Holographic Description of a Cosmological Singularity

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Do cosmological singularities represent a true beginning (or end) of time, or is there a bounce?

True beginning: Hartle and Hawking, …

Bounce: Veneziano; Steinhardt and Turok, …

T. Hertog and G. H., hep-th/0406134,

hep-th/0412169, and hep-th/0503071

AdS/CFT Correspondence

Maldacena (1997)

AdS: Anti de Sitter spacetime

Maximally symmetric solution to Einstein's equation with negative cosmological constant. The metric is:

$$ds^2 = -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega$$

If you rescale by $1/r^2$, infinity is a 3D cylinder at a finite distance. Light can reach infinity in finite time.

CFT: Ordinary (nongravitational) quantum field theory that is conformally invariant.

The AdS/CFT correspondence states that string theory on spacetimes that asymptotically approach AdS x K is completely equivalent to a CFT living on the boundary.

If there is smooth asymptotically AdS initial data which evolve to a big crunch, then one can use the CFT to study what happens near the singularity in a full quantum description.

Big crunch: a spacelike singularity which reaches infinity in finite time.

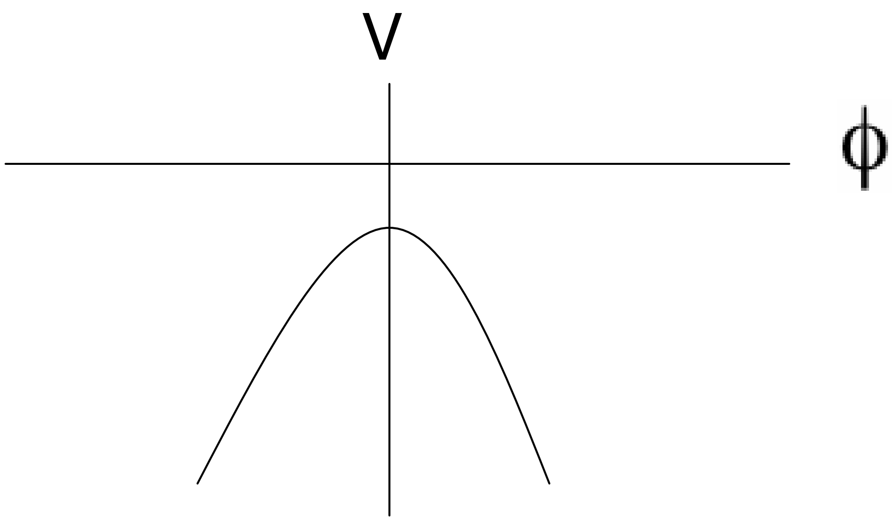
Outline:

- Show that there indeed exist solutions which evolve into a big crunch.
- Discuss what is known about the dual CFT description.
- Surprising spin-off: designer gravity

Consider $AdS_4 \times S^7$ boundary conditions.

M theory reduces to N=8 supergravity in four dimensions. We can set all matter fields to zero, except for a single scalar ϕ. The potential is

$$V(\phi) = -2 - \cosh\sqrt{2}\phi$$



AdS can be stable even with potentials that are unbounded from below

(Breitenlohner and Freedman,1982)

E = (positive gradient term) +

(negative potential term)

The gradient term includes

$$g^{rr}(\partial_r\phi)^2 \approx r^2(\partial_r\phi)^2$$

which can dominate potential term.

For all asymptotically AdS solutions, the scalar field falls off asymptotically like

$$\phi = \frac{\alpha}{r} + \frac{\beta}{r^2}$$

If $\alpha=0$ or $\beta=0$, AdS is stable. Consider a new boundary condition $\beta = k\alpha^2$ (Hertog and Maeda). This is also invariant under all asymptotic AdS symmetries.

Claim: For all nonzero k, there are solutions that evolve to a big crunch.

