"Non-SUSY smooth geometries + D1-D5-P bound states"

Simon Ross - talk at Fields 6/5/06

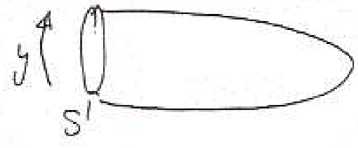
work with Vishnu Jejjala, Owen Madden + Gina Titchener,
hep-th/0504181

- Introduction + review
- Soliton properties
- Finding solitons
- Relation to CFT
- Wave equation + propagation
- Discussion: future directions.

Introduction: · IIB Sugra on $$\overbrace{\underset{z_i}{T^4} \times \underset{y}{\overset{D1}{S^1}}}^{D5} \times \underset{(t,r,\theta,\phi,\psi)}{\mathbb{R}^5}$$

· Smooth AF SUSY solutions:

Balasubramanian, de Boer, Keski-Vakkuri + SFR, '00
Maldacena + Maoz



$S^1 \to 0$ at $\rho = 0$ $R\partial_y = \partial_\phi$

i.e., y at fixed $\tilde{\phi} = \phi + y/R$

Topologically $T^4 \times \underset{r,y}{\mathbb{R}^2} \times \underset{t}{\mathbb{R}} \times \underset{(\theta,\tilde{\phi},\psi)}{S^3}$

$y \sim y + 2\pi R$ $Q_1, Q_5; Q_p = 0$

If $R^2 \gg Q_1, Q_5$, 'near-core' region global AdS

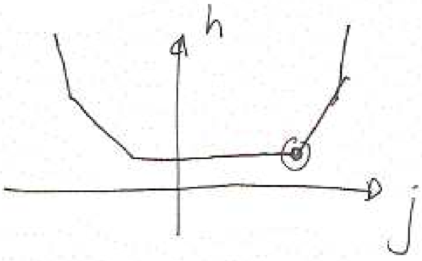
$T^4 \times \underset{t,r,y}{AdS_3} \times \underset{(\theta,\tilde{\phi},\psi)}{S^3}$ $\ell = (Q_1 Q_5)^{1/4}$

SUSY: fermions periodic around asymptotic S^1.

Dual CFT: $(T^4)^{n_1 n_5}/S_{n_1 n_5}$ $\quad c = 6 n_1 n_5$

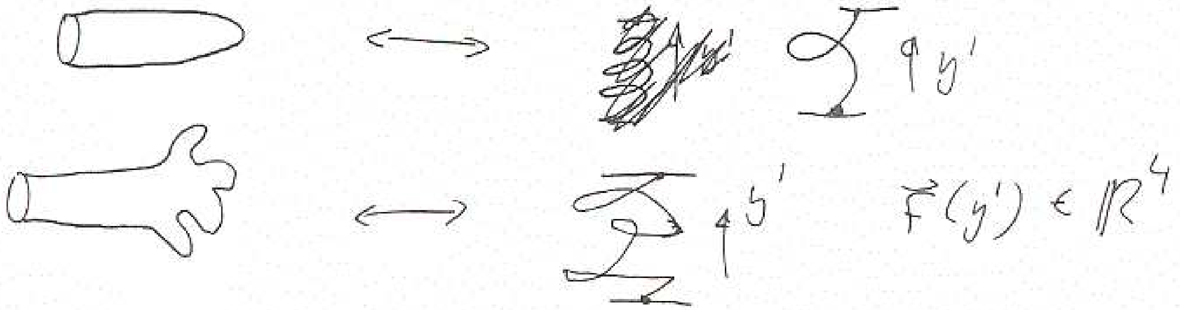
Geometry id with RR ground state

$\tilde{\phi} = \phi + y$ corr to spectral flow.



