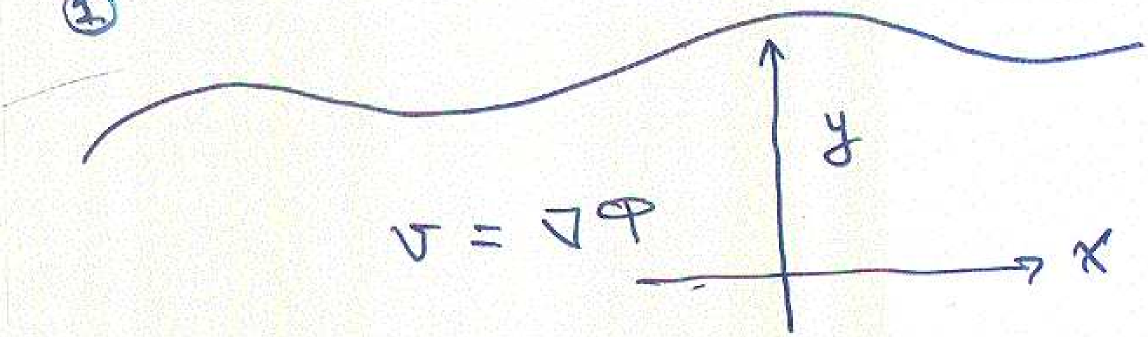
Surface waves in conformal [illegible]

by V. E. Zakharov and A. I. Dyachenko.

1



$$v = \nabla\varphi$$

$$-\infty < y < \eta(x,t)$$

$$\frac{\partial\varphi}{\partial y} \to 0, \quad y \to -\infty$$

$$\varphi\big|_{z=\eta} = \Psi$$

$$\Delta\varphi = 0$$

$$\boxed{\frac{\partial\eta}{\partial t} = \frac{\delta H}{\delta\psi}} \qquad \boxed{\frac{\partial\psi}{\partial t} = -\frac{\delta H}{\delta\eta}}$$

$$H = T + U$$

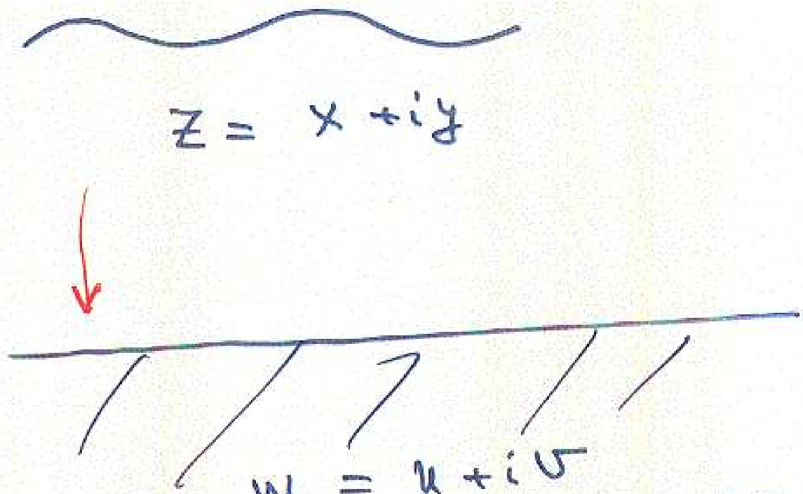
$$T = \frac{1}{2}\int_{-\infty}^{\infty} dx \int_{-\infty}^{\eta} |\nabla\varphi|^2 \, dy$$

$$\int_{-\infty}^{\infty} \eta \, dx = 0$$

$$U = \frac{g}{2}\int_{-\infty}^{\infty} \eta^2 \, dx$$

$$L = \int_{-\infty}^{\infty} \psi \eta_t \, dx - H$$

$$S = \int L \, dt \qquad \boxed{\delta S = 0}$$



$$\eta(x,t) \rightarrow \begin{cases} y(u,t) \\ x(u,t) \end{cases}$$ – parametric representation

$$y = \hat{H}(x-u)$$

$$\hat{H}f = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{f(u')}{u'-u}\,du'$$

Hilbert transform

$$x = u - i\hat{H}y$$

$$L = \int_{-\infty}^{\infty} \psi(y_t x_u - x_t y_u)\,du - H$$

$$y_t x_u - x_t y_u = \frac{\delta H}{\delta \psi}$$

$$\psi_t x_u - x_t \psi_u - \hat{H}(y_u \psi_t - y_t \psi_u) = -\frac{\delta H_{pot}}{\delta y}$$

$$H = T + U_\infty = T + H_{pot}$$

$$T = -\frac{1}{2}\int_{-\infty} \psi \hat{H} \psi_u\,du$$

$$H_{pot} = U = \frac{g}{2}\int y^2 x_u\,du$$

(*)

$$\boxed{\begin{aligned} &y_t x_u - x_t y_u = -\hat{H}\psi_u \\ &\psi_t x_u - x_t \psi_t + \hat{H}(\psi_t y_u - y_t \psi_u) = -g(y x_u + \hat{H} y y_u) \end{aligned}}$$

Kuzentzov, Spector, Zakharov 1992

$$\hat{K} = -\hat{H}\frac{\partial}{\partial u}$$

$$\psi = \hat{K}^{-1}(y_t x_u - x_t y_u)$$

$$T - H_{pot}$$

$$L = T - U \qquad F = -x_t y_u + y_t x_u$$

$$T = \frac{1}{2}\int_{-\infty}^{\infty} F \hat{K}^{-1} F\, dx$$

$$\boxed{\text{A. Balk 1991}}$$

$$H_{pot} = U = \frac{g}{2}\int_{-\infty}^{\infty} y^2 x_u\, dx$$

After substitution $\psi \to$ (*), one get the Euler-Lagrange equations

$$\int \delta L\, dx = 0$$

For stationary stokes waves

$$\frac{\partial}{\partial t} = c\frac{\partial}{\partial u} \qquad c\,y = -H\psi$$

$$\boxed{\psi = c H y}$$

$$\boxed{c^2 H y = -g\left(y x_u + \hat{H}(y y_u)\right)}$$

Equivalent to the Nekrasov-Levi-Civita equation for stationary waves.

Implicit equation can written in the Hamiltonian form

$$\boxed{\begin{aligned} \Omega_{11}\dot{y} + \Omega_{12}\dot{\psi} &= \frac{\delta H}{\delta y} \\ -\Omega_{12}^{+}\dot{y} \qquad &= \frac{\delta H}{\delta \psi} \end{aligned}}$$

$$\boxed{\begin{aligned} \Omega_{11} &= -\left(\psi_u \hat{H} + \hat{H}\psi_u\right) \\ \Omega_{12} &= -x_u + H y_u \end{aligned}}$$

The system can be resolved with respect to time-derivatives:

$$\boxed{\begin{aligned} \dot{\psi} &= R_{11}\frac{\delta H}{\delta \psi} - R_{12}\frac{\delta H}{\delta y} \\ \dot{y} &= R_{21}\frac{\delta H}{\delta \psi} \end{aligned}}$$

$$R_{11} = -\left(\psi_u \hat{H}\frac{1}{M} + \frac{1}{M}\hat{H}\psi_u\right)$$

$$R_{12} = \frac{1}{M}\left(x_u' + \hat{H}y_u\right)$$

$$R_{21} = R_{12}^{+} = \left(x_u - y_u\hat{H}\right)\frac{1}{M}$$

$$M = \left|\frac{\partial z}{\partial u}\right|^2 - \text{Jacobian.}$$

The forms of Hamiltonian (symplectic and implectic) equations are universal and good for any Hamiltonian (not only for Hydrodynamics!

③ Complexification

$\Phi = \psi + i\hat{H}\Psi \qquad z = x + iy$

$H \to H[\Phi, \bar{\Phi}, z, \bar{z}]$

Φ, z are analytic in the lower half-plane. Projection operators

$$P^{\pm} = \frac{1}{2}\left(1 \mp i\hat{H}\right)$$

For any Hamiltonian:

$$\boxed{\dot{z} = iUz_u}$$

$$\boxed{\dot{\Phi} = iU\Phi_u - B - P}$$

— standard complex form

$$U = 4P^{-}\left\{\frac{1}{y}\left(P^{-}\frac{\delta H}{\delta \bar{\Phi}} + P^{+}\frac{\delta H}{\delta \Phi}\right)\right\}$$

$$B = -4iP^{-}\left\{\frac{1}{y}\left(P^{-}\left(\bar{\Phi}_u \frac{\delta H}{\delta \bar{\Phi}}\right) - P^{+}\left(\Phi_u \frac{\delta H}{\delta \Phi}\right)\right)\right\}$$

$$P = -4iP^{-}\left\{\frac{1}{y}\left[P^{-}\left(\bar{z}_u \frac{\delta H}{\delta \bar{z}}\right) - P^{+}\left(z_u \frac{\delta H}{\delta z}\right)\right]\right\}$$

These equations good for any Hamiltonian

Dyachenko form!

$$R = \frac{1}{z'}, \qquad V = iR\Phi_u = i\frac{\partial \Phi}{\partial z}$$

$$\boxed{\begin{aligned} \frac{\partial R}{\partial t} &= i(UR_u - RU_u) \\ \frac{\partial V}{\partial t} &= i(UV'_u - R(B+P)_u) \end{aligned}}$$

Returning to Hydrodynamics

$$\begin{cases} U = i\hat{P}^{-}\frac{1}{|z_u|^2}(\Phi_u - \bar{\Phi}_u) = -\hat{P}^{-}\frac{H\Psi_u}{|z_u|^2} \\ B = \hat{P}^{-}\frac{|\Phi_u|^2}{|z_u|^2} \end{cases}$$

In Dyachenko variables

$$\boxed{\begin{aligned} U &= \hat{P}^{-}(R\bar{V} + \bar{R}V) \\ B &= \hat{P}^{-}(V\bar{V}) \end{aligned}}$$

To determine P we have to determine H_{pot}

① . gravity waves

$$H_{pot} = \frac{g}{2}\int \eta^2 dx = -\frac{g}{16}\int_{-\infty}^{\infty}(z-\bar{z})^2(z_u + \bar{z}_u)\,du$$

$$P = ig(z - w)$$

② capilliary waves

$$H_{pot} = \sigma\int_{-\infty}^{\infty}\left(\sqrt{y'^2 + x'^2} - x'\right)dx =$$

$$= \sigma\int_{-\infty}^{\infty}\left(\sqrt{|z_u|^2} - \frac{z_u + \bar{z}_u}{2}\right)du$$

$$P = 2\sigma\hat{P}^{-}(Q_u\bar{Q} - \bar{Q}_u Q)$$

③ $H_{pot} = -\frac{1}{2}\int_{-\infty}^{\infty} y\,(x'-1)\,du$

This is potential energy of the ideal dielectric fluid with ideally conducting free surface placed into electric fixed (inside a capacitor).

Now

$$P = \alpha P^{-}(R\bar{R}-1)$$

In absence of ~~gravity~~ capilliarity Dyachenko equation read

$$\begin{cases} R_t = i(U R_u - R U_u) \\ V_t = i(U V_u - R B' - R P') + g(R-1) \end{cases}$$

$$U = P^{-}(R\bar{V}+\bar{R}V)$$

$$B = P^{-}(V\bar{V})$$

If $g=0$ this equation can be simplified to one Zubarev! evolutional equation

$$V = qR \qquad q = \sqrt{\alpha}$$

Finally for $\alpha=0$ (gravity waves] Dyachenko equation read

$$R_t = i(U R_u - R U_u)$$

$$V_t = i(U V_u - R B_u^*) + g(R-1)$$

Summarizing we conclude that there are three different forms of free-surface equations in conformal variables

1. Implicit Hamiltonian equation = = Euler-Lagrange equations
2. Explicit Poissonian equation (back to Ovsyannikov 1973)
3. Dyachenko form of equations

Constants of motion

$$\begin{cases} z_t = i\, U z_u \\ \varphi_t = i\, U \varphi_u - B - P \end{cases}$$

Let

$$z = a \log(w - \lambda) + \ldots \qquad z_u = z_w = \frac{a}{w-\lambda} + \ldots$$

$$\varphi = b \log(w - \lambda) \qquad \operatorname{Im} \lambda > 0$$

$$z_t = a_t \log(w - \lambda) + \frac{a \lambda_t}{w - \lambda} + \ldots$$

If U is regular at $w = \lambda$

$$\frac{da}{dt} = 0 \qquad \frac{\partial \lambda}{\partial t} = i\, U \Big|_{w=\lambda}$$

B and P also have to be regular

note that

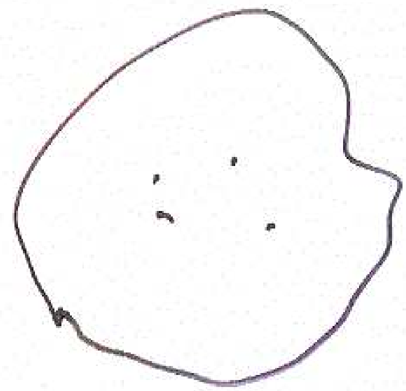
$$U = i\,\bar{P}\left(\frac{\varphi_u - \bar{\varphi}_u}{z_u \bar{z}_u}\right) \qquad B = \bar{P}\left(\frac{|\varphi_u|^2}{|z_u|^2}\right)$$

since φ and z have singularities in the same points, U and B are regular

$$V\big|_{w=\lambda} = \frac{b}{a}$$

$$\boxed{\frac{d}{dt}\frac{b}{a} = g}$$

more generally



$$\frac{d}{dt}\oint_\Gamma z'\,dw = 0$$

$$\frac{d}{dt}\oint_\Gamma \varphi'\,dw = g\oint z'\,dw = \text{const}$$

if $a_1 \dots a_n$ are poles

$$\{a_i, a_j\} = 0$$

If $g = 0$

b_i are constants of motion

$\{b_i, b_j\}$ and $\{a_i, b_j\}$ still unknown

For Dyachenko equations zero of R is structurally stable

$$R = a(w-\lambda)$$

$$\frac{\partial \lambda}{\partial t} = i\,U\big|_{w=\lambda} \qquad \frac{da}{dt} = 0$$

$\vdots$

If one has concentration of zeros, than $\frac{1}{R} \to \infty \to$ LGE

$W \simeq W_0 = \text{const}$ — equivalent.

