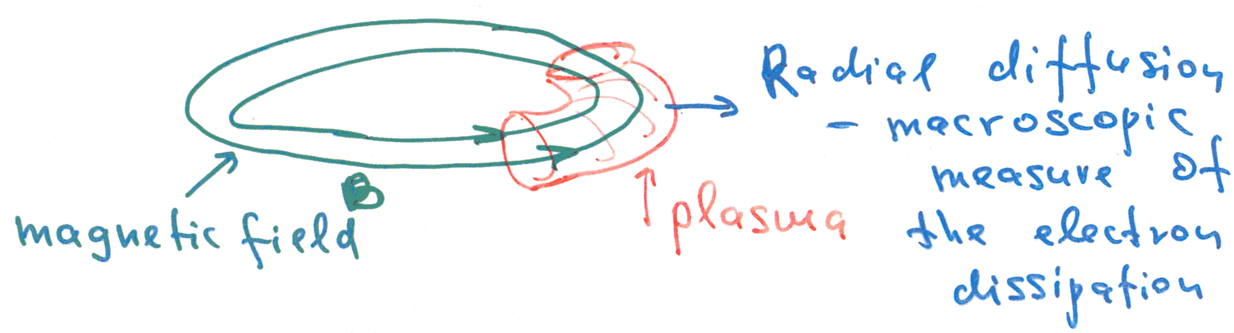
- Discuss analogy between Landau damping (boundary value problem) and ultrasound damping in neutral gases

Related talks at this workshop:

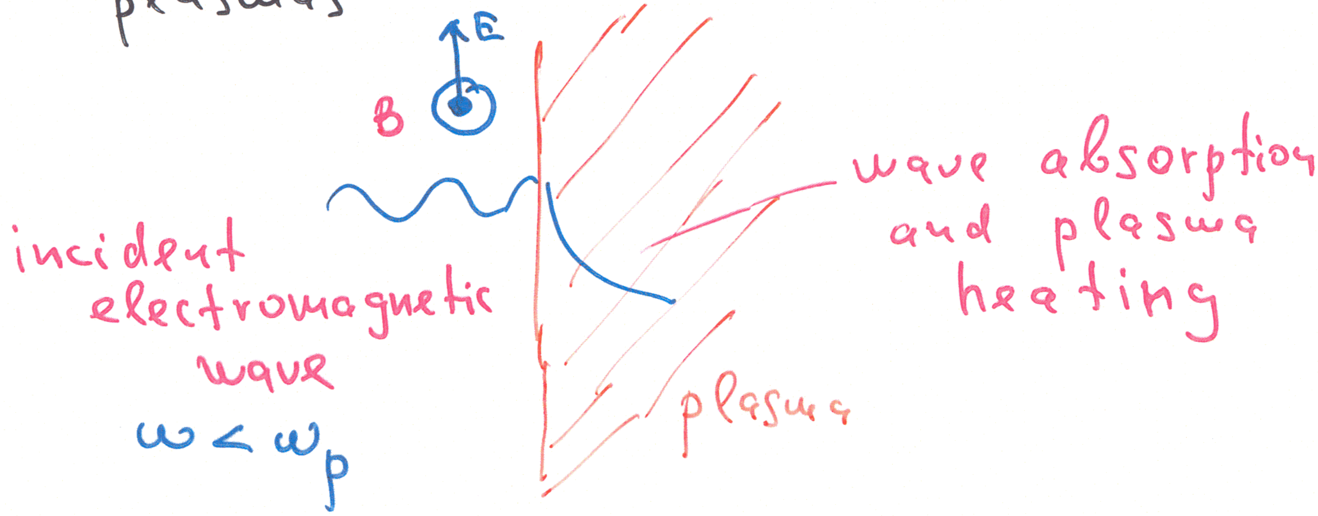
D. Levermore
G. Hammet
B. Dorland
T. Passot

Collisionless Dissipation in Plasmas

- Collisional dissipation in tokamaks is too weak to explain electron diffusion across the magnetic surfaces
  - in absence of dissipation electron fluid is frozen-in the magnetic field



- Plasma heating due to collisionless absorption of the electromagnetic wave in the inductively coupled plasmas



Anomalous power absorption or collisionless heating

$$S(z) = \int_0^z P(x)\,dx$$ - total absorbed power

Heating

Collisional (Ohmic) | Collisionless (resonant)

Collisionless heating dominates!

Effective resonant wave-particle interaction (Landau damping)