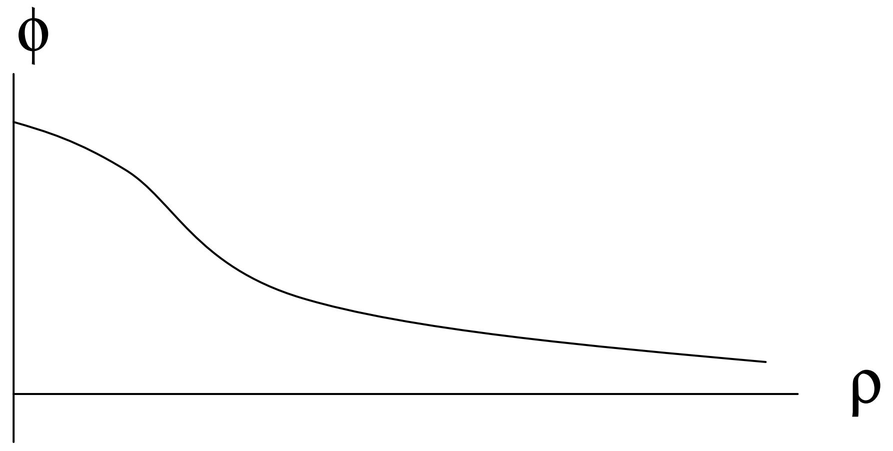
Start by solving the Euclidean field equations with SO(4) symmetry

$$ds^2 = \frac{d\rho^2}{b^2(\rho)} + \rho^2 d\Omega_3$$

Get ODE's. Pick ϕ at the origin and integrate out. Asymptotically find

$$\phi = \frac{\alpha}{\rho} + \frac{\beta}{\rho^2}$$

Define k by $\beta = k\alpha^2$. Find that k is large for small initial ϕ and vice versa.



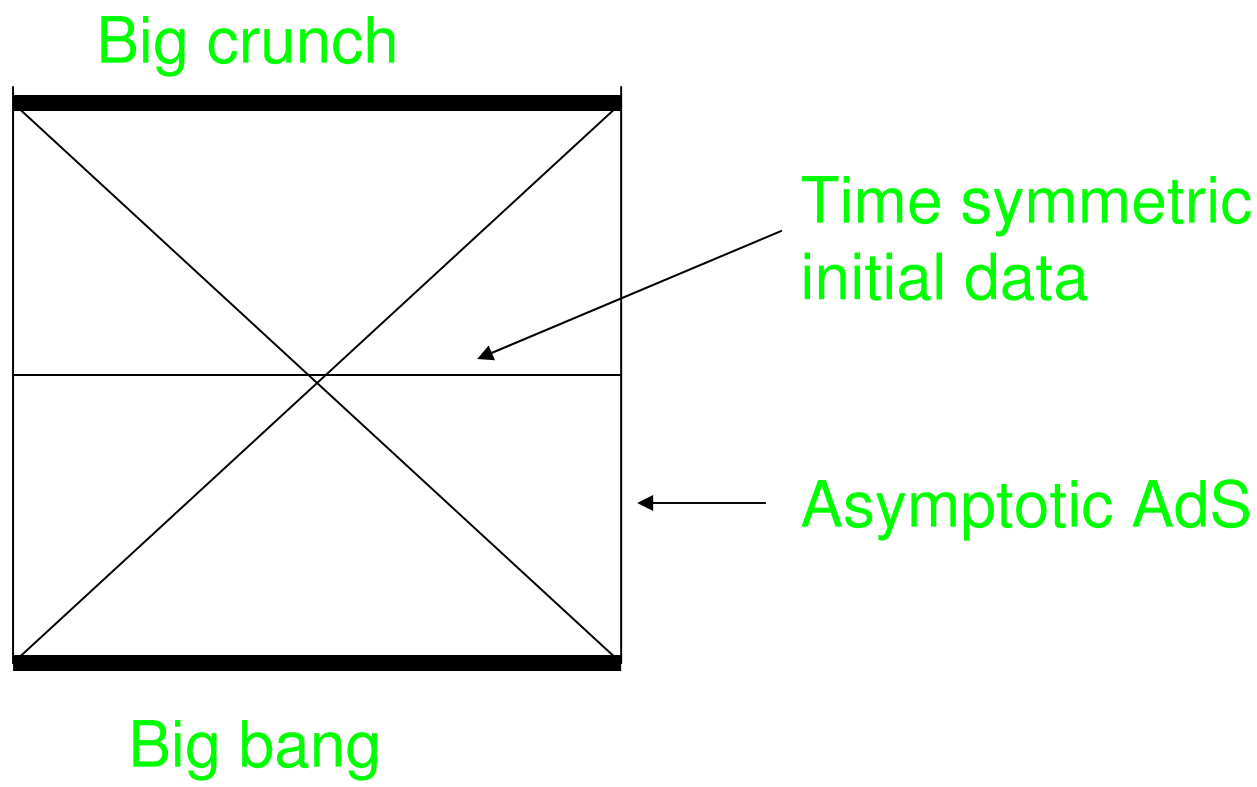
Restricting to the equator of the three-sphere, we get initial data for a Lorentzian solution. The evolution of this initial data can be obtained by analytic continuation.