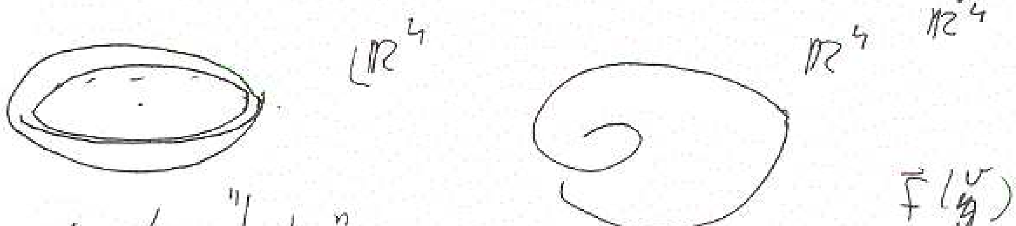
- More general solutions: Lunin + Mathur

F-string picture: D1-D5 U-dual to F1-P



- Supertube picture: Lunin, Maldacena + Maoz
Mateos + Townsend

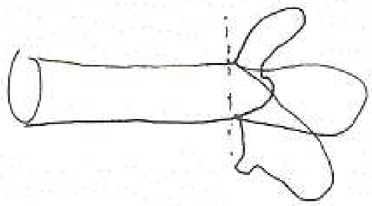
write solutions as $ds^2 = H^{-1}(-(dt-A)^2 + (dy+B)^2) + H \underset{\mathbb{R}^4}{\underset{\uparrow}{d\vec{x}^2}}$



circular "tube"
for simple solutions.

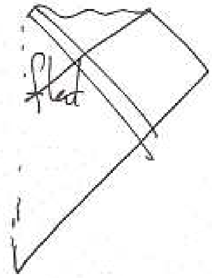
- Id with more general RR ground states.

- Radical proposal for black holes: Mathur + ...
  - "effective horizon" arises from coarse-graining over geometry



  - Different from AdS/CFT: microstates described directly in geometric language
    - curvatures large for typical states

I don't advocate this picture: although solitons exist, don't see how they can replace black hole dynamically.

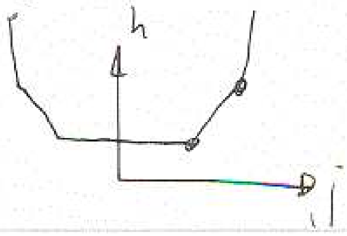


Horizon teleological concept!

+ Transition from $\mathbb{R}^4 \times S^1 \to S^3 \times \mathbb{R}^2$ classically forbidden — Geroch

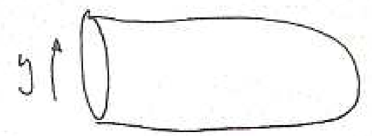
- Extensions of these solitonic solutions?
  - Giusto, Mathur + Saxena:
    some SUSY 3-charge solutions.

$S^1 \to 0$ is $R\partial_y - (n+1)\partial_\phi + n\partial_\psi$



- We extend it to non-SUSY 3-charge solutions

<u>Soliton properties</u>



$S^1 \to 0$ is $Rk\,\partial_y - m\,\partial_\phi + n\,\partial_\psi$

i.e., y at fixed $\tilde\phi = \phi + \frac{m}{k} y/R$
$\tilde\psi = \psi - \frac{n}{k} y/R$

$\partial_t, \partial_y, \partial_\phi, \partial_\psi$ isometries – "$U(1) \times U(1)$ invt"

Topologically $T^4 \times \underset{r,y}{\mathbb{R}^2} \times \underset{t}{\mathbb{R}} \times \underset{(\theta, \tilde\phi, \tilde\psi)}{S^3}$

Charges Q_1, Q_5,

$Q_p = \frac{nm}{k^2} \frac{Q_1 Q_5}{R^2}$ $\quad J_\phi = -\frac{m}{k} \frac{Q_1 Q_5}{R}$

$J_\psi = \frac{n}{k} \frac{Q_1 Q_5}{R}$

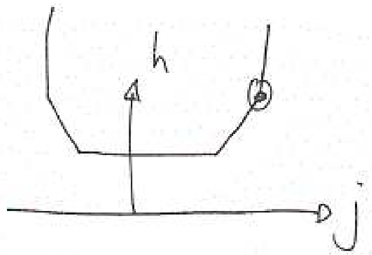
M_{ADM} also fixed by $Q_1, Q_5, R, (n, m, k)$

SUSY only if $m = n+1$.