In the implicit form

$$\mathrm{Im}\,(\dot{z}\,\bar{z}') = V_0$$

$$U = \hat{P}^{-}\left\{\frac{V_0}{|z'|^2}\right\}$$

formation of fingers

Approximation of almost flat surface

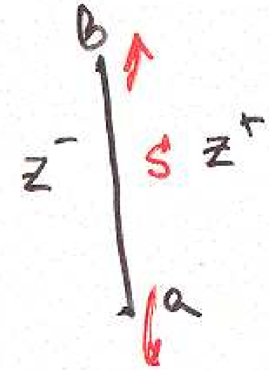
$R \simeq 1 \qquad u = V$

$$V_t = i\left(\tfrac{1}{2}V^2 - B\right)' \qquad B = \hat{P}^{-}(V\bar{V})$$

$$v = \alpha + i\beta \qquad |v|^2 = \alpha^2 + \beta^2$$

$$\boxed{\frac{\partial \beta}{\partial t} = -2\beta\beta_u}$$ — real Hopf equation

motion of cuts



Cut in upper half-plane is structuraly stable!

$$z^- - z^+ = \rho(s)$$

$$f(s) = -\frac{i}{2}(z^- + z^+) = \frac{1}{2\pi}\int_a^b \frac{\rho(s')}{s'-s}\,ds'$$

$$A(s) = \frac{1}{2\pi}\int_a^b \frac{\rho(s')}{s'+s}\,ds'$$

$$\varphi^- - \varphi^+ = i\,\varphi(s)$$

$$g(s) = \frac{1}{2}\hat{H}\varphi(s)$$

$$B(s) = \frac{1}{2\pi}\int_a \frac{\varphi(s')}{s'+s}\,ds'$$

$$\frac{\partial}{\partial t}\left[\rho(1+A_s)\right] = \frac{\partial}{\partial s}\left(-\varphi + \rho A_t\right)$$

$$\frac{\partial}{\partial t}\left[\varphi(1-f(s)) + \rho C_s\right) = \frac{\partial}{\partial s}\left(-\varphi f_t + \rho C_t\right.$$

$$C(s) = \frac{1}{\pi}\int^b \frac{\varphi(s')}{s'^2 - s^2}\,ds'$$

Narrow cut. In Dyachenko equations

If we start with a simple pole in R

$$\bar{V} = V_c$$
$$\bar{R} = R_c$$

$$W = \xi + i\int_0^t V_c\, dt \qquad R_c = \text{const}$$

$$\dot{V} = i R_c V \frac{\partial V}{\partial \xi}$$ – complex Hopf equation

$$\Delta \lambda \simeq \sqrt{At}$$

Numerical algorithm with the use Tschebyshev polinomials could be used for numerical study of the cut evolution:

$$Z = Z_0 + \delta Z \, e^{i\Omega t - ipx}$$

$$\cancel{\operatorname{Im}} \; \operatorname{Im}\Omega = \pm \operatorname{Im} \frac{1}{Z_0} \sqrt{gp}$$

Classical conformal variables

$$\cancel{\quad} \quad (\Omega - s)^2 = |R_0'|^2 \, \cancel{\sqrt{g}} \, (g + \delta g) p$$

Freak waves as a result of modulational instability

V. E. Zakharov, A. I. Dyachenko

Freak waves are a well-documented hazard for mariners. They are observed in the whole world ocean, but some areas are more preferable for them. The Ahulas current SE of Durban, South Africa is the most notorious place.

Explanation: linear focusing of swell on the curved inhomogenous current. This is the first part of the theory. Second part is nonlinear. Freak waves appear as a nonlinear stage of the modulational istability of the Stokes waves

1a

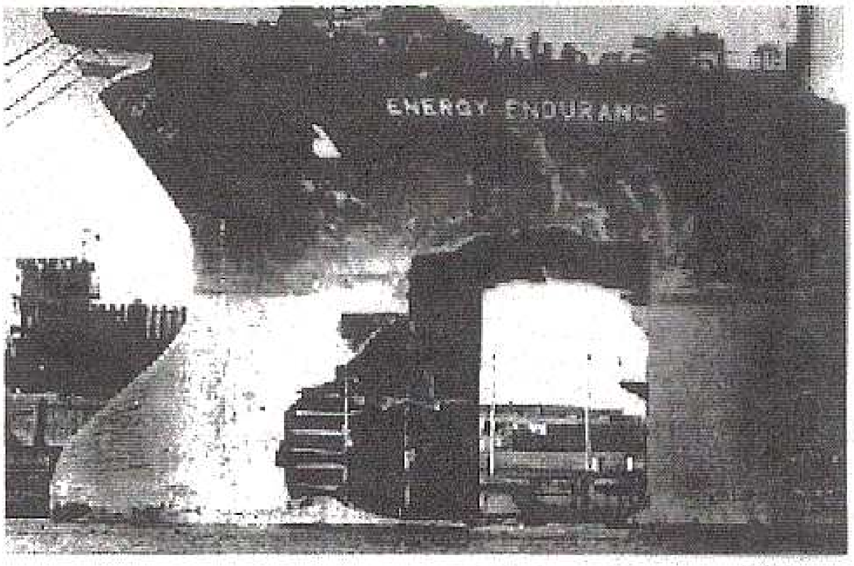


Figure 1.4: Bow damage to Energy Endurance.

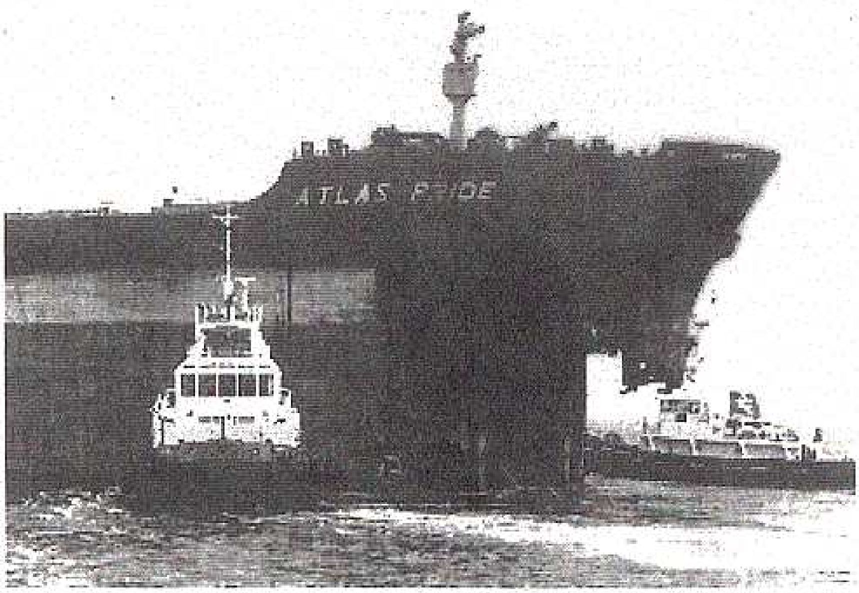


Figure 1.5: Bow damage to Atlas Pride.

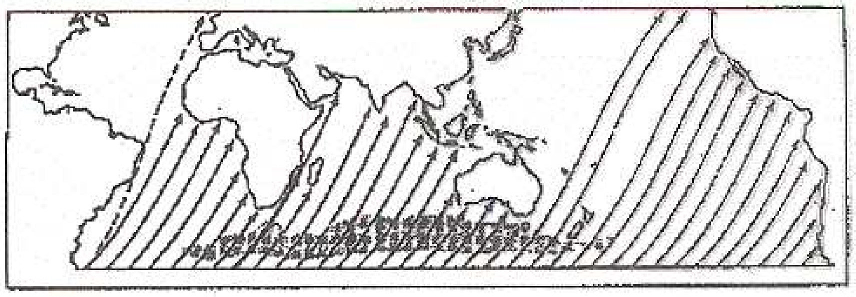


Figure 1.1: Typical swell paths. In the shaded area - part of the 'roaring forties' - average wave heights exceed 4.5 m. (Data from Chelton, Hussey and Parke, 1981).

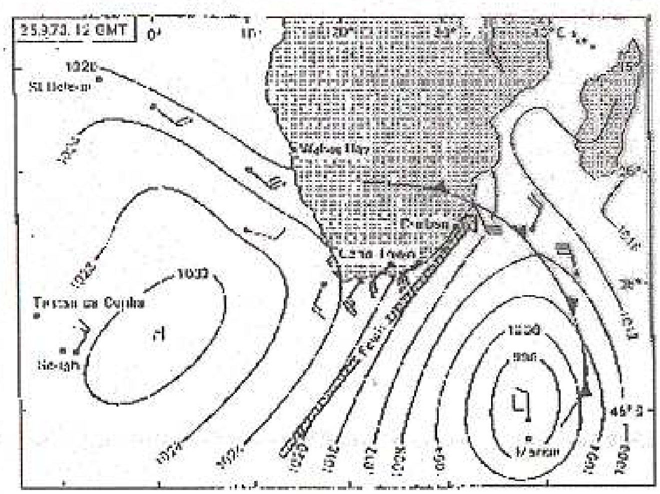
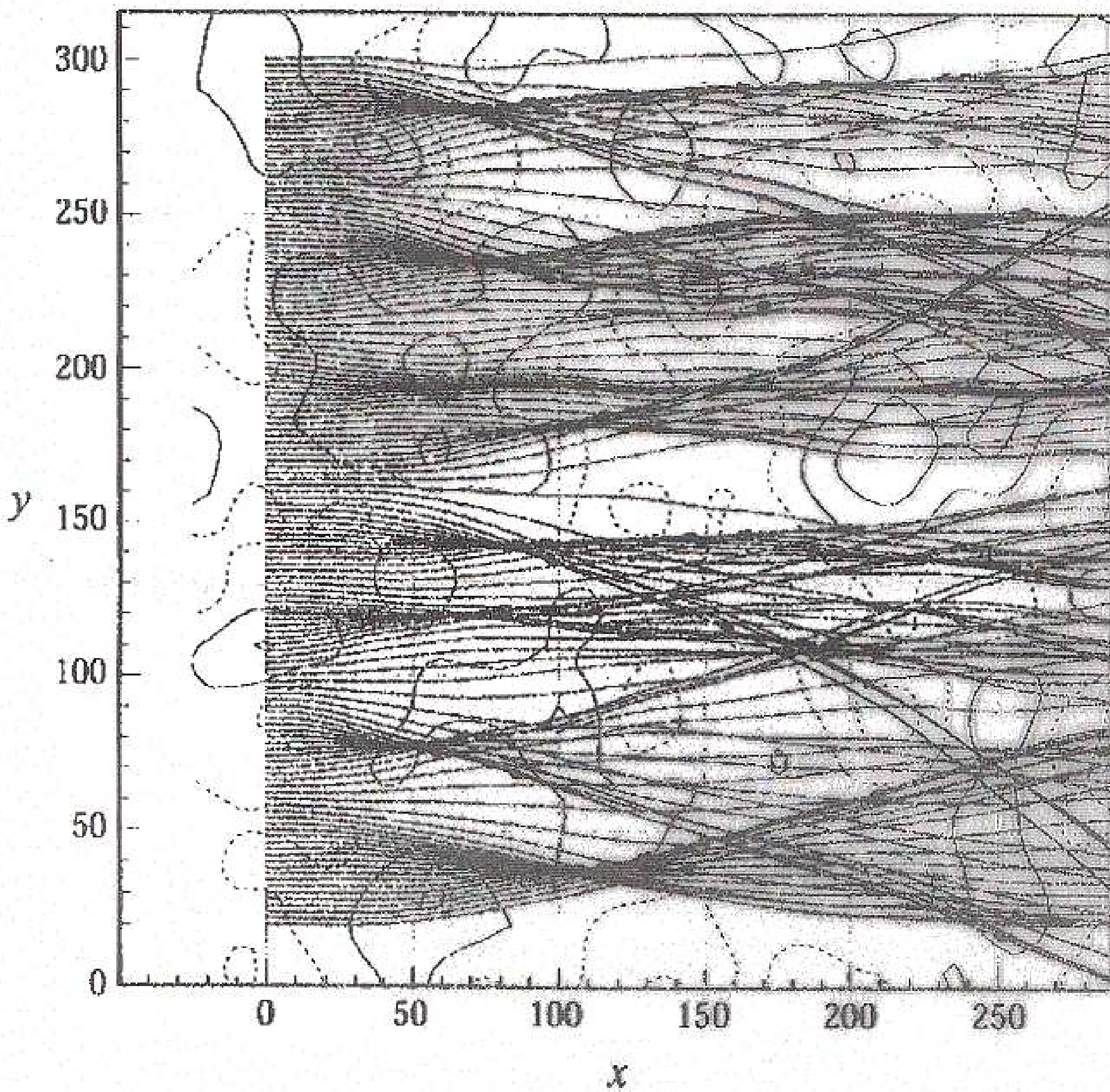


Figure 1.2: Weather situation most likely to generate freak waves (long 'fetch' strengthening the incoming swell; illustration from Mallory, 1974).