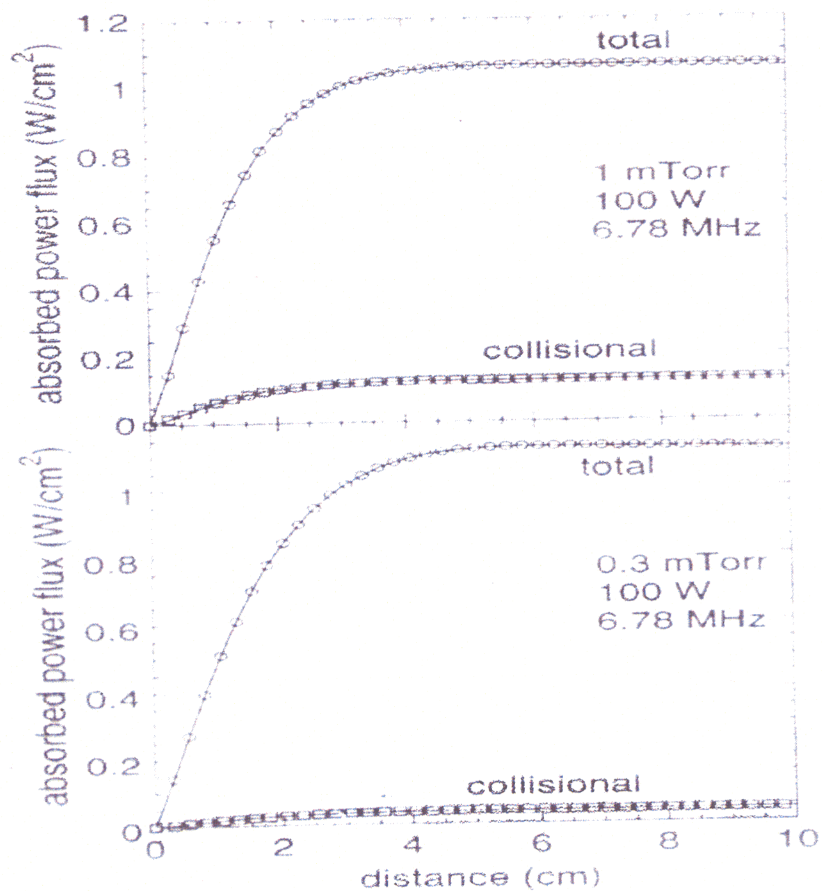


FIG. 1. Absorbed power flux for 0.3 and 1.0 mTorr.

Godyak *et. al.*, Phys. Rev. Lett. **80**, 3264 (1998)

Fluid models + Collisionless Absorption

- Kadomtsev (1970): Use standard collisional fluid model, replace collisional frequency ν with v_{Te}/λ: $\nu \to v_{Te}/\lambda$

  Landau resonance: $\omega \simeq k v_{Te}$

- Kadomtsev (1984): Electron viscosity and heat flux are responsible for collisionless dissipation, calculate q and π exactly from the kinetic equation, $q_{\parallel} = q_{\parallel}(E_{\parallel})$, $\pi_{\parallel} = \pi_{\parallel}(E_{\parallel})$

- Model dissipative terms (viscosity/heat flux): Lee, Diamond (1986), Hamaguchi, Horton (1990) Waltz (1988)

- Closure models

$$\frac{\partial}{\partial t}(mnV) + \frac{\partial}{\partial z}(mnVV) = -\frac{\partial p}{\partial z} + enE - \frac{\partial}{\partial z}\pi$$

$$\frac{\partial}{\partial t}p + \frac{\partial}{\partial z}(pV) = -2(p+\pi)\frac{\partial V}{\partial z} - \frac{\partial}{\partial z}q$$

Postulated form $\Big\}\to$ $q = -\chi \dfrac{\partial T}{\partial z}$ $\qquad \pi = -\mu \dfrac{\partial V}{\partial z}$

χ, μ - are chosen to fit the linear kinetic response (in Fourier-space)

Hammet, Perkins (1990)
Dorland, Hammet (1993) $\leftarrow$ toroidal, FLR
Snyder, Hammet, Dorland (1997) $\leftarrow$ MHD

The equation for f which can be solved by a standard method, gives

$$\frac{\partial \psi}{\partial t} = c\frac{\mathbf{B}}{B_0}\left(\nabla\phi - \frac{\nabla p_e}{en}\right) + \frac{c^2\hat{\eta}}{4\pi}\Delta_\perp \psi, \tag{4}$$

where $\hat{\eta}$ is the collisionless resistivity of the plasma, which in the Fourier representation is found from the relation

$$\eta^{-1}_{k\omega} = \sigma_{k\omega} = -i\frac{e^2 n\omega}{T_e k_\parallel}\left\{1 + \frac{\omega}{k_\parallel v_e} Z\left(\frac{\omega}{k_\parallel v_e}\right)\right\}. \tag{5}$$

Here $v_e = \sqrt{2T_e/m}$, m is the mass of the electron, and Z is the so-called dispersion function.[8] Since in what follows we shall need only the real part of $\hat{\eta}$, we set approximately

$$\hat{\eta} \cong \mathrm{Re}\,\hat{\eta} \approx \begin{cases} \dfrac{mk_\parallel v_e}{e^2 n} & \text{for} \quad \omega^2 \leqslant k_\parallel^2 v_e^2, \\ 0 & \text{for} \quad \omega^2 > k_\parallel^2 v_e^2. \end{cases} \tag{6}$$

Kadomtsev, 1984

$$\frac{dn}{dt}=\frac{c}{4\pi eB_0}(\mathbf{B},\nabla)\Delta_\perp\psi; \tag{1}$$

an equation for the generalized vorticity[1] $\Gamma=-\rho_i^{-2}(1-\exp(\rho_i^2\Delta_\perp))I_0(-\rho_i^2\Delta_\perp)c\varphi/B_0$:

$$\frac{d\Gamma}{dt}=\frac{1}{4\pi Mn_0}(\mathbf{B},\nabla)\Delta_\perp\psi; \tag{2}$$

a generalized Ohm's law,

$$\frac{d\psi}{dt}=ce_z\nabla\varphi-\frac{c}{eB_0}\mathbf{B}\nabla T_e\ln\frac{n}{n_0}+\frac{c^2\hat\eta}{4\pi}\Delta_\perp\psi; \tag{3}$$

and a heat transfer equation,

collisionless dissipatia

$$\frac{1}{2}\frac{d}{dt}nT_e=\frac{(\mathbf{B},\nabla q_\parallel)}{B_0}+E_\parallel j_\parallel. \tag{4}$$

Here $d/dt=\partial/\partial t+(c/B_0)[e_z\nabla\varphi]\cdot\nabla$; M and m are the ion and electron masses; $B_p=[e_z\nabla\psi]$; n is the density; n_0 is the average density; $\rho_i^2=T_i/M\omega_{Bi}^2$ is the ion Larmor radius; the operator $\hat\eta=\hat\sigma^{-1}$ represents the specific collisionless resistance; and

$$q_\parallel=-\int\frac{m}{2}(v_\perp^2+v_\parallel^2)v_\parallel f(v)\,dv=\left(\frac{mv_e^2\hat\sigma}{2e}+\hat\mu\right)E_\parallel.$$

The operators $\hat\eta$ and $\hat\mu$ in an inhomogeneous plasma can be found from the electron drift kinetic equation

$$\frac{d}{dt}f(v_\parallel)+\frac{v_\parallel\cdot\mathbf{B}}{B_0}\nabla f(v_\parallel)+\frac{e}{m}E_\parallel\frac{\partial f}{\partial v_\parallel}=0. \tag{5}$$

Kadomtsev, 1986.