(Coleman and De Luccia, 1980)

The Euclidean origin becomes a Lorentzian lightcone. Outside the lightcone everything is smooth and bounded. Inside the lightcone the solution evolves like an open FRW universe:

$$ds^2 = -dt^2 + a(t)^2 d\sigma_{-1}$$

The field equations imply that the scale factor vanishes in finite time, producing a big crunch.



This looks like Schwarzschild AdS, but:

(1) Infinity is not complete.

(2) Two-spheres shrink to zero size at the center.

We must check that this solution satisfies our boundary conditions. Outside the lightcone (for large ρ)

$$ds^2 = \frac{d\rho^2}{\rho^2} + \rho^2(-d\tau^2 + \cosh^2\tau d\Omega)$$

In terms of standard global coordinates:

$$\rho^2 = r^2\cos^2 t - \sin^2 t$$ so $$\frac{1}{\rho} = \frac{1}{r\cos t} + O(r^{-3})$$

The asymptotic scalar field is: $$\phi = \frac{\alpha}{\rho} + \frac{k\alpha^2}{\rho^2} = \frac{\tilde{\alpha}}{r} + \frac{k\tilde{\alpha}^2}{r^2}$$

with $$\tilde{\alpha} = \frac{\alpha}{\cos t}$$

Note that k is unchanged.

CFT Description

This is the 2+1 theory on a stack of M 2-branes. Infrared limit of a theory containing an SU(N) gauge field and 7 scalars φ_i. With β=0 boundary conditions, the bulk scalar ϕ is dual to the dimension one operator

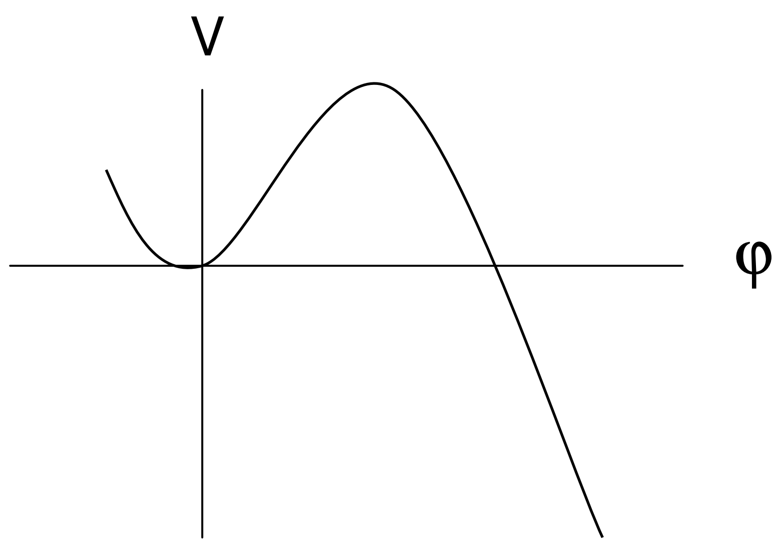
$$O = Tr(\varphi_1^2 - \varphi_2^2)$$

Our new boundary condition corresponds to adding to the field theory action the term:

$$\frac{k}{3}\int O^3$$

This is like a 3D field theory with potential

$$V(\varphi) = \frac{1}{2}m^2\varphi^2 - \frac{k}{3}\varphi^6$$



φ=0 is a perturbatively stable vacuum. But nonperturbatively, it decays.