The problem with this explanation of freak waves is the following: To have a significant effect from this process it is required that the waves enter the zone of variable currents with a single direction. If they have natural directional distribution one would end up with the same situation as observed at the bottom of the swimming pool when the sun goes behind a cloud such that the light becomes more diffuse; the effect disappears.

Nonlinear focusing. As opposed to the effects above, this one cannot be explained by linear theory. It was shown in the middle of the 1960s that if you generate uniform periodic waves in one end of a long wave tank, the waves will spontaneously split into groups, which get more prominent as they propagate along the tank. According to linear theory these waves should remain uniform and periodic. One developed a wave equation (the so-called nonlinear Schrödinger equation) capable of explaining this strange behavior qualitatively. This equation has later been modified and improved to also give good quantitative agreement with experiments.

They are one more example of " wave collapses", in line with self-focusing of light in non-linear dielectrics and collapse of Langmuir waves in plasma.

Modulational instability of the Stokes waves was discovered by Lighthill in 1965. According to Lighthill for a finite amplitude Stokes wave

$$\omega_k = \sqrt{gk}\left(1 + \frac{1}{2}(ka)^2 + \dots\right)$$

The Lighthill criteria for instability

$$\omega'' \Delta\omega < 0$$

$$\omega'' = -\frac{1}{4k^2}\sqrt{gk} \qquad \Delta\omega = \frac{1}{2}(ka)^2\sqrt{gk}$$

is satisfied. Further theory was developed by Zakharov (1966, 1967) and by Benjamin and Feir (1967) who also observed this instability in the experiment

The Nonlinear Schrödinger equation (NLSE) was derived by Zakharov in 1968. In proper variables NLSE read

$$i\psi_t + \psi_{xx} - \frac{1}{2}\psi_{yy} + |\psi|^2\psi = 0$$

The stationary solution $\psi = A$ is instabl

$$\psi = A + \delta\psi$$

$$\delta\psi \simeq e^{i\Omega t + ipx}$$

$$\Omega^2 = -2A^2p^2 + p^4$$

Instability leas to formation of "solitonic turbulence". Solitons are stable and do no collapse. NLSE model cannot explain the freak wave phenomenon.

Conjecture: Solitons of a certain critical amplitude are unstable and collapse up to the level $ka \sim 1$. k - wave number, a - amplitude.

Has to proven numerically!

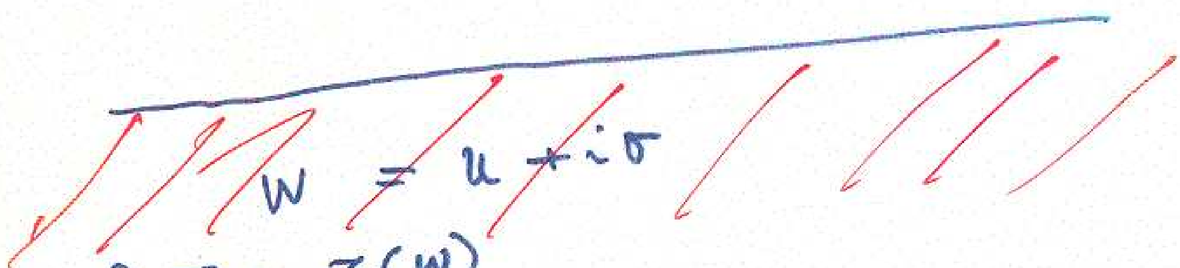
Conformal mapping to the lower half-plane

$$\eta = \eta(x, y)$$

$$Z = x + iy \qquad \Delta\varphi = 0$$

$$v = \nabla\varphi$$

$$\varphi = \mathrm{Re}\,\Theta$$



$$W = u + iv$$

$$\begin{cases} Z = Z(w) \\ \Theta = \Theta(w) \end{cases}$$

$$R = \frac{1}{Z'_u} = R(w)$$

$$V = i\,\frac{\partial\Theta}{\partial Z} = V(w)$$

analytic functions in the lower half-plane. Two versions of the dynamic equations:

① Ovsyannikov - Kuznetsov - Spector - Zakharov version

$$z_t = i U z_u$$

$$\Theta_t = i U \Theta_u + i g (z-u) - B$$

$$U = i \hat{P}^- \left(\frac{\Theta_u - \bar{\Theta}_u}{|z_u|^2} \right)$$

$$B = P^- \left(\frac{\Theta_u \bar{\Theta}_u}{|z_u|^2} \right)$$

$$P^- = \frac{1}{2} \left(1 + i \hat{H} \right)$$

$$\hat{H} f = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{f(u')}{u'-u} du'$$

Hilbert transform

$\boxed{\text{ill - posed equations!}}$

② Dyachenko version!

Well - posed.

$$R_t = i (U R_u - U_u R)$$

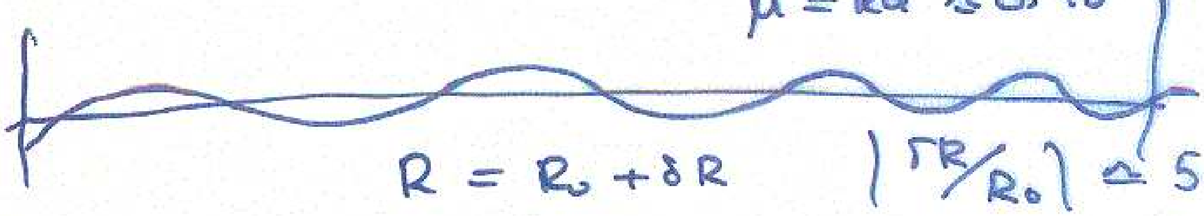
$$V_t = i (U V_u - R B_u) + g (R - 1)$$

$$U = P^- (R \bar{V} + \bar{R} V)$$

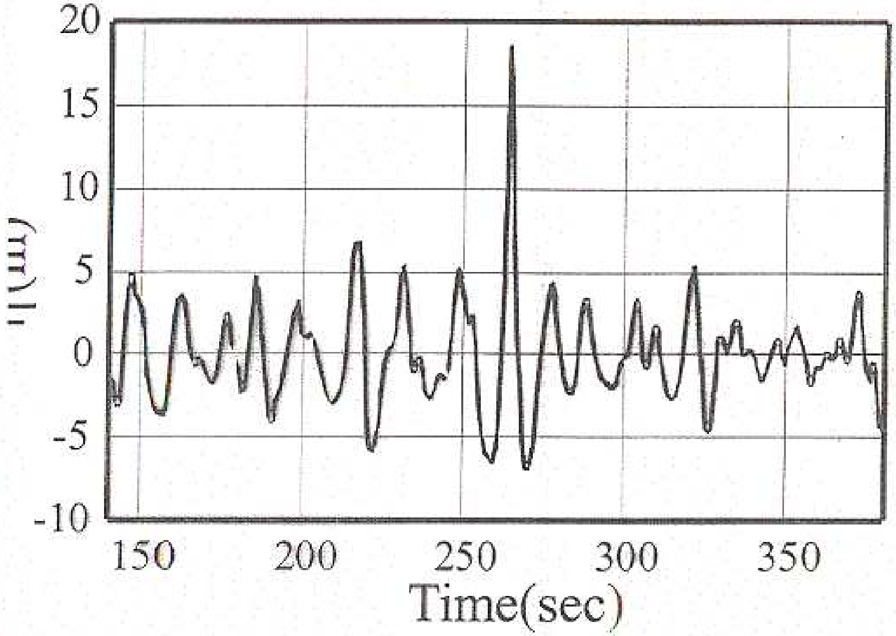
$$B = P^- (V \bar{V})$$

Stokes wave

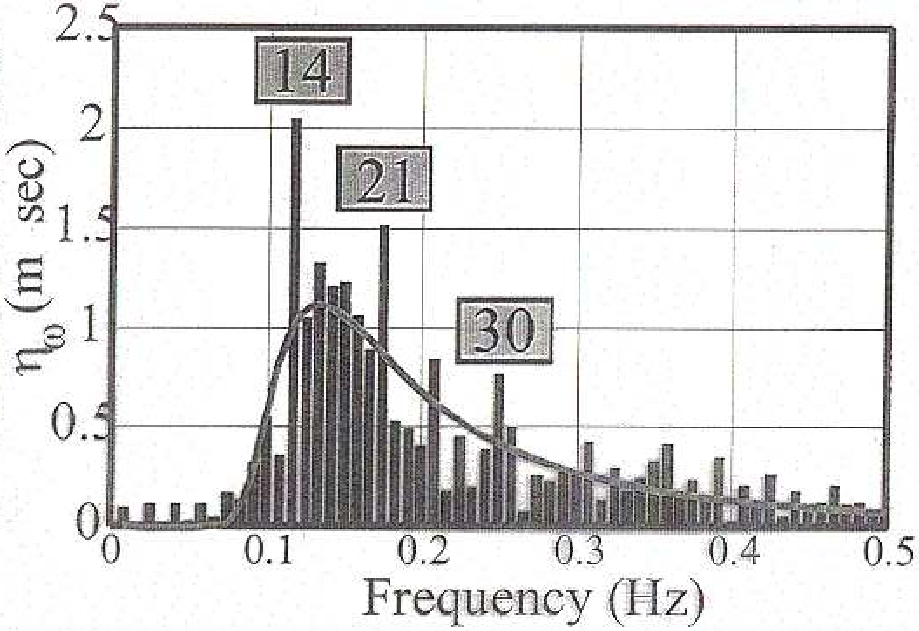
$\mu = ka \approx 0.15$



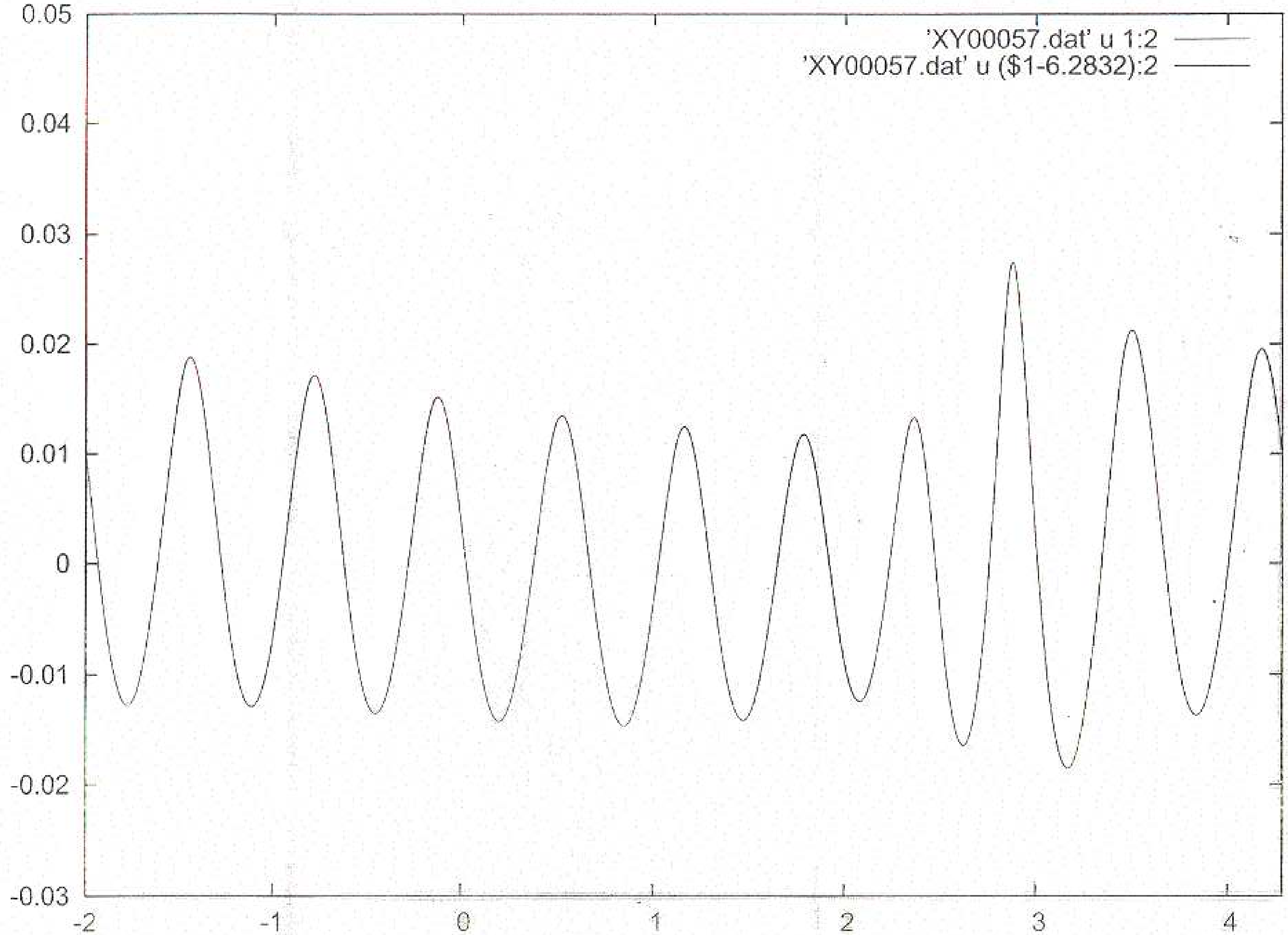
$R = R_0 + \delta R$ $\quad \left| \delta R / R_0 \right| \simeq 5 \cdot 10^{-2}$