Chapman-Enskog like ~~equations~~ closures

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{V}) = 0,$$

$$mn\left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right)\mathbf{V} = en\mathbf{E} - \nabla p - \nabla\cdot\pi,$$

$$\frac{3}{2}n\frac{\partial T}{\partial t} + \frac{5}{2}p\nabla\cdot\mathbf{V} + \nabla\cdot\mathbf{q} + \pi : \nabla\mathbf{V} = 0,$$

$$\boxed{f(\mathbf{x},t) = n(\mathbf{x},t)\left(\frac{m}{2\pi T(\mathbf{x},t)}\right)^{3/2}\exp\left(-\frac{m(v-\mathbf{V})^2)}{2T(\mathbf{x},t)}\right) + \tilde{F}.}$$

Chapman-Enskog Anzatz

$$\int \tilde{F}\{1, \mathbf{v}, mv^2/2)\}d\mathbf{v} = 0$$

$$\frac{\partial f}{\partial t} + v\cdot\nabla f + \frac{q}{m}E\frac{\partial f}{\partial v} = C(f)$$

nonlinear

$$\frac{\partial \tilde{F}}{\partial t} + \mathbf{v}\cdot\nabla\tilde{F} - C(f) + \frac{q}{m}\mathbf{E}\cdot\frac{\partial\tilde{F}}{\partial\mathbf{v}} = \frac{m}{T}\left(\mathbf{v}'\mathbf{v}' - \frac{\mathbf{v}'^2}{3}\mathbf{I}\right) : \nabla\mathbf{V} f_M + \mathbf{v}'\cdot\nabla\cdot\pi\frac{f_M}{p_0}$$
$$+ \left(\frac{m\mathbf{v}'^2}{3T} - 1\right)(\pi : \nabla\mathbf{V} + \nabla\cdot\mathbf{q})\frac{f_M}{p_0} - \left(\frac{m\mathbf{v}'^2}{2T} - \frac{5}{2}\right)\mathbf{v}'\cdot\nabla T\frac{f_M}{T_0}$$

Solved in Fourier space

$$\tilde{F} = \tilde{F}\left(\pi, q, V, \nabla_\parallel T\right)$$

$$\left.\begin{aligned} \pi &= \pi\left(V_\parallel, \nabla_\parallel T\right) \\ q &= q\left(V_\parallel, \nabla_\parallel T\right) \end{aligned}\right\}$$

closure equations
Chang, Callen (1992)

$$\tilde F = k_{\|}u_{\|}\frac{4}{3v_t^2}\frac{f_M}{\omega - k_{\|}v_{\|}}(v_{\|}^2 - \frac{v_{\perp}^2}{2}) - k_{\|}\Pi_{\|}\frac{2}{3p}\frac{v_{\|}f_M}{\omega - k_{\|}v_{\|}}$$
$$-k_{\|}q_{\|}\frac{2}{3p}\frac{f_M}{\omega - k_{\|}v_{\|}}(\frac{v^2}{v_t^2} - \frac{3}{2}) + k_{\|}\frac{\tilde T}{T}\frac{v_{\|}f_M}{\omega - k_{\|}v_{\|}}(\frac{v^2}{v_t^2} - \frac{5}{2})$$

Closures relations

$$\boxed{\begin{aligned} \frac{\pi_{\|}}{p_0} &= A_u\frac{u_{\|}}{v_t} + A_T\frac{\tilde{T}^{\dagger}}{T_0} \\ \frac{q_{\|}}{p_0 v_t} &= B_u\frac{u_{\|}}{v_t} + B_T\frac{\tilde{T}^{\dagger}}{T_0} \end{aligned}}$$

$$\begin{aligned}
A_u &= \frac{-12sZ(s) + 6Z^2(s) - 12s^2Z^2(s)}{2s - 5Z(s) + 2s^2Z(s) - 4sZ^2(s)} \\
A_T &= \frac{6s + 3Z(s) + 6s^2Z(s) + 6sZ^2(s))}{2s - 5Z(s) + 2s^2Z(s) - 4sZ^2(s)} \\
B_u &= \frac{4s + 2Z(s) + 4s^2Z(s) + 4sZ^2(s)}{2s - 5Z(s) + 2s^2Z(s) - 4sZ^2(s)} \\
B_T &= \frac{-9 + 3s^2 - 33/2sZ(s) + 3s^3Z(s) - 6s^2Z_P^2(s)}{2s - 5Z(s) + 2s^2Z(s) - 4sZ^2(s)}
\end{aligned}$$

$$s = (\omega + i\nu)/kv_T$$

$$\begin{cases} ik\tilde\pi = -n_0 m \int q(-k^2\tilde V) - \frac{2}{5}n_0 ik_{\|}\tilde T \\ \tilde q = \frac{2}{5}p_0\tilde V - n_0 f_{\|}\, ik_{\|}\tilde T \end{cases}$$

Onsager symmetry

Collisionless case $\quad kV_T \gg (\omega, \nu)$

Transport theory in the collisionless limit (Hazeltine, 1998)

- Magnetically confined plasmas are approximately maxwellian
  - confinement time is much longer compared to collision times
    $\tau_n >> \tau_c \sim 1/\nu$

- Weak Maxwellian sources are assumed

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = C(f) + S(f)$$ — source

$$S(f) = \hat{f}_m \left(S_0(x) + (s^2 - \tfrac{1}{2}) S_2(x) \right)$$

↑ particles ↑ energy

$$\frac{\partial}{\partial t} n + \frac{\partial}{\partial x} nV = S_0$$ ← sources

$$\frac{\partial}{\partial t} p + \frac{\partial}{\partial x}(pV) = -\Gamma p \frac{\partial u}{\partial x} + \frac{\partial q}{\partial x} + \left(\frac{S_2}{2} - S_0\right)$$