In a semiclassical approximation, φ tunnels through the barrier and rolls down to infinity in finite time.

(In 1/N approximation of full theory, $\langle\varphi\rangle$ diverges in finite time.)

It appears that field theory evolution ends in finite time, and hence there is no bounce through the big crunch!

This model field theory works surprisingly well.

We saw that $\alpha(t) = c/\cos t$.
Since $\alpha = \langle O \rangle$, we might try
$\varphi(t) = \tilde{c}/\cos^{1/2} t$
It turns out that this is an exact
solution provided $m^2 = \frac{1}{8}R$
as expected for conformal coupling.

What about the full quantum theory?

If we restrict to homogeneous field configurations, the field theory reduces to ordinary quantum mechanics with a potential

$$V(x) = \frac{1}{2}m^2x^2 - \frac{k}{3}x^6$$

There is a one parameter family of Hamiltonians (Farhi et al). Picking one, evolution continues for all time. <x> can diverge and come back.

This looks more like a bounce.

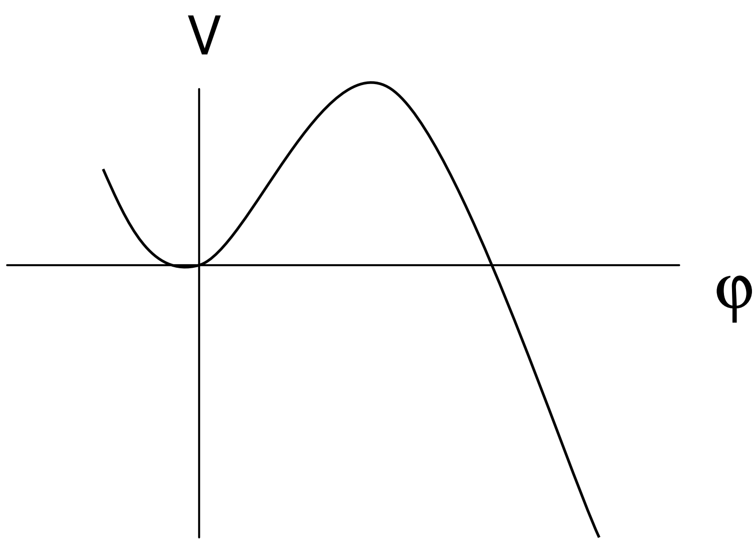
This almost never happens in field theory.

The CFT has infinitely many degrees of freedom which become excited when the field falls down a steep potential (tachyonic preheating). Like particles decaying into lower and lower "mass" particles.

In the QM problem, the different self-adjoint extensions correspond to different ways to cut off the potential. If the same is possible in the CFT, one would expect to form a thermal state. $\langle\varphi\rangle$ would not bounce.

"Minisuperspace" is a bad approximation.

This leads to a natural time asymmetry between a big bang and a big crunch (cf: Penrose)