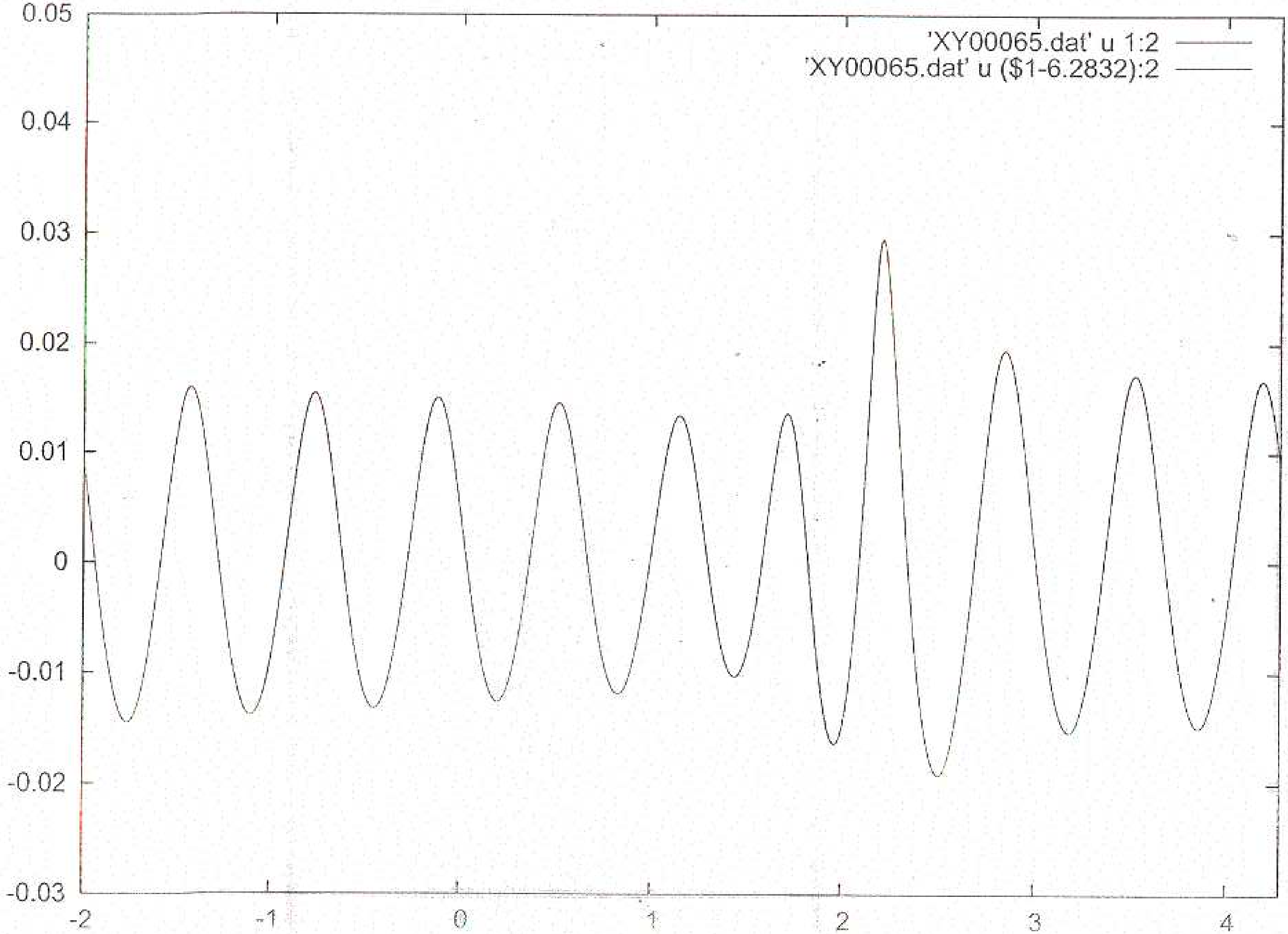


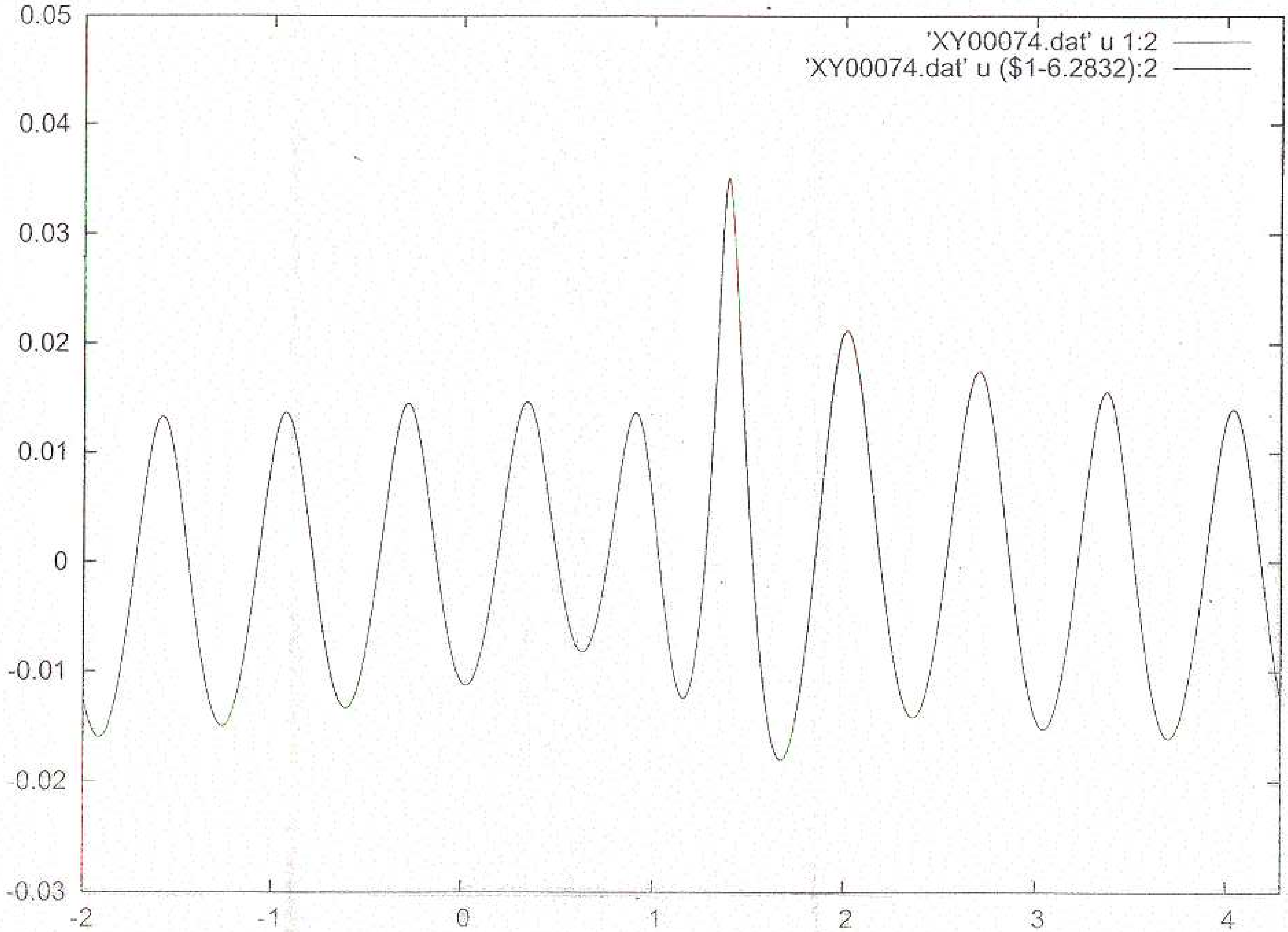
20
15
10
5
0
-5
-10
η(m)
150
200
250
300
350
Time(sec)

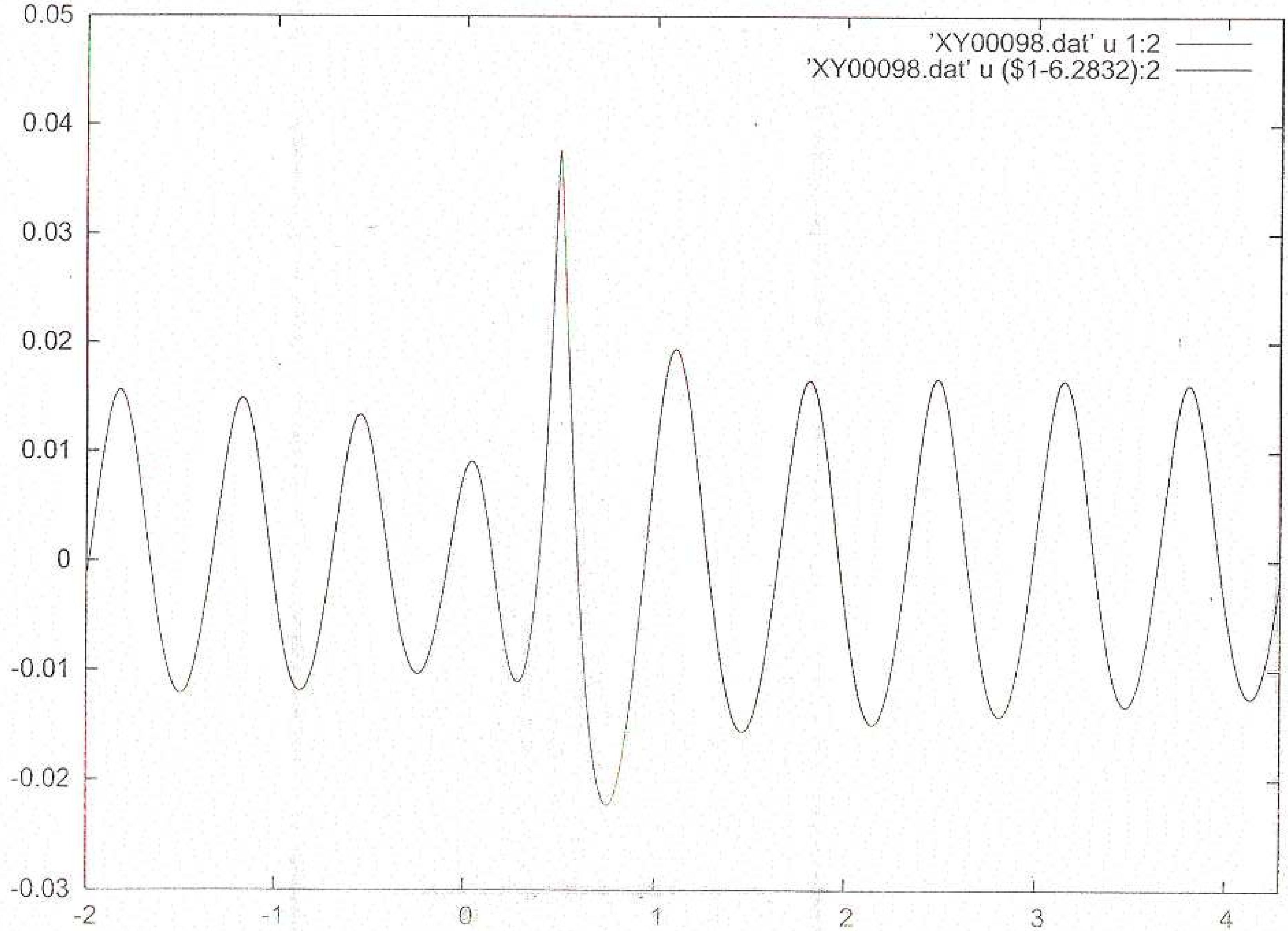


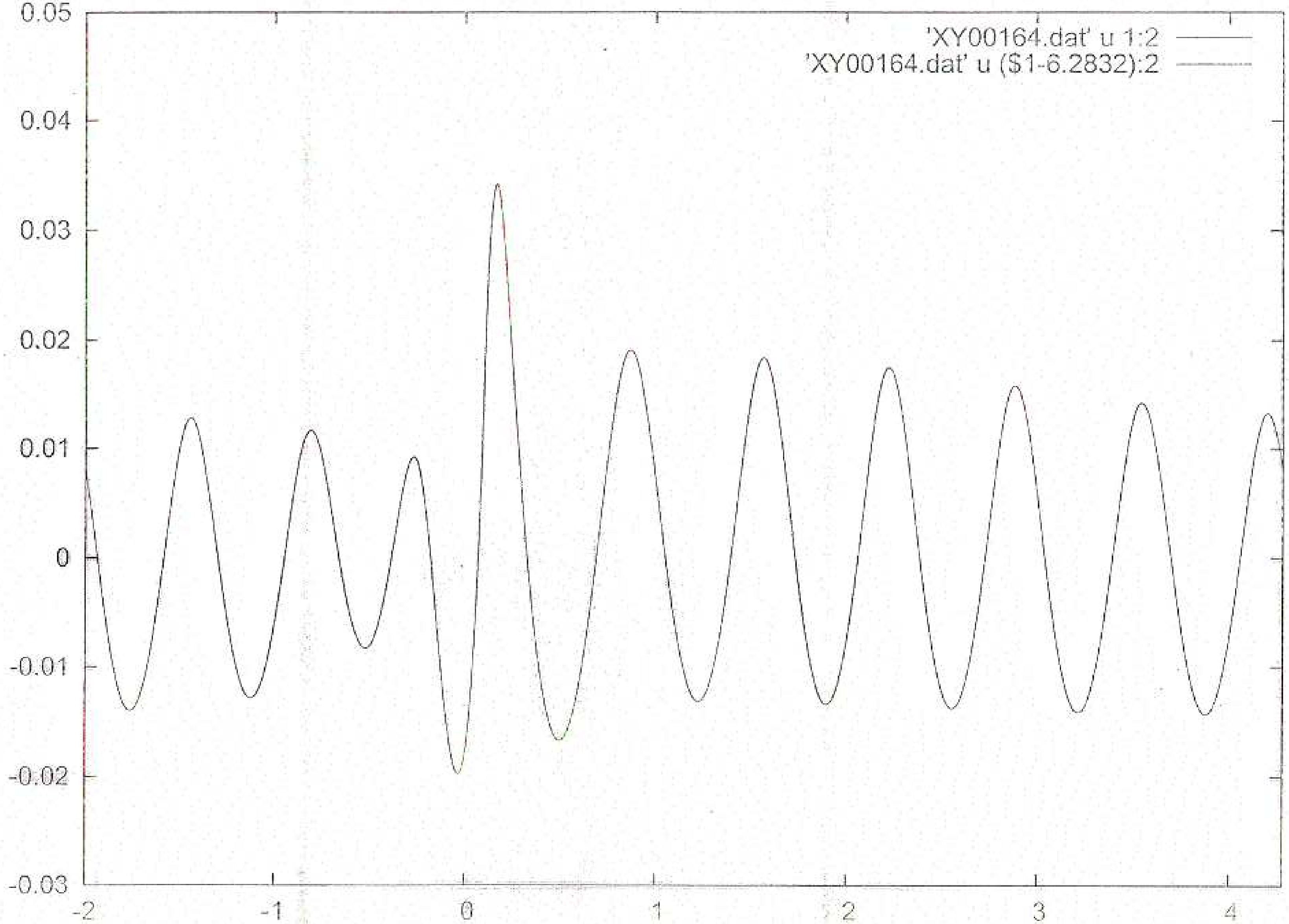
14
21
30
2.5
2
1.5
1
0.5
0
η_ω (m sec)
0
0.1
0.2
0.3
0.4
0.5
Frequency (Hz)