$q = (\tilde{p}, S_2, S_0 \ldots)$ $\quad q_k = -\frac{\nu \lambda^2}{2} W(\xi)\, n_0 A_{2k}$ $\quad A_2 = \frac{\partial \ln T}{\partial x}$

$\Gamma = (S_0, \tilde{n})$ $\quad W(\xi) = \xi \dfrac{2(\xi^2 - 1) + \xi(2\xi^2 - 1)\tilde{Z}}{2\xi + (2\xi^2 - 1)\tilde{Z}}$ $\quad \xi = -i/k\lambda$

$$\Gamma_k = -\frac{1}{2}\nu\lambda^2 n_0 \nabla \ln p = -\frac{1}{2}\frac{v_t^2}{\nu} n_0 \nabla \ln p$$

singular for $\nu \to 0$

The Boltzmann kinetic equation

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = C(f)$$

Constraints of particle, momentum, and energy conservation

$$\int C(f)dv = 0,$$

$$\int vC(f)dv = 0,$$

$$\int v^2 C(f)dv = 0$$

A linearized model operator of the relaxation type

$$C(f) = -\nu\left(f - f_r\right),$$

The moments of the distrubution function

$$n = \int f dv$$

$$n\mathbf{V} = \int \mathbf{v} f dv$$

$$p = \frac{m}{3}\int v^2 f dv$$

where $\mathbf{v}' = \mathbf{v} - \mathbf{V}$ is the random velocity.

Relaxation distribution function

$$f_r = f_M\left[\frac{\tilde{n}}{n_0} + 2\frac{\mathbf{V}\cdot\mathbf{v}}{v_t^2} + \frac{\tilde{T}}{T_0}\left(\frac{v^2}{v_t^2} - \frac{3}{2}\right)\right],$$

Closure in terms of the source functions

(Hazeltine, 1998)
Kinetic equation

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = C(f) + S(f)$$

particles, energy

$$S(f) = \frac{f_M}{n_0}\left(S_0 + 2S_x\frac{v_x}{v_t^2} + S_2\left(\frac{v^2}{v_t^2} - \frac{3}{2}\right)\right)$$

momentum

Conservation laws

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{V}) = S_0$$

$$mn\frac{d\mathbf{V}}{dt} = -\nabla p - \nabla\cdot\pi + S_1$$

$$\frac{3}{2}n\frac{dT}{dt} + \frac{5}{2}p\nabla\cdot\mathbf{V} + \nabla\cdot\mathbf{q} + \pi : \nabla\mathbf{V} = S_2$$

Sources drive the fluxes!

$$\tilde{f}_k = \frac{\hat{f}_M}{-i\omega + ikv + \nu}F_k$$

$$F_k = 2\sigma_{0k} + 2s_x\left(\sigma_{1k} + \nu\hat{V}_k\right) + \left(s^2 - \frac{3}{2}\right)\left(\sigma_{2k} + \nu\hat{T}_k\right)$$

$$\hat{T}_k = \frac{2}{3}\int\left(s^2 - \frac{3}{2}\right) f_k d^3v$$

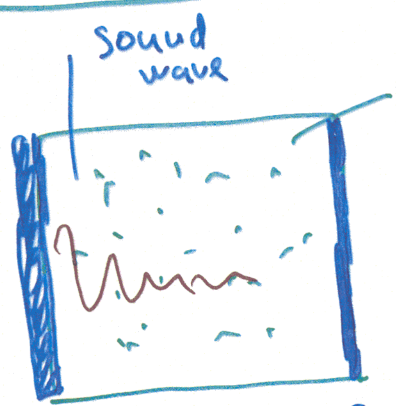
$$\hat{n}_k = \int f_k d^3v$$

$$\hat{V}_k = \int v f_k d^3v$$

$\boxed{\text{Closures}}$
$$\begin{cases} \pi_k = \pi_k(\hat{V}_k, \hat{T}_k) \\ q_k = q_k(\hat{V}_k, \hat{T}_k) \end{cases}$$

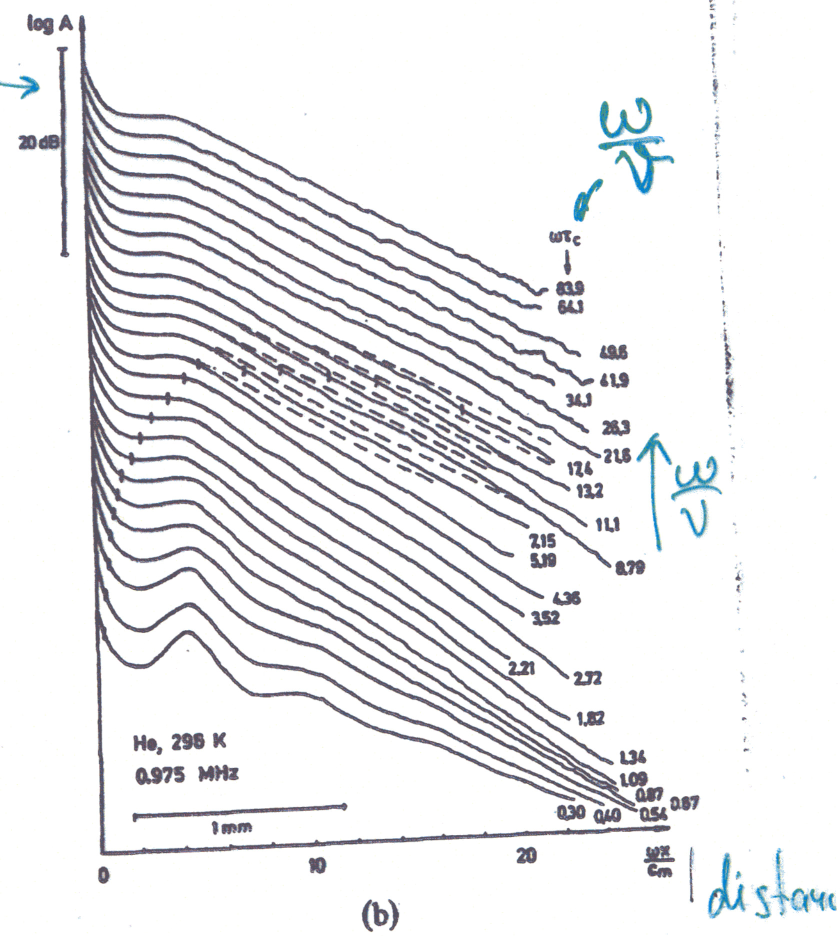
Similar to Chang, Callen (identical?)