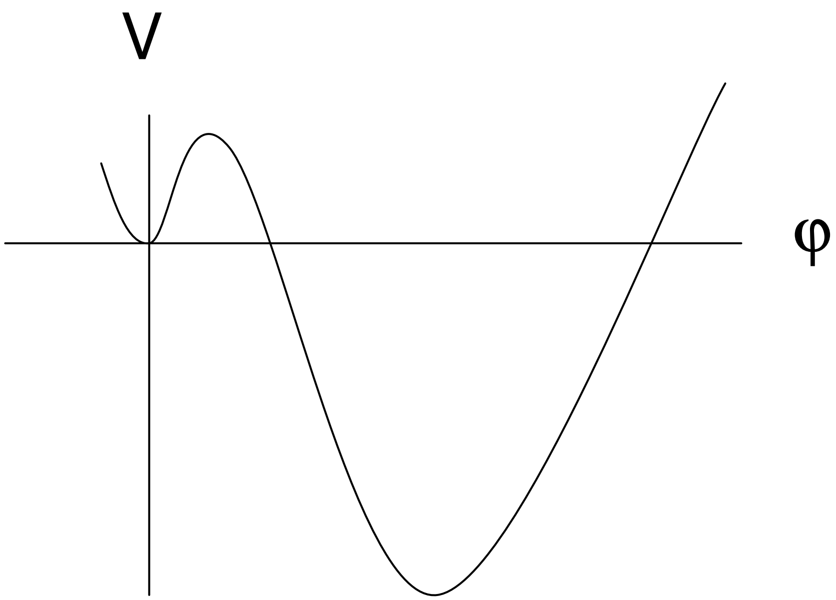
The universe starts in an approximately thermal state with all the Planck scale degrees of freedom excited. Very rarely there is a fluctuation in which most of the energy gets put into the zero mode which goes up the potential. This is the big bang.

This could help explain the origin of the second law of thermodynamics!

What happens if we regulate the potential?

Suppose we add to the CFT action

$$W = \frac{k}{3}\int O^3 + \frac{\varepsilon}{4}\int O^4$$



This theory has a stable vacuum with a large negative energy. Expect the decay of $\varphi=0$ vacuum to produce a thermal state (like inflation).

What does this correspond to on the gravity side?

General rule (Witten, Sever and Shomer):

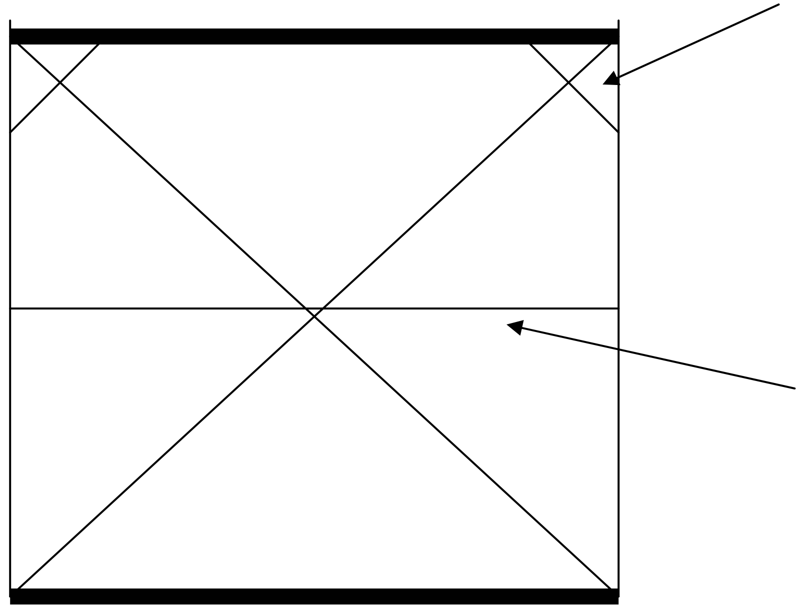
If you add W(O) to the CFT, the boundary conditions are

$$\beta = W'(\alpha)$$

where $\alpha = \langle O \rangle$

This means that we must now evolve our initial data with the boundary conditions

$$\beta = k\alpha^2 + \epsilon\alpha^3$$



Evolution is unchanged until α becomes large, so only affects spacetime in the corners. But this can stop the singularity from being a big crunch. Can you form a black hole?