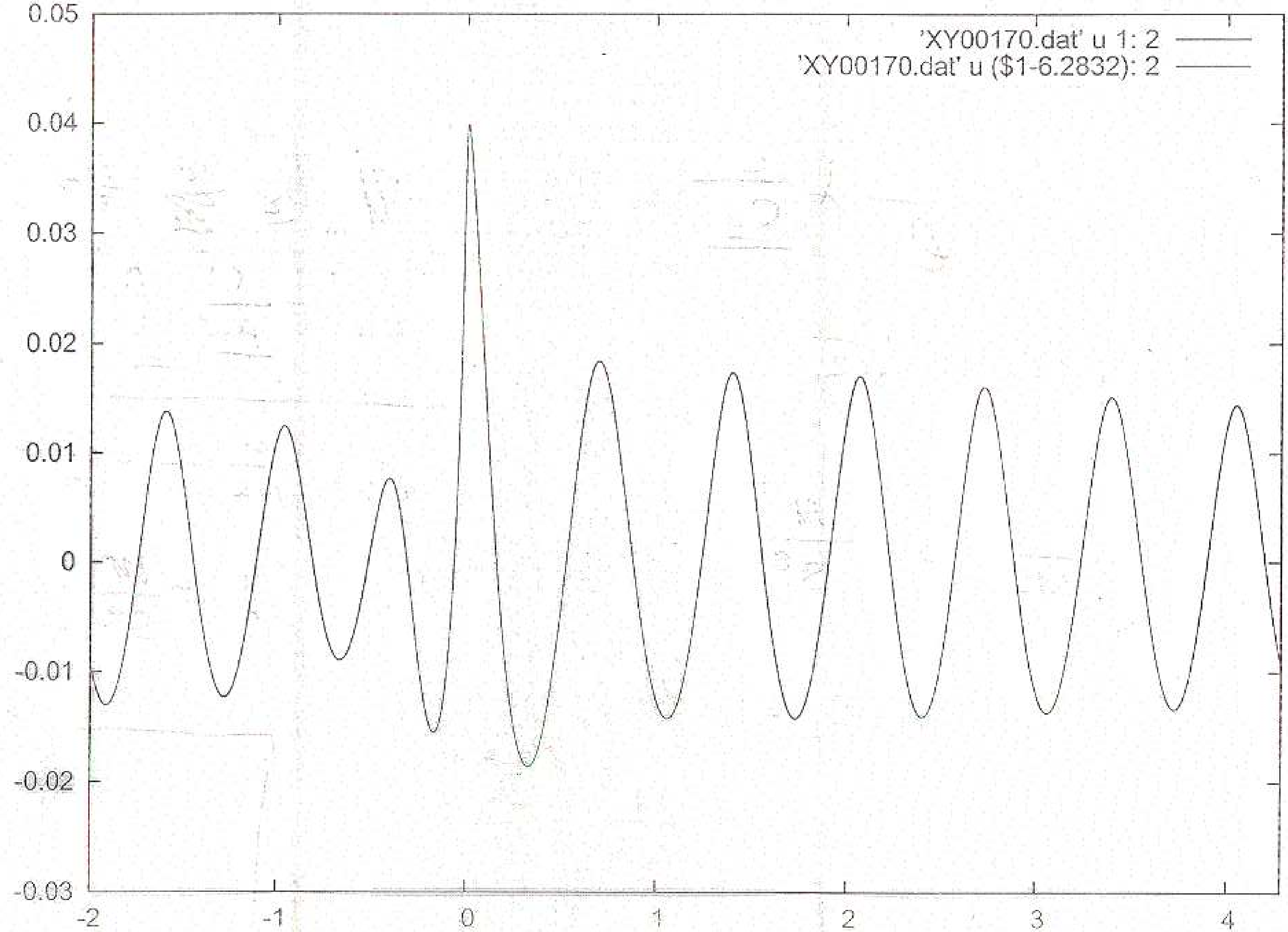

'XY00057.dat' u 1:2
'XY00057.dat' u ($1-6.2832):2
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-2
-1
0
1
2
3
4

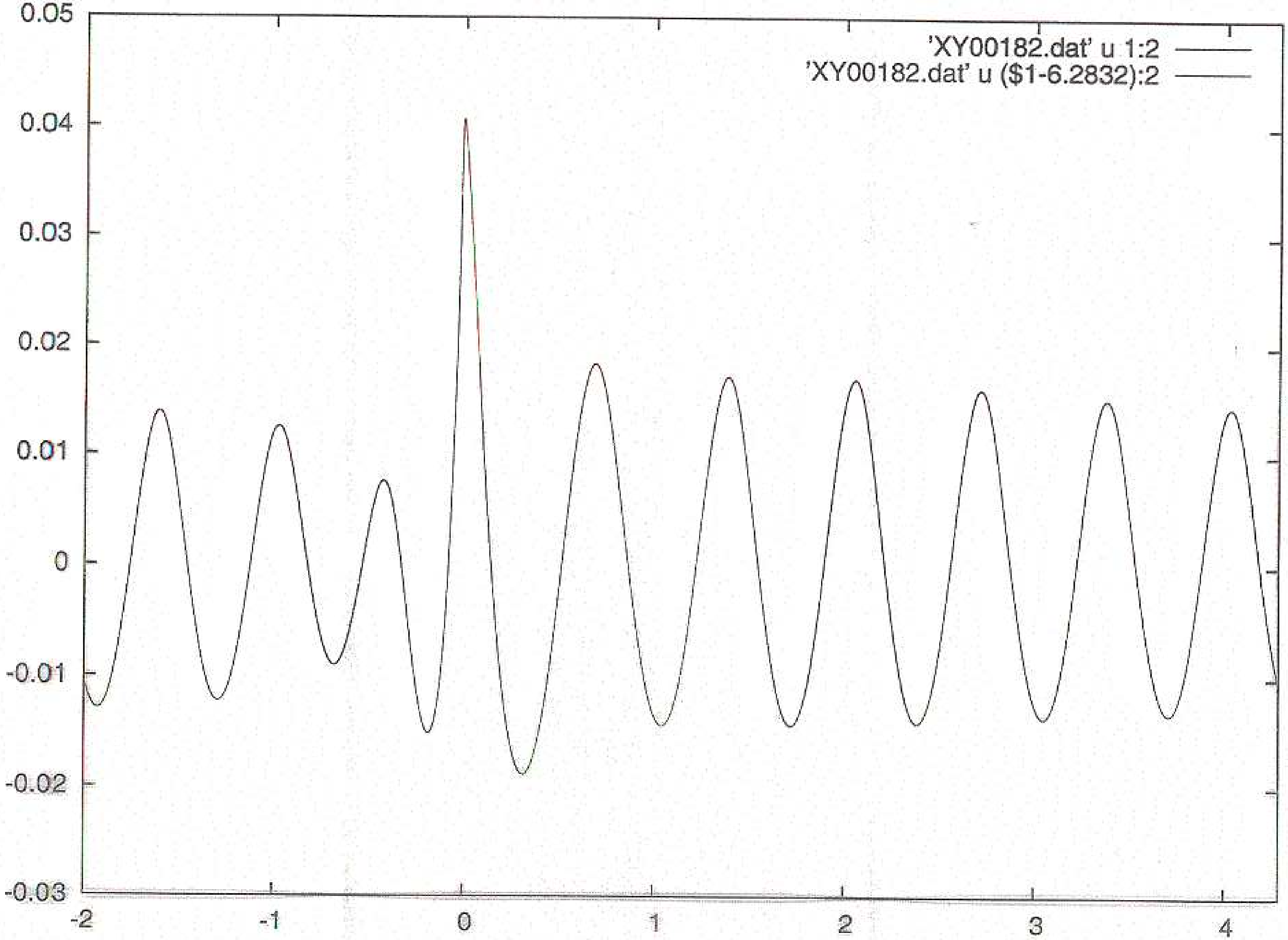

'XY00065.dat' u 1:2
'XY00065.dat' u ($1-6.2832):2
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-2
-1
0
1
2
3
4

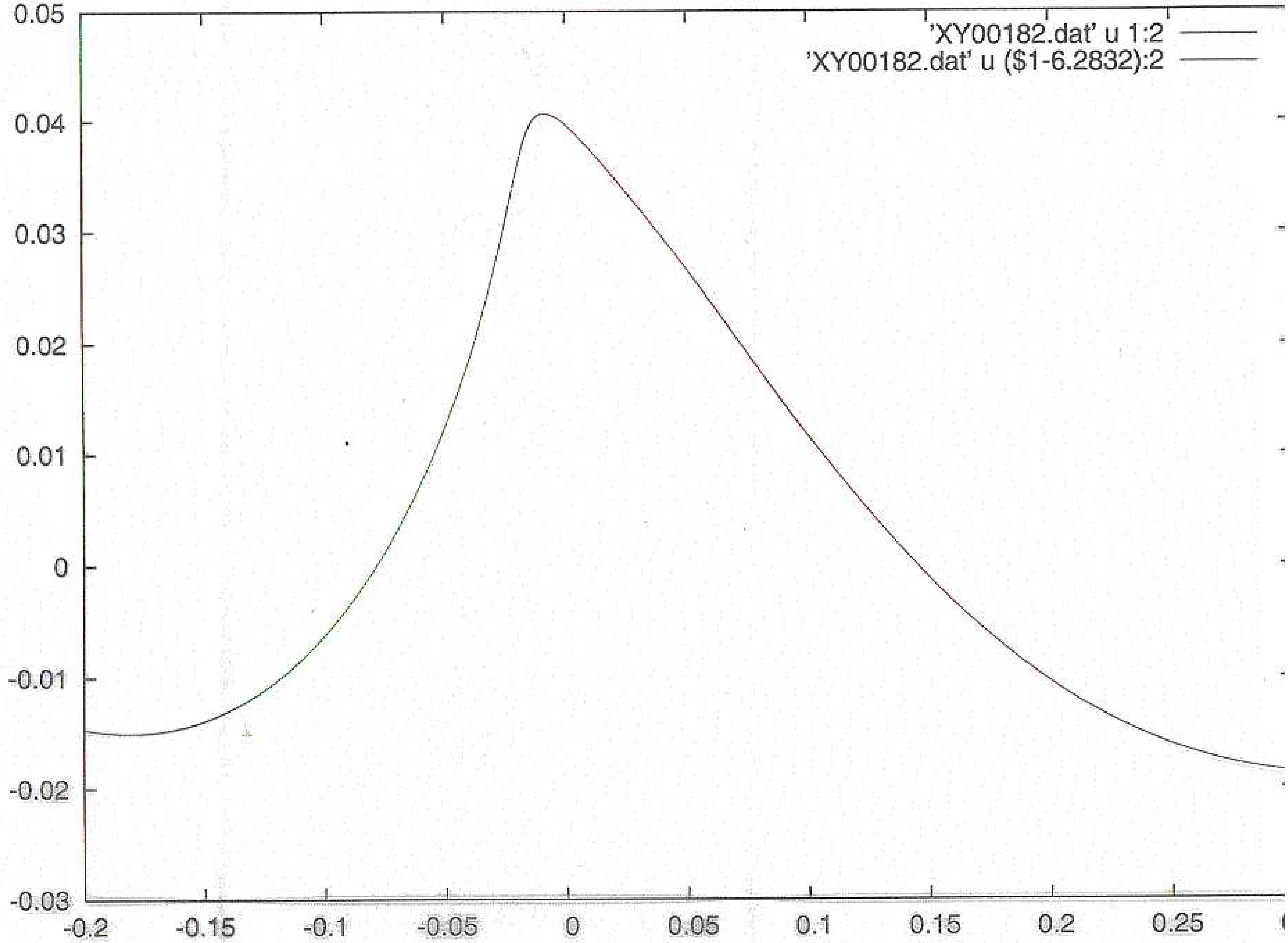

'XY00074.dat' u 1:2
'XY00074.dat' u ($1-6.2832):2
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-2
-1
0
1
2
3
4