Collisionless damping of sound waves in neutral gases



- Forced wave (boundary value) problem, $\omega = \text{const}$ (real)
  $k = k_R + i k_I$
  ↑ damping
- Non-exponential decay ($k_R, k_I \neq \text{const}$)
  (note that it also typical for classical boundary value Landau problem in plasmas)
- Receiver effects (e.g. standing wave)

amplitude



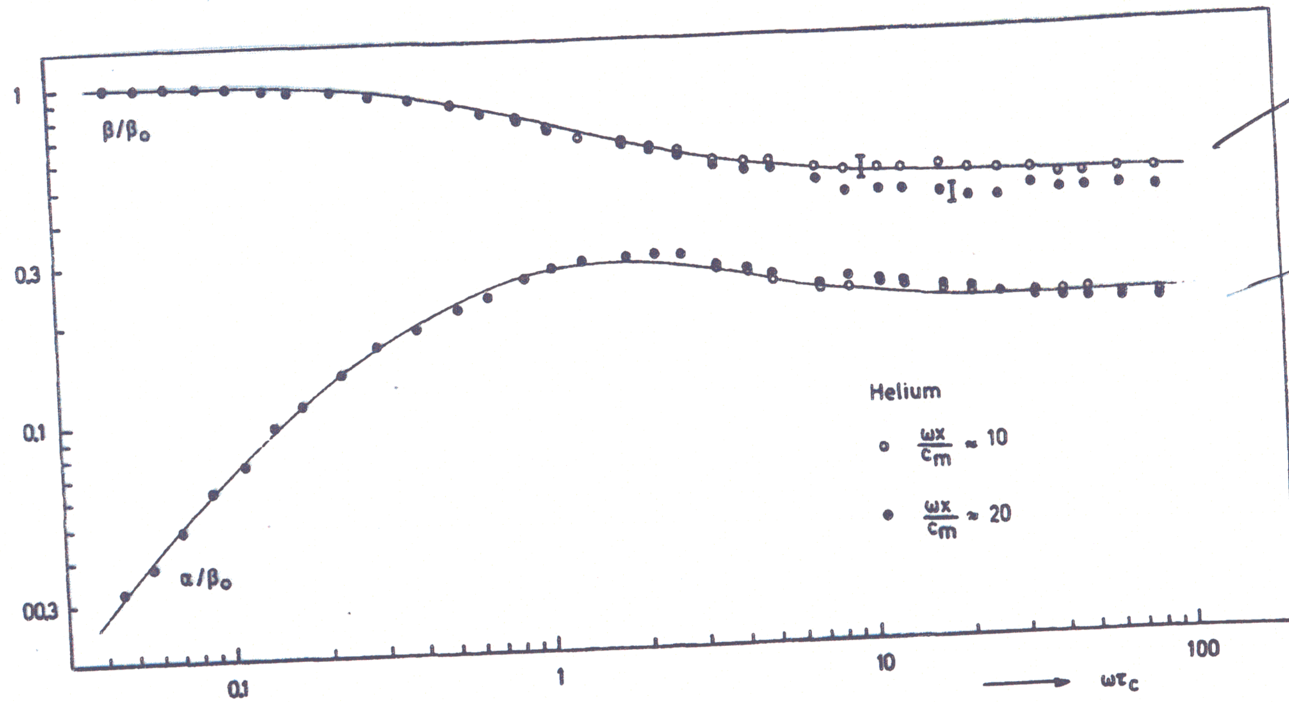
(b)

FIG. 3. Experimental recordings of the phase (a) and the logarithm of the amplitude (b) versus normalized transmitter–receiver distance with arbitrary origin of the ordinate axis in helium. | End of one mean free path from the transmitter. ------ Average of the recordings for low pressures ($\omega\tau_c$-independent).

Schotter, 1974

also Meyer, Sessler, 1957
Greenspan, 1956
Uchida, 2002

$K_0 = \frac{\omega}{c_s}$



$\frac{K_R}{K_0}$

$\frac{K_I}{K_0}$

$\frac{\omega}{\nu}$

Theory (solid line) by Buckner, Ferziger, see also Sirovich et. al.