Energy

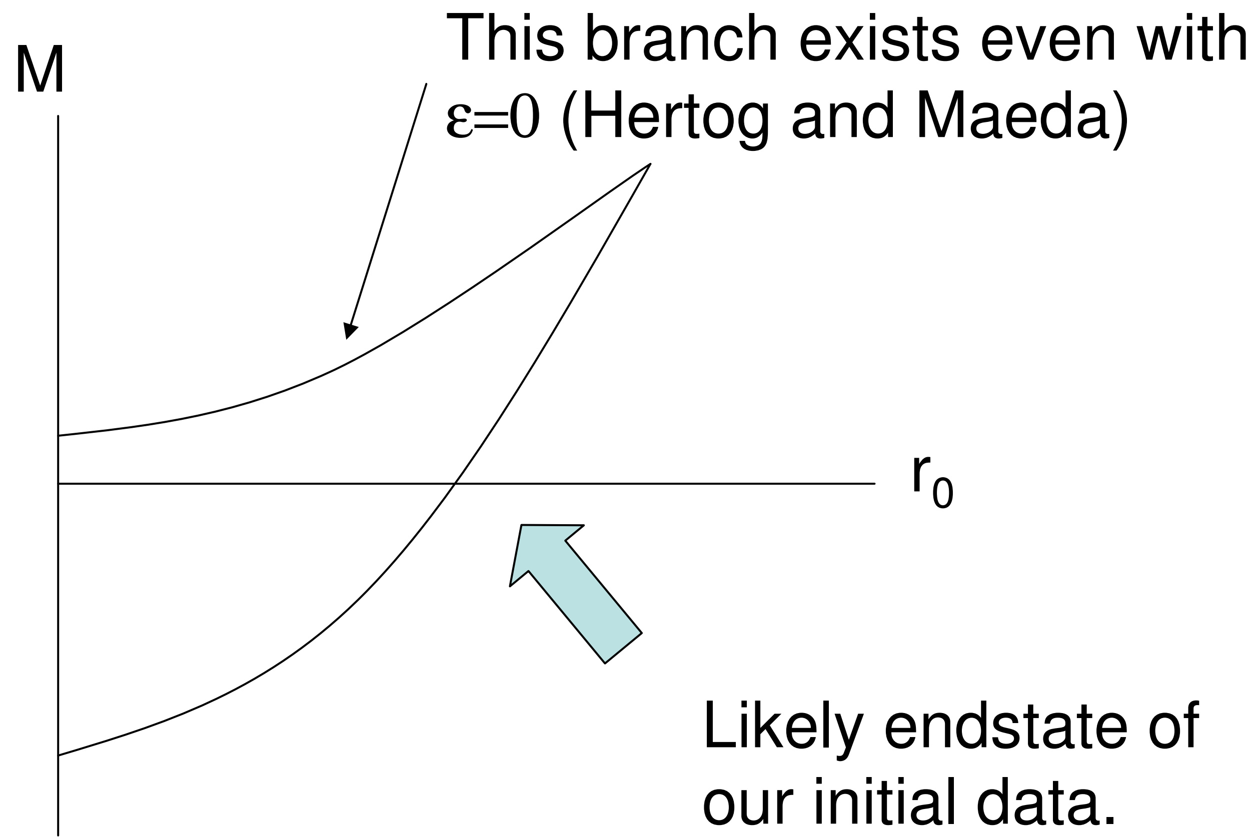
The usual notion of energy is finite only when $\alpha=0$. One can always define a conserved energy, but it depends on the boundary conditions for the scalar field. This energy is associated with asymptotic time translations, but it is not positive definite.

The energy of our initial data is zero.

Do zero mass black holes exist?

Yes!