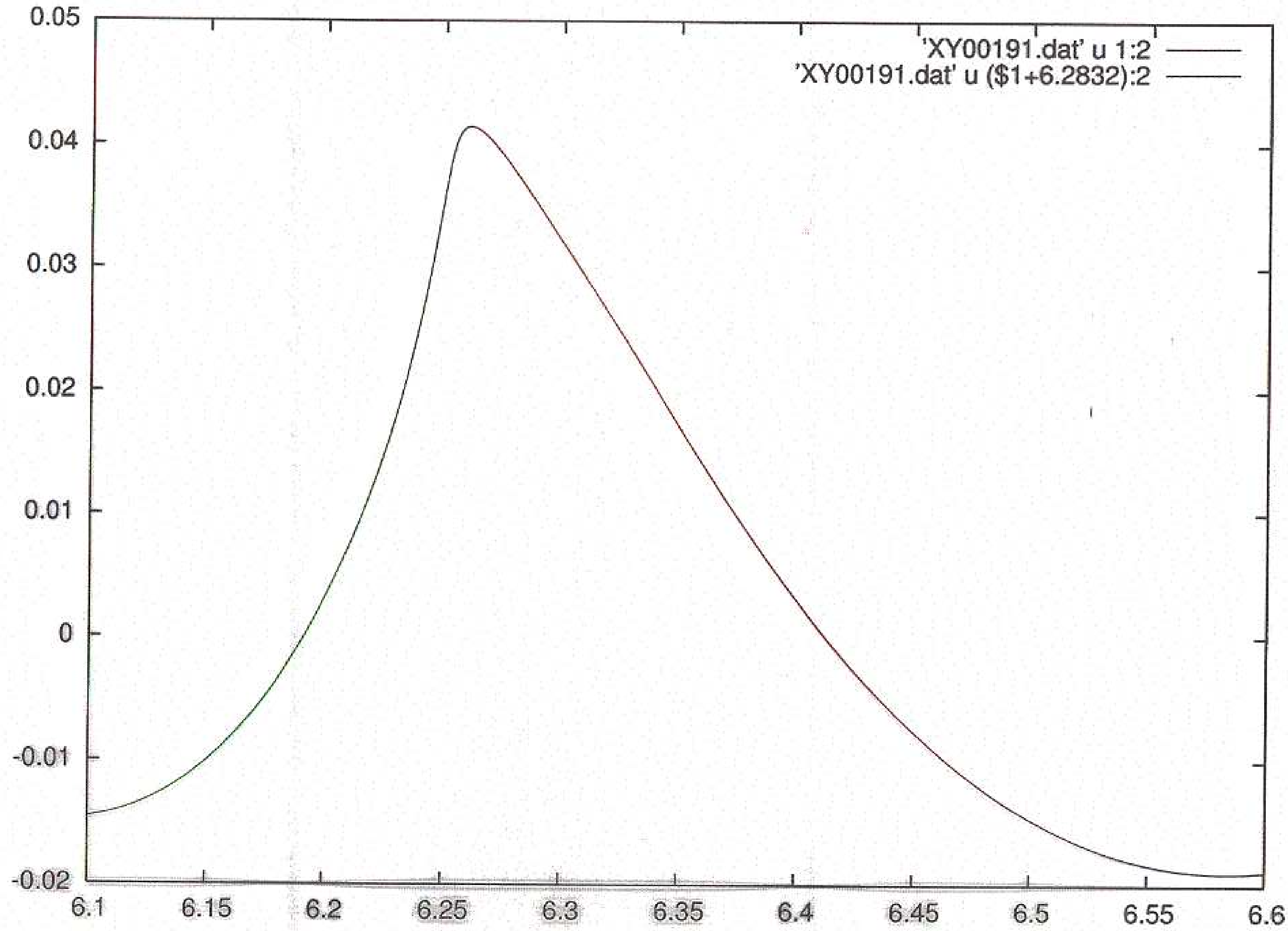

'XY00098.dat' u 1:2
'XY00098.dat' u ($1-6.2832):2
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-2
-1
0
1
2
3
4


'XY00164.dat' u 1:2
'XY00164.dat' u ($1-6.2832):2
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-2
-1
0
1
2
3
4


'XY00170.dat' u 1: 2
'XY00170.dat' u ($1-6.2832): 2
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-2
-1
0
1
2
3
4

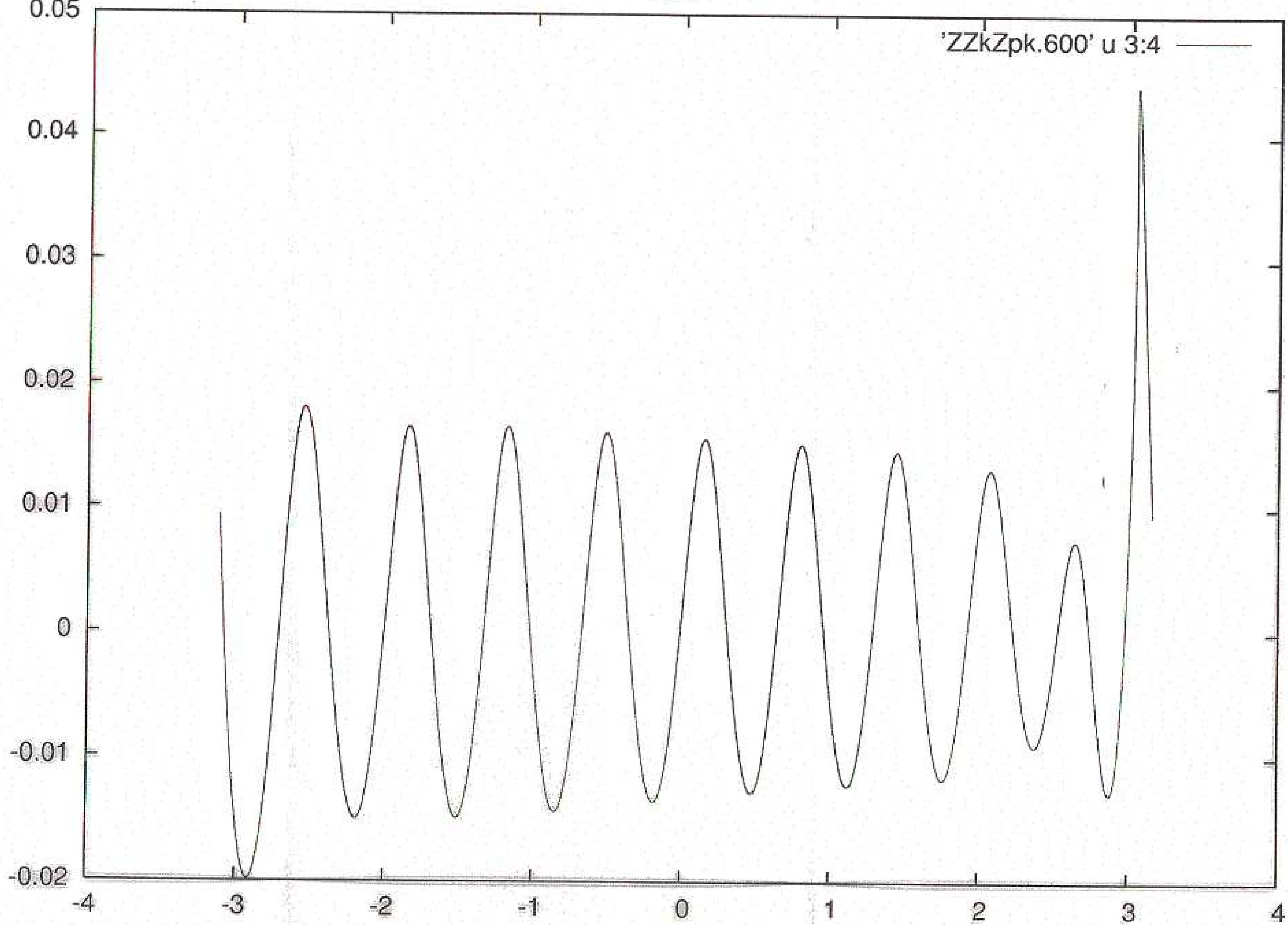

'XY00182.dat' u 1:2
'XY00182.dat' u ($1-6.2832):2
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-2
-1
0
1
2
3
4

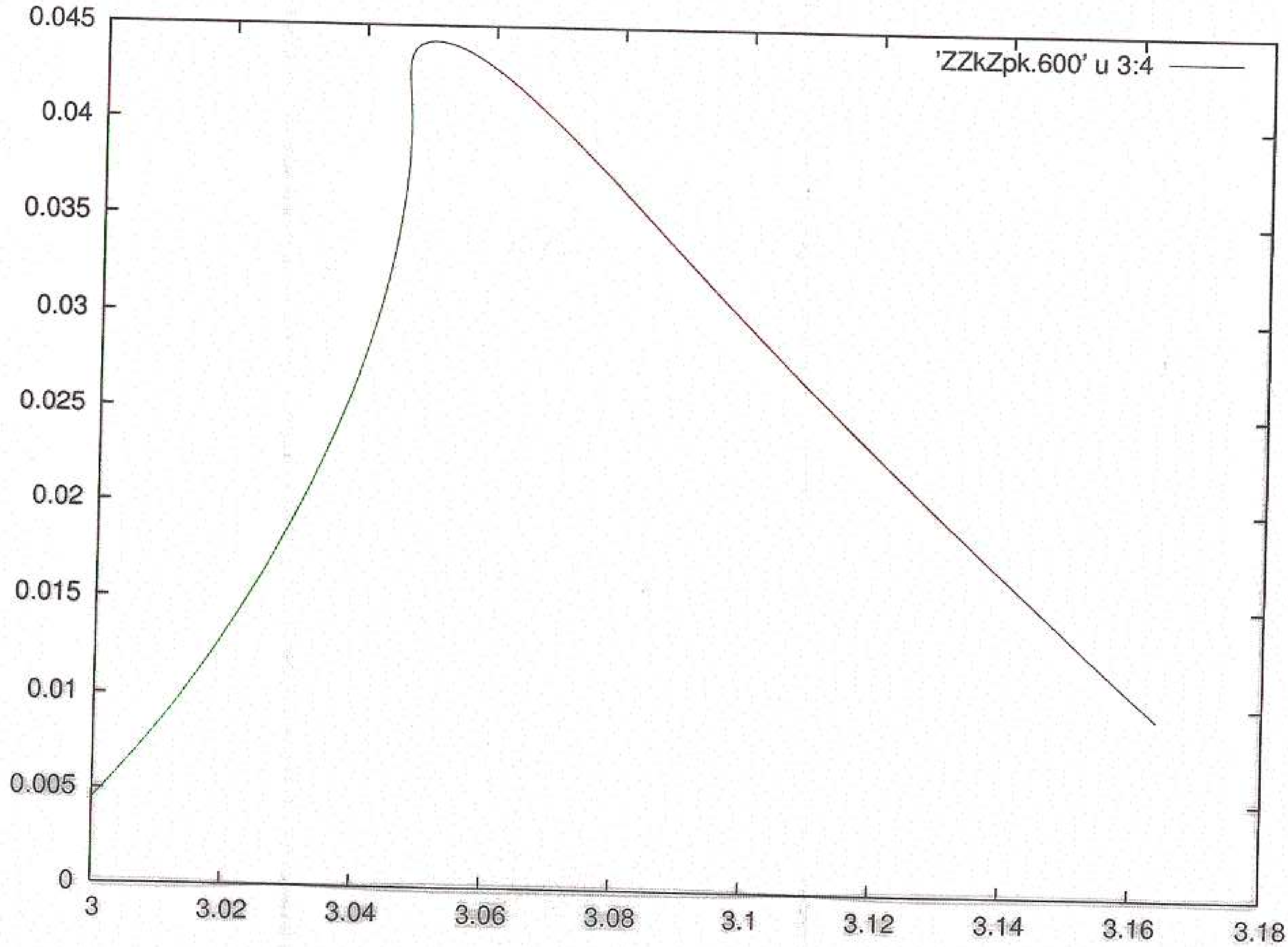

'XY00182.dat' u 1:2
'XY00182.dat' u ($1-6.2832):2
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
0.25