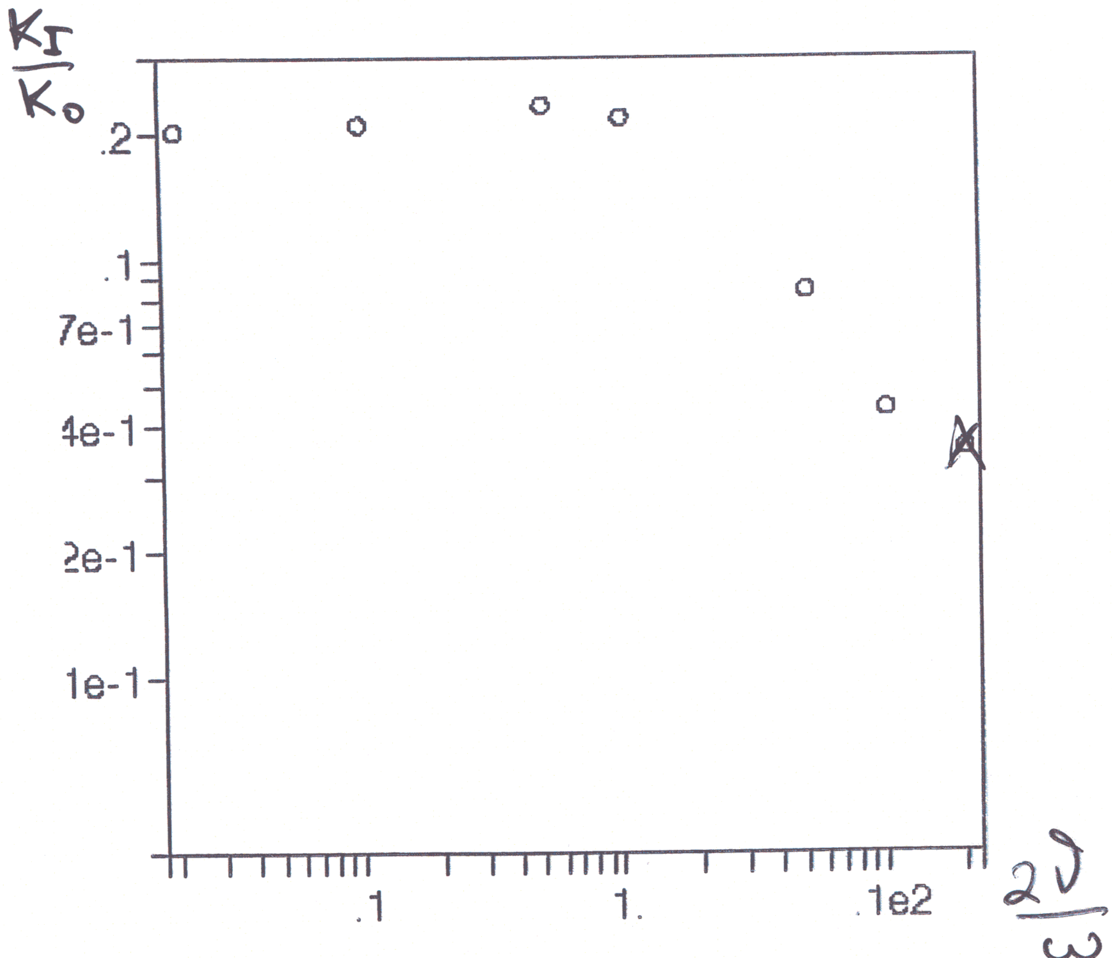
FIG. 4. Propagation characteristics for helium taken at $\omega x/c_m = 10$ (O) and 20 (●).——Theory of Buckner and Ferziger.[5] The vertical bars indicate the pressure where the propagation parameters were taken one mean free x path from the receiver.

... of overlap. With

R. Schotter, Phys. Fluids, 1974

Dissipation at low ν is a result of phase mixing of "quasimodes" (Van-Campen-Case modes) which are Landau damped

$K_0 = \omega / v_T$



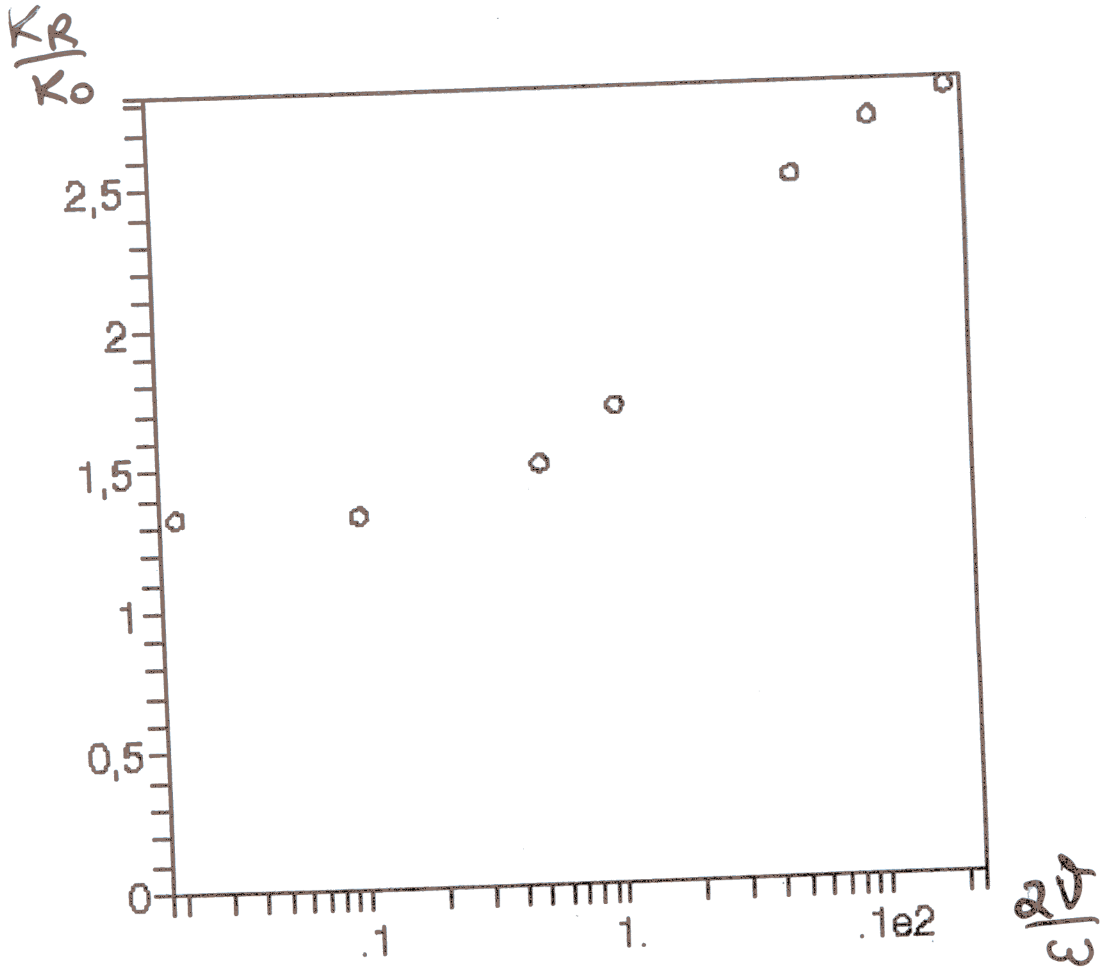
Damping in neutral gas

$$\frac{\partial}{\partial t} n + \frac{\partial}{\partial x} nV = 0$$

$$m \frac{\partial}{\partial t} V = -\frac{\partial}{\partial x} p - \frac{\partial}{\partial x} \pi$$