Black holes with scalar hair:



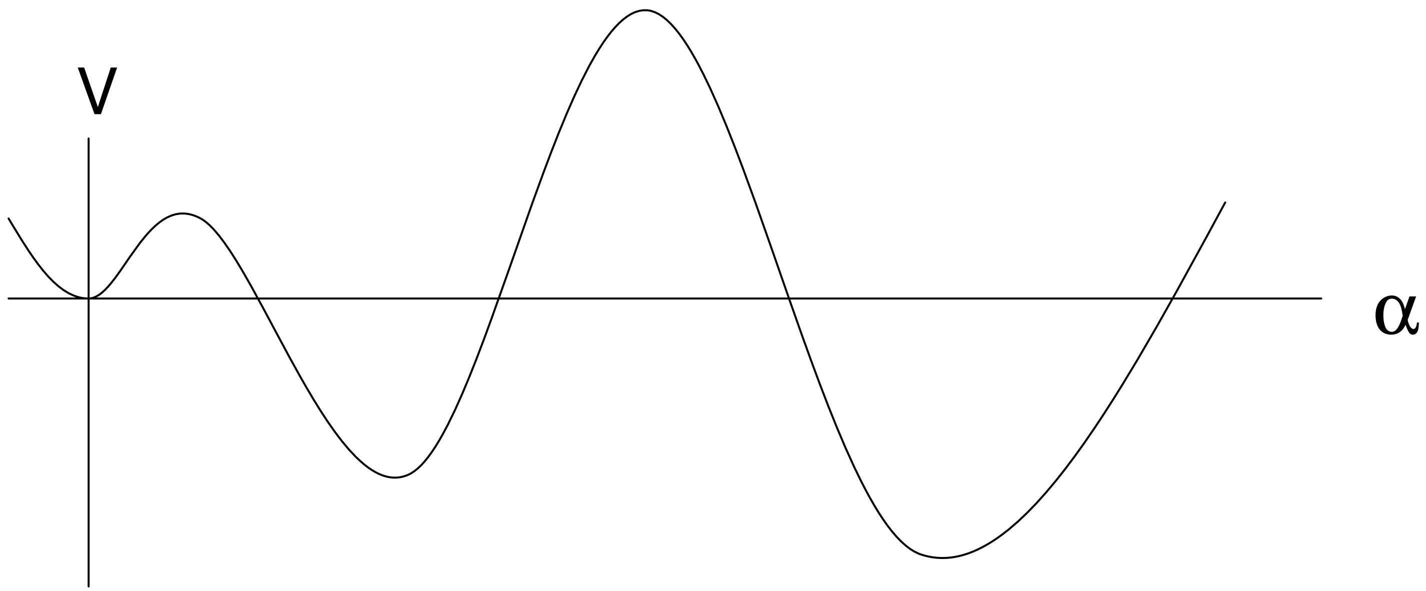
As you take $\varepsilon \to 0$, radius of final black hole diverges and you recover the big crunch.

Regulated field theory produces large black holes and not a big crunch (consistent with previous AdS/CFT ideas).

If you take the radius of hairy black hole to zero, you find a nonsingular soliton. These solitons correspond to extrema of the potential. This can be generalized:

Designer Gravity

Given any potential $V(\alpha)$, there are boundary conditions so that the gravity theory has solitons precisely at the extrema of V, with masses given by the value of V at the extrema.



The bulk theory is unchanged. Only the boundary conditions are affected. This has several consequences:

- there are new “positive” energy conjectures for asymptotically AdS spacetimes. The total energy of all solutions is probably greater than the global minimum of V.

- one can compute an effective potential for the operator O in the dual field theory.

In a little more detail:

There is a soliton for each choice of ϕ at the origin. Get a curve $\beta_{SG}(\alpha)$. Define

$$W_0(\alpha) = -\int_0^{\alpha} \beta_{SG}(\tilde{\alpha})d\tilde{\alpha}$$

For any V, write $V=W_0+W$ and choose boundary conditions $\beta=W_{,\alpha}$. Then $\beta=\beta_{SG}$ is an extremum of V and one can show the mass is just given by V.

Summary:

- A small change in the standard asymptotically AdS boundary conditions allow smooth initial data to evolve to a big crunch in string theory.
- The dual CFT has a potential unbounded from below. Semiclassically, evolution ends. Full quantum description of zero mode exists and describes a bounce. Analog in field theory should be a type of thermal state. (If you regulate the potential by hand, you form a large black hole.)
- There exist supergravity solutions with pre-ordered solitons (and black holes).