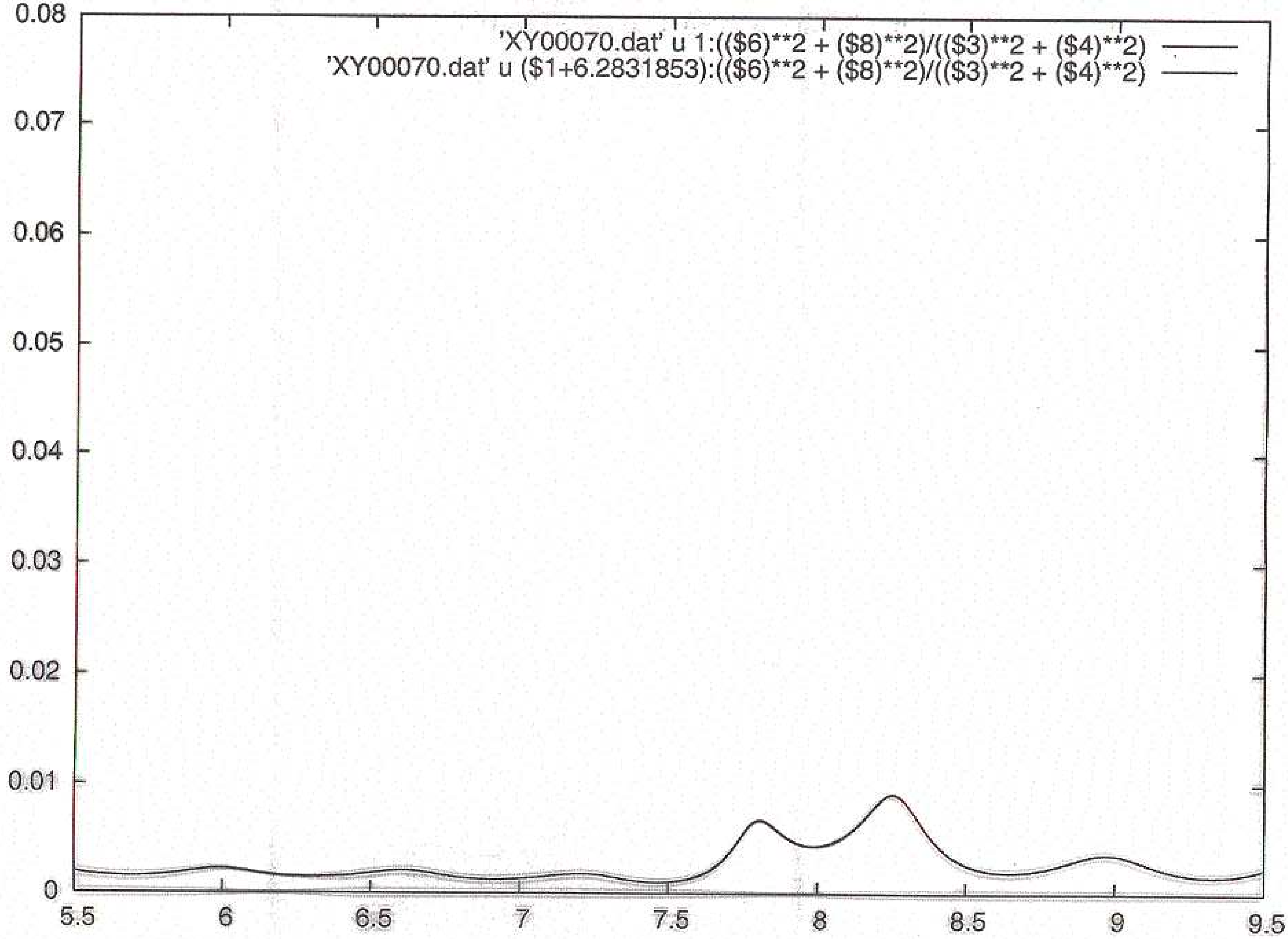


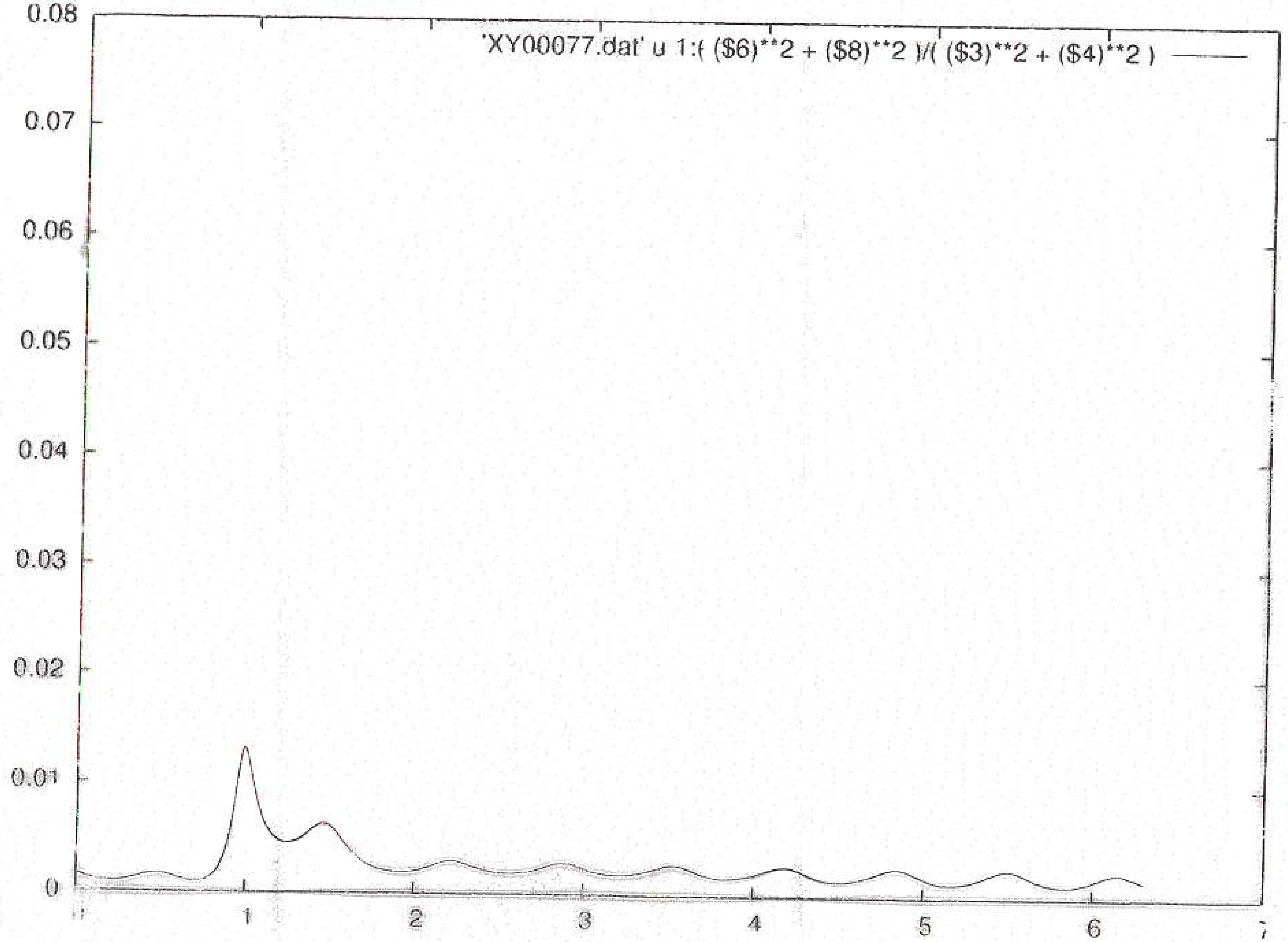
'XY00191.dat' u 1:2
'XY00191.dat' u ($1+6.2832):2
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
6.1
6.15
6.2
6.25
6.3
6.35
6.4
6.45
6.5
6.55
6.6

-16-

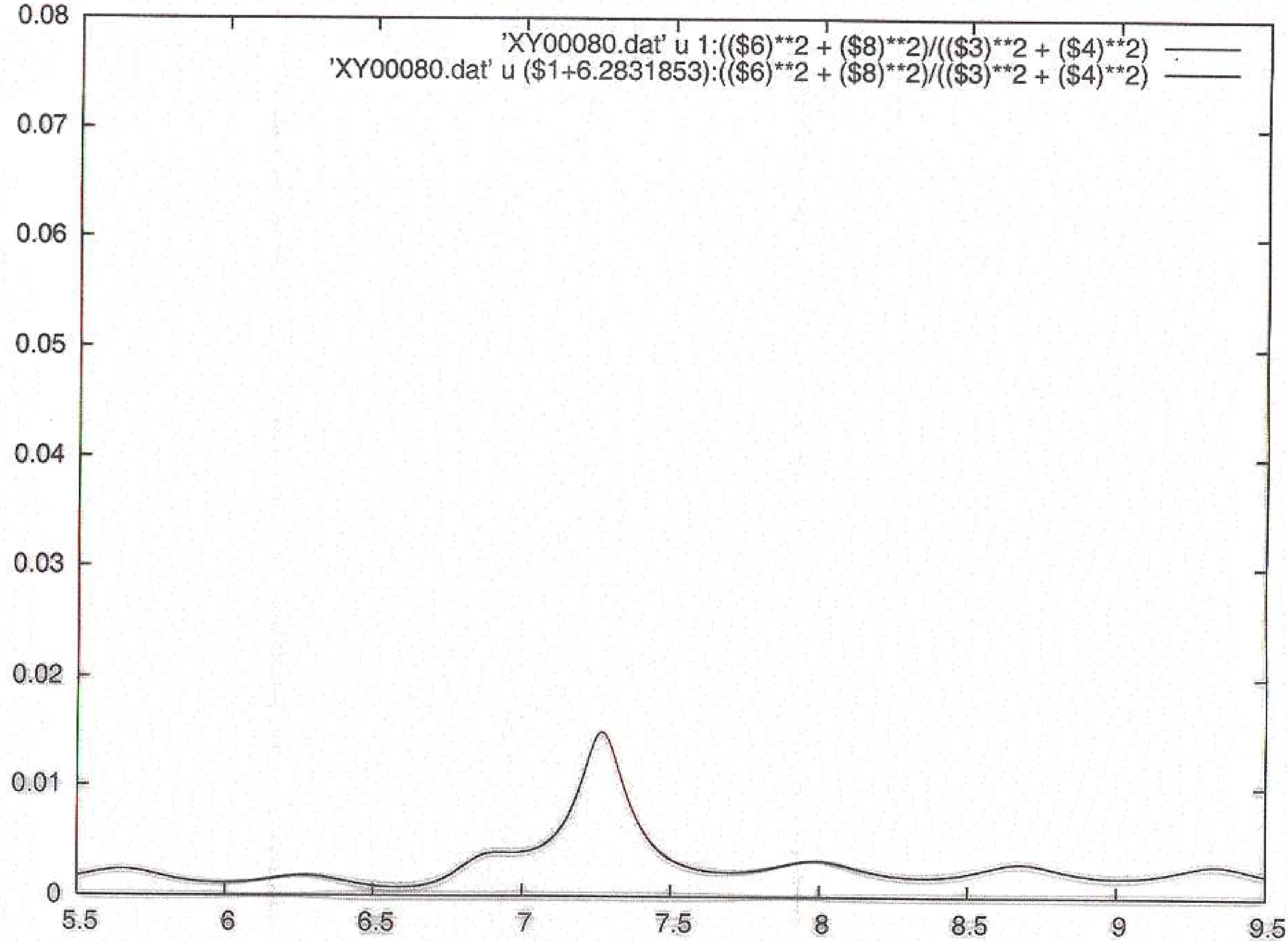


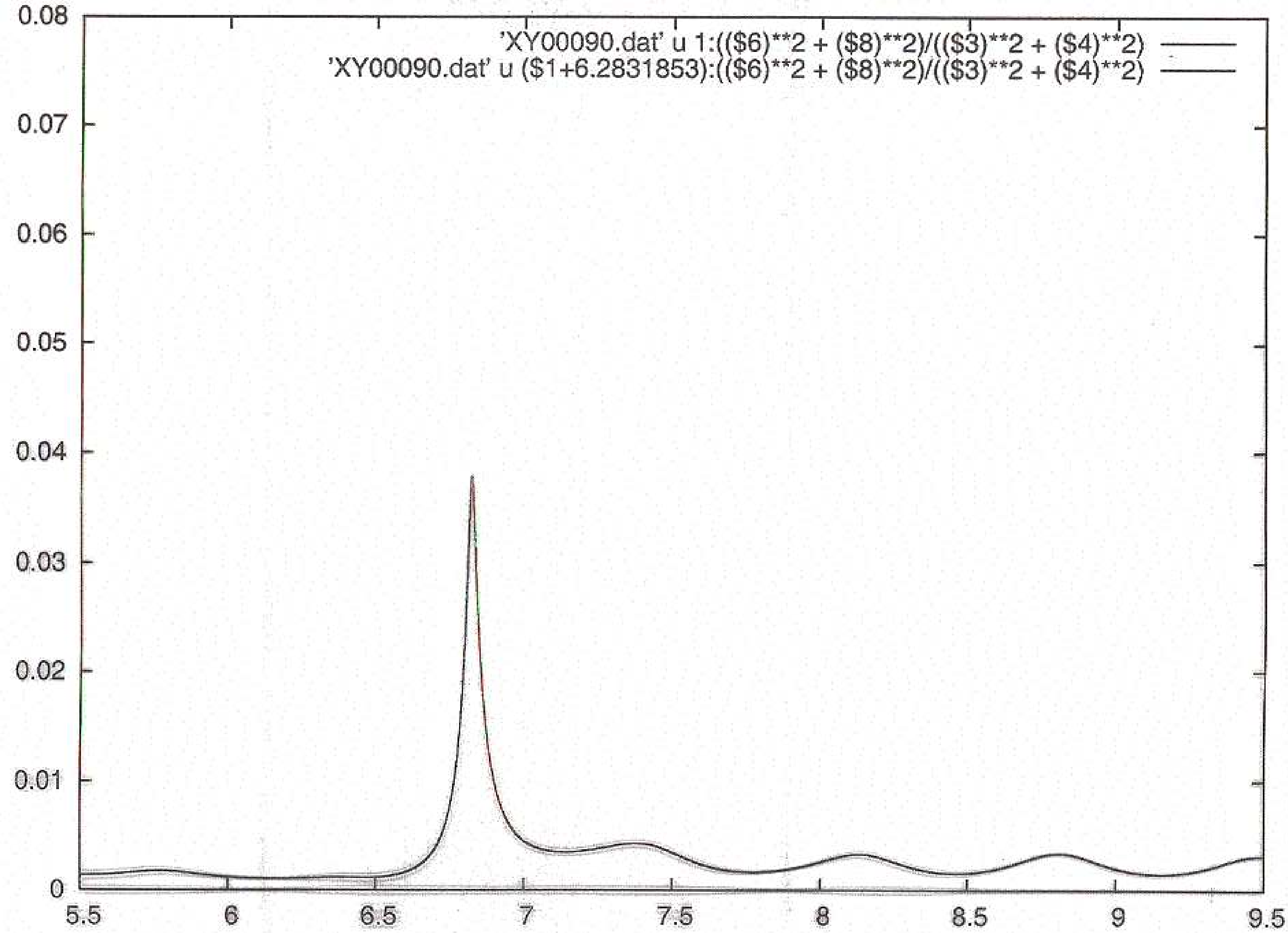

'ZZkZpk.600' u 3:4
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
3
3.02
3.04
3.06
3.08
3.1
3.12
3.14
3.16
3.18