$$\frac{\partial}{\partial t} p = -2p \frac{\partial}{\partial x} V - \frac{\partial}{\partial x} q$$

$$\left.\begin{aligned} \pi &= \hat{\eta}\, V + \hat{\alpha}_1 T \\ q &= +\hat{\chi}\, T + \hat{\alpha}_2 V \end{aligned}\right\}$$ Chen, Callen type closure



KR/Ko
2,5
2
1,5
1
0,5
0
.1
1.
.1e2
2ϑ/ω

"Higher Order Approximations" from the Chapman-Enskog like closures

$$\pi = \hat{A}_\pi \tilde{V} + \hat{B}_\pi \tilde{T}$$
$$q = \hat{A}_q \tilde{V} + \hat{B}_q \tilde{T}$$

$\hat{B}_\pi, \hat{A}_q = 0$ in strongly collisional limit (Navier-Stokes)

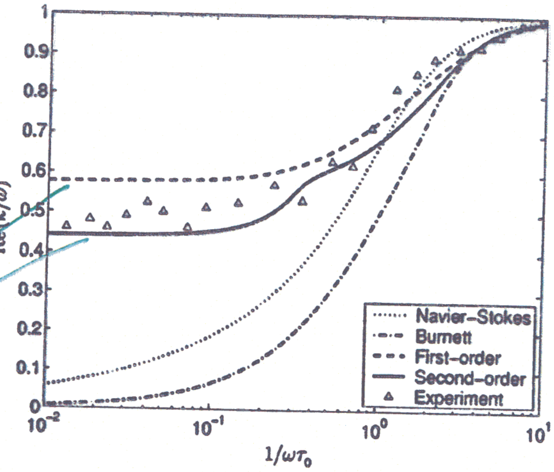
expansion in $\frac{\omega}{\nu}, \frac{kv}{\nu}$

$$\pi = -\mu_0 \left(\nabla u + \widetilde{\nabla u} + \partial_t \ln T \right) + O\left(\frac{\omega^2}{\nu^2}, \frac{kv^2}{\nu^2} \right)$$

$$q = -\chi_0 T \nabla \ln T - \frac{5}{2} \mu_0 \nu^{-1} \partial_t u + \ldots$$

First Order Terms

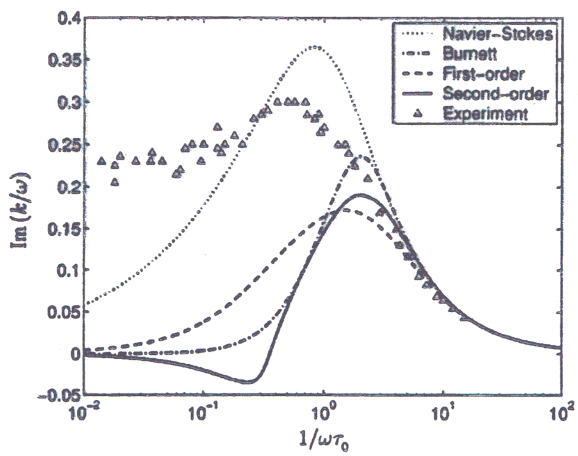
E.A. Spiegel et al. 2000, 2003
- ultrasound damping
- shock wave structure
- stellar dynamics



Higher Order Approximations

FIG. 1. The real part of the inverse phase speed, k/ω, vs $1/\omega\tau_0$. The phase speed is given in units of the adiabatic sound speed, $(\gamma p_0/\rho_0)^{1/2}$. The first- and second-order dispersion relations of this paper (with $\sigma=2/3$) are compared to Navier–Stokes, Burnett, and the experiment of Meyer and Sessler (Ref. 25).

Fail to describe damping



IG. 2. The same as for Fig. 1, but for the imaginary part of the inverse hase speed.

E.A. Spiegel
J.-L. Thiffeault, Phys. Fluids
2003

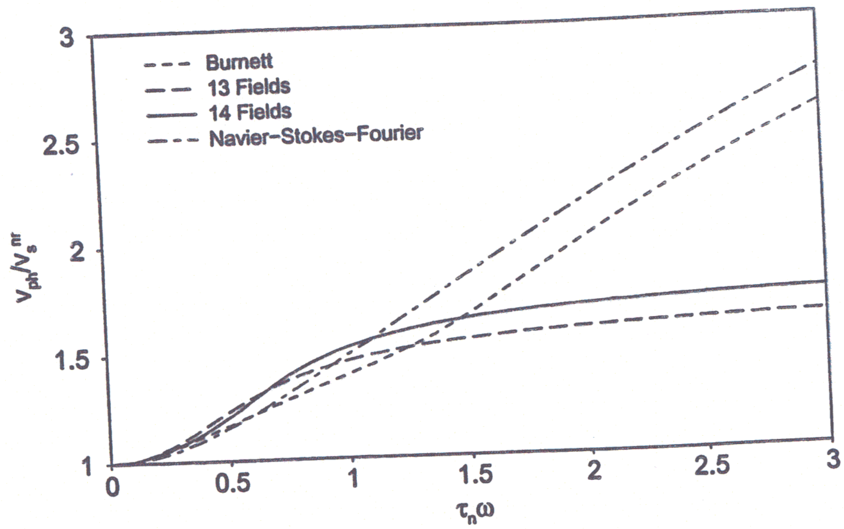
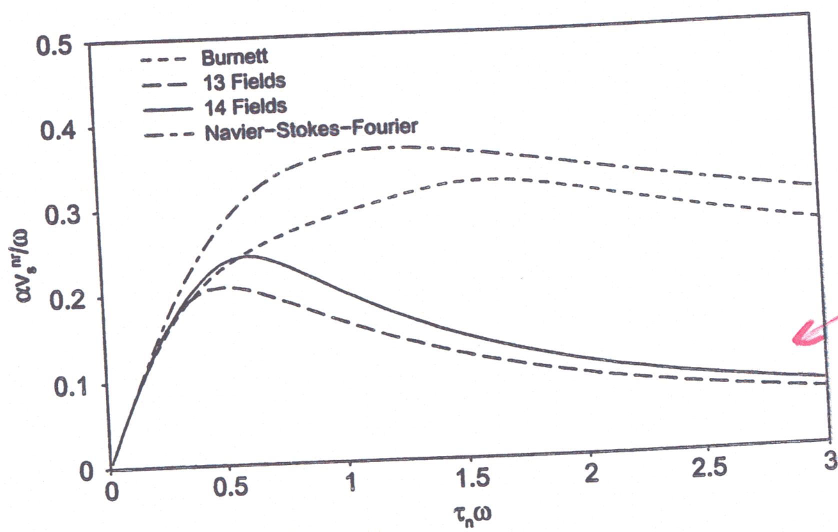