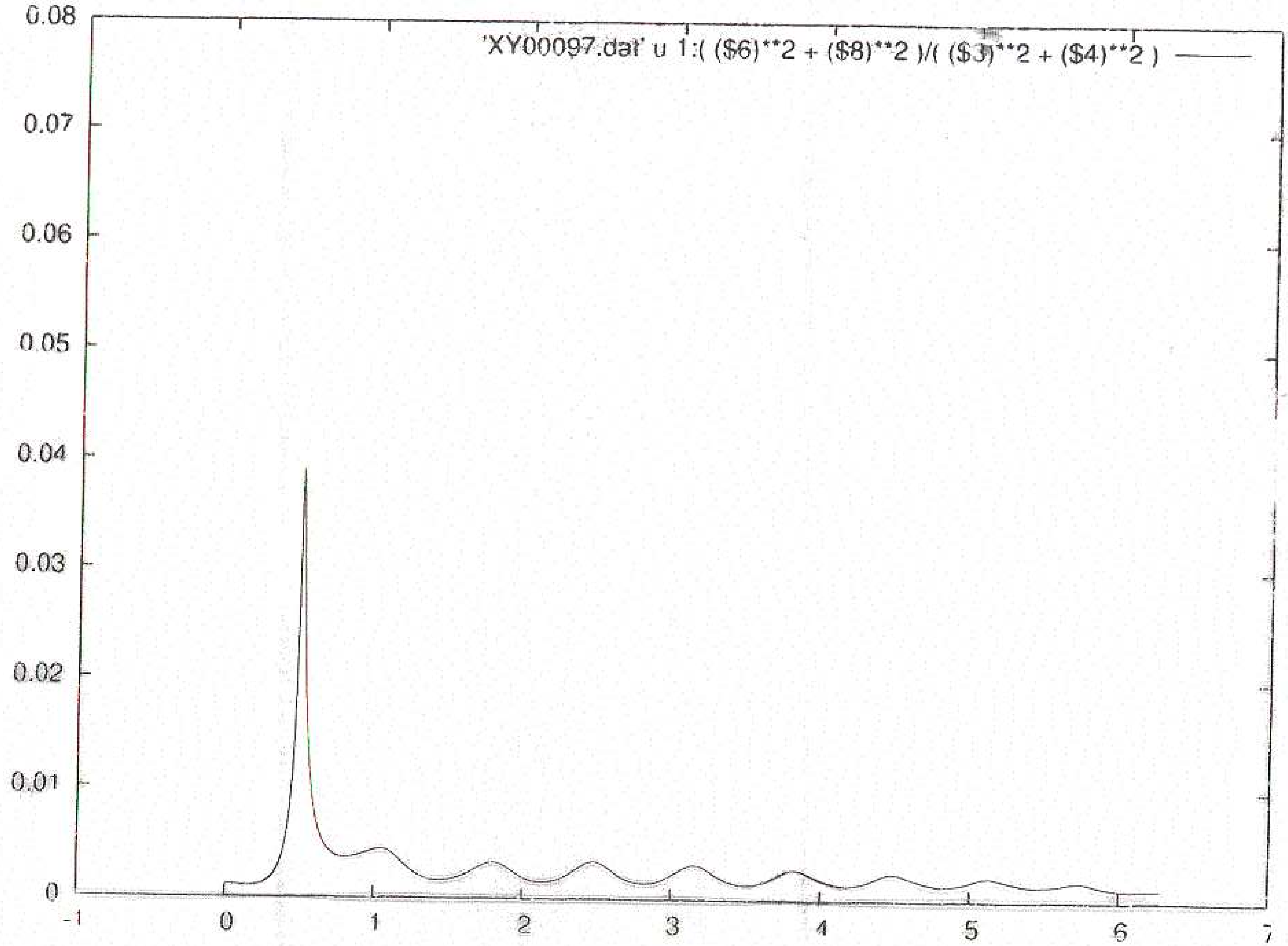

'XY00070.dat' u 1:(($6)**2 + ($8)**2)/(($3)**2 + ($4)**2)
'XY00070.dat' u ($1+6.2831853):(($6)**2 + ($8)**2)/(($3)**2 + ($4)**2)
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
5.5
6
6.5
7
7.5
8
8.5
9
9.5

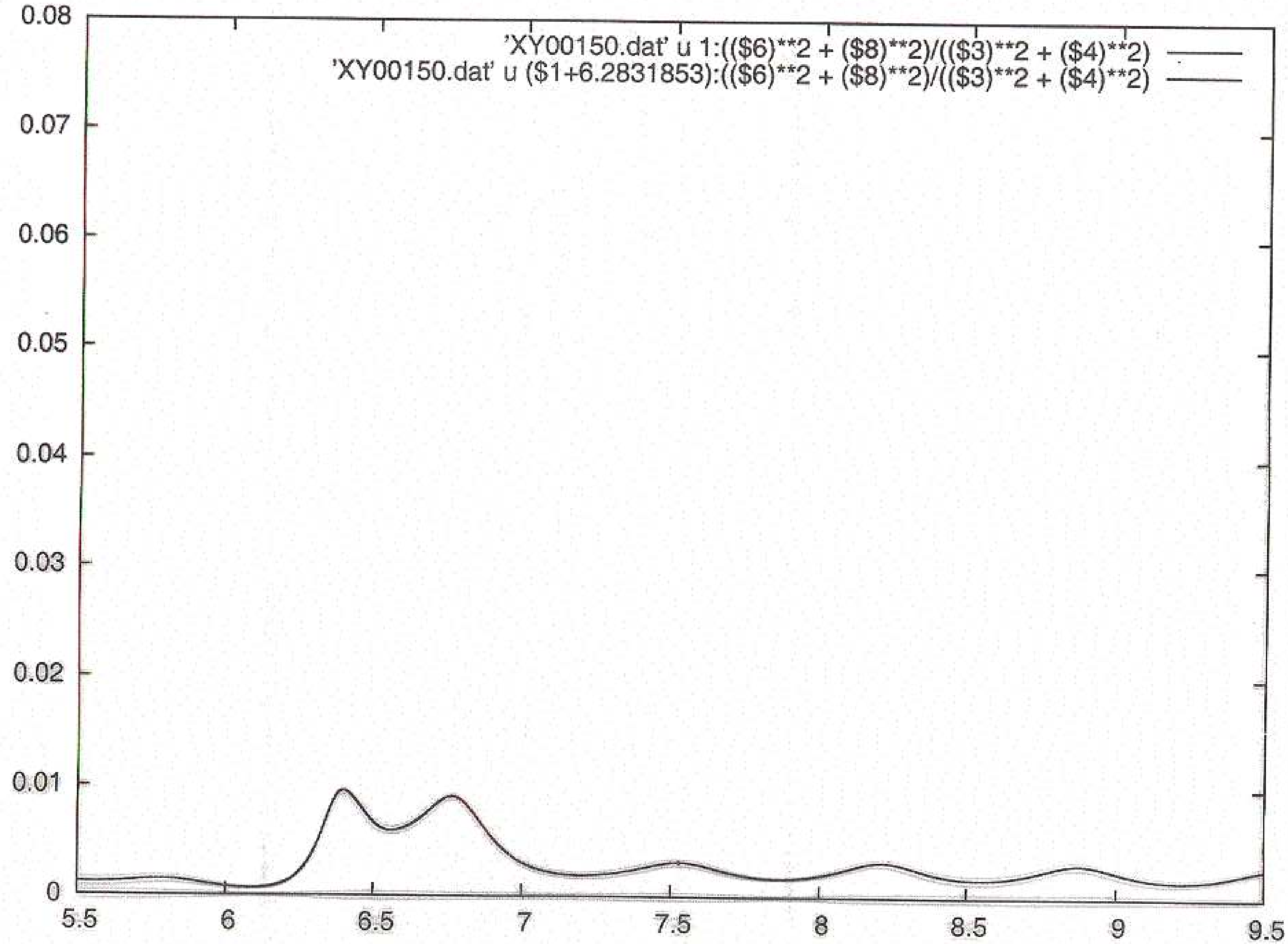

'XY00077.dat' u 1:(($6)**2 + ($8)**2)/(($3)**2 + ($4)**2)
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
1
2
3
4
5
6
7

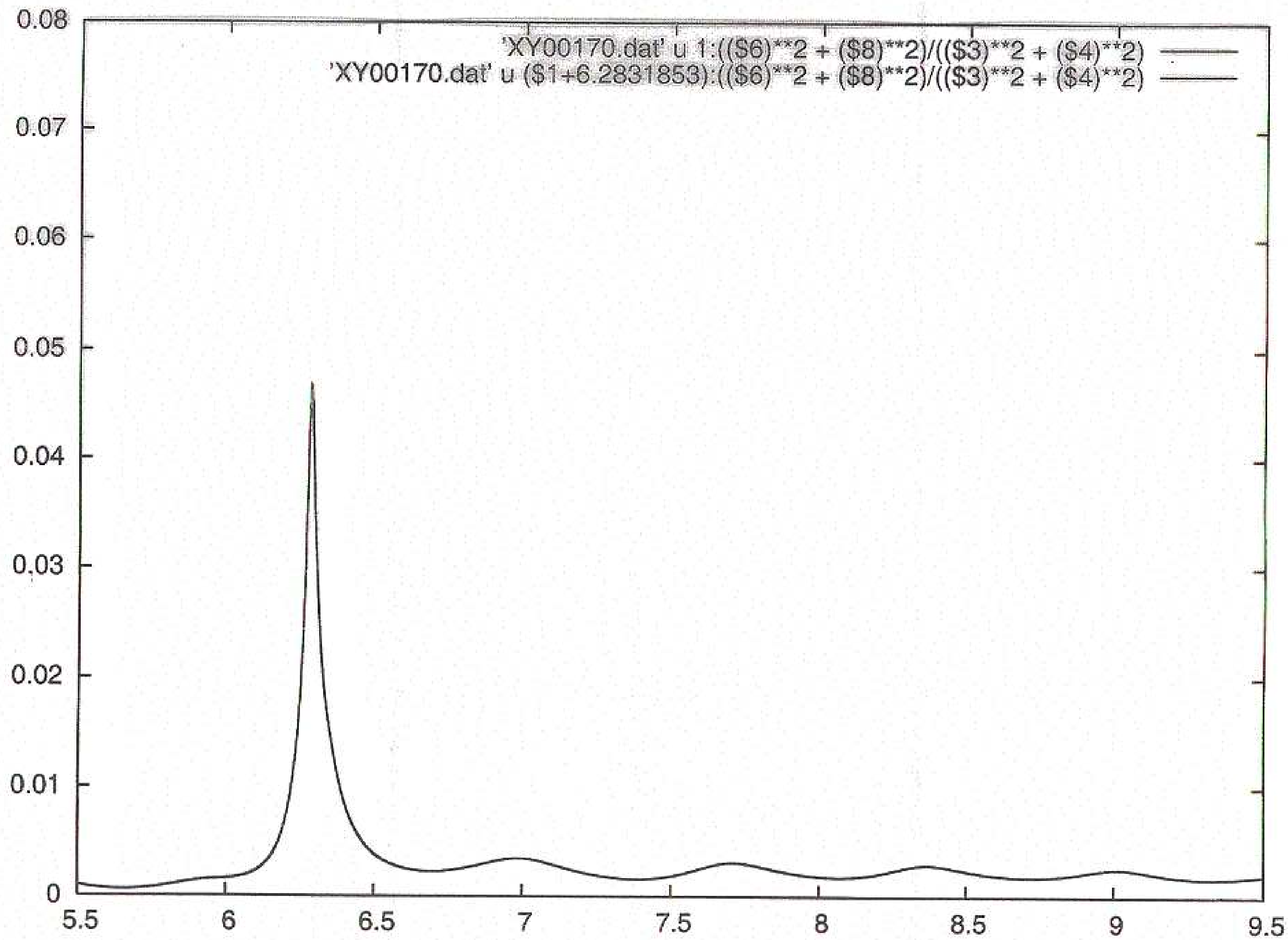
2d


'XY00080.dat' u 1:(($6)**2 + ($8)**2)/(($3)**2 + ($4)**2)
'XY00080.dat' u ($1+6.2831853):(($6)**2 + ($8)**2)/(($3)**2 + ($4)**2)
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
5.5
6
6.5
7
7.5
8
8.5
9
9.5


'XY00090.dat' u 1:(($6)**2 + ($8)**2)/(($3)**2 + ($4)**2)
'XY00090.dat' u ($1+6.2831853):(($6)**2 + ($8)**2)/(($3)**2 + ($4)**2)
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
5.5
6
6.5
7
7.5
8
8.5
9
9.5


'XY00097.dat' u 1:(($6)**2 + ($8)**2)/(($3)**2 + ($4)**2)
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-1
0
1
2
3
4
5
6
7


'XY00150.dat' u 1:(($6)**2 + ($8)**2)/(($3)**2 + ($4)**2)
'XY00150.dat' u ($1+6.2831853):(($6)**2 + ($8)**2)/(($3)**2 + ($4)**2)
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
5.5
6
6.5
7
7.5
8
8.5
9
9.5


'XY00170.dat' u 1:(($6)**2 + ($8)**2)/(($3)**2 + ($4)**2)
'XY00170.dat' u ($1+6.2831853):(($6)**2 + ($8)**2)/(($3)**2 + ($4)**2)
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
5.5
6
6.5
7
7.5
8
8.5
9
9.5

-25-