Fig. 1. Phase velocities as functions of the circular frequency in the non-relativistic limiting case



damping

Fig. 3. Attenuation coefficients as functions of the circular frequency in the non-relativistic limiting case

C. Cergignani . 2001

13,14 fields models - Grad type equations

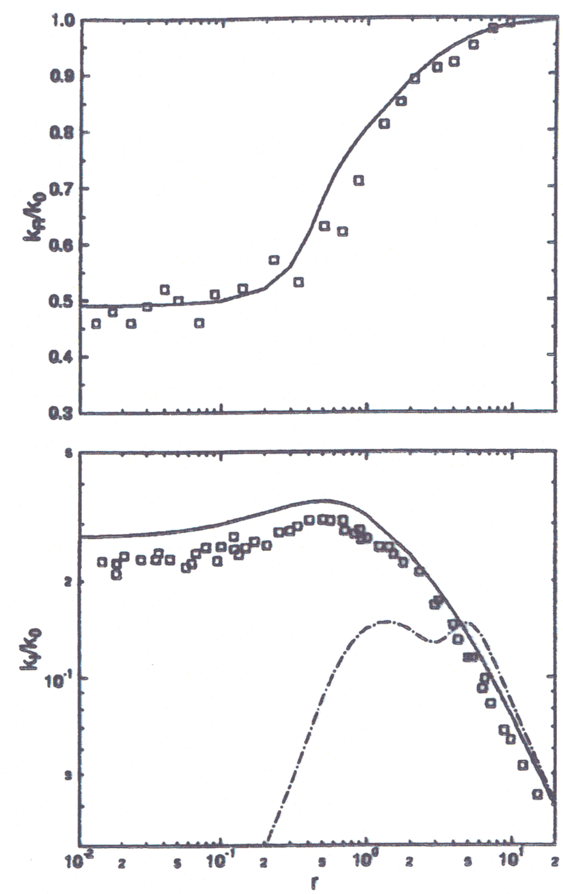


FIG. 2. k_R/k_0 (upper panel) and k_I/k_0 (lower panel) vs r. The theoretical results from Sukhorukov and Stubbe (Ref. 16) are shown by solid curves. The experimental results from Meyer and Sessler (Ref. 17) are shown by white squares. Fluid damping results, following from Eq. (44), are shown by the dashed-dotted curve in the lower panel.

P. Stubbe et. al. 1994

$\tilde{p} = \gamma \tilde{\rho}$

Generalized adiabatic constant (complex calculated kinetically)

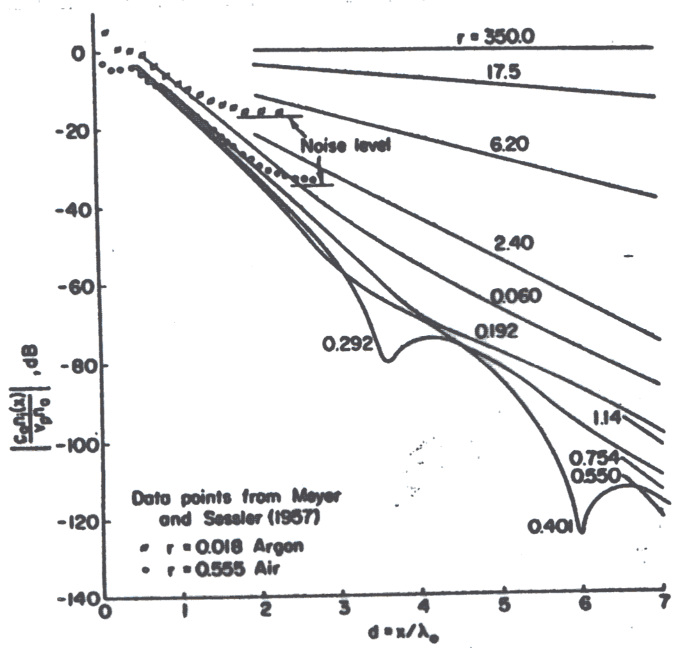


FIG. 11. Approximate total Maxwellian density disturbance.

R. J. Manson

In Conclusion

- Closure in terms of the source profile functions is similar (identical?) to Chapman-Enskog-like closures

- Chapman-Enskog like closure gives adequate description of sound damping in rarefied neutral gas

- Linear Landau damping
  - Evolution of the continuos spectrum (phase mixing)
  - Exists both in plasmas and neutral gas

- Nonlinear Landau damping (particles orbit modification/ particle trapping) requires electric field, e.g. O'Neil problem