'Near-core' limit $T^4 \times (AdS_3 \times \underset{(\theta, \tilde\phi, \tilde\psi)}{S^3}) / \mathbb{Z}_k$

orbifold is freely-acting if k rel prime to m, n.

For $k=1$, corr CFT states



Finding solitons

General 5d black hole metric Cvetic + Youm '9.
Cvetic + Larsen

7 parameters: $M, a_1, a_2, \delta_1, \delta_5, \delta_p, R$ assume $a_1 \geq a_2$

$$Q_i = M \sinh \delta_i \cosh \delta_i$$

$$J_\psi = -M\left(a_1 \cosh\delta_1 \cosh\delta_5 \cosh\delta_p - a_2 \sinh\delta_1 \sinh\delta_5 \sinh\delta_p \right)$$
$$J_\phi = -M\left(a_2 \quad -''- \quad - a_1 \quad -''- \right)$$

$$M_{ADM} = \frac{M}{2}\left(\cosh 2\delta_1 + \cosh 2\delta_5 + \cosh 2\delta_p \right)$$

Metric breaks down at $\tilde{H}_1 = 0$, $\tilde{H}_5 = 0$,

$$g(r) = (r^2+a_1^2)(r^2+a_2^2) - Mr^2 = (r^2-r_+^2)(r^2-r_-^2) = 0$$

Require $r^2 = r_+^2$ to be a smooth origin in $\mathbb{R}^2$

$\therefore$ a) need $\|\xi\|^2 = 0$ at $r^2 = r_+^2$ for some

$$\xi = R\,\partial_y + n\,\partial_\psi - m\,\partial_\phi$$

b) need $m, n \in \mathbb{Z}$ so that ξ has closed orbits

c) need an appropriate period to get smooth solution.

i) $\underline{a_1 a_2 = 0}$ ⇒ $r_+^2 = 0$, $\|\xi\|^2 = 0$ if

$\sinh\delta_1 \sinh\delta_5 \sinh\delta_p = 0$ ⇒ $\underline{\underline{\delta_p = 0}}$

$m = \frac{a_1 R}{M \sinh\delta_1 \sinh\delta_5}$, $n = 0$

smooth ⇒ $R = \frac{M \sinh\delta_1 \sinh\delta_5}{\sqrt{a_1^2 - M}}$, so $m = \frac{a_1}{\sqrt{a_1^2 - M}}$

- a) fixes $\delta_p = 0$ ⇒ $Q_p = 0$
- b) m fixes $a_1/\sqrt{M}$; if $m=1$, $M=0$.
- c) relates R to local parameters
  ↳ fix M in terms of Q_1, Q_5, R, m

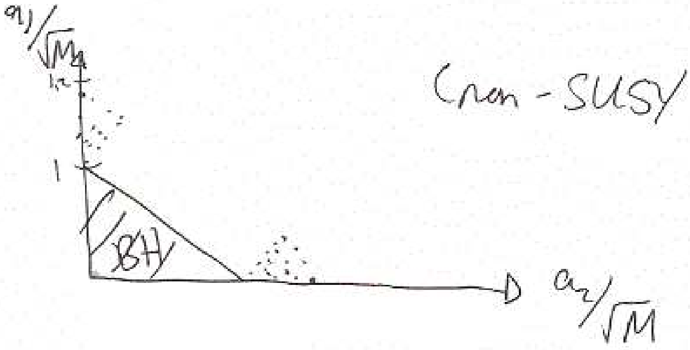
ii) $\underline{a_1 a_2 \neq 0}$ – three charge solutions

a) $\|\xi\|^2 = 0$ ⇒ $M = a_1^2 + a_2^2 - a_1 a_2 \left(\prod \coth\delta_i + \prod \tanh\delta_i\right.$

b) → $m = (\ )R$, $n = (\ )R$

c) smooth → $R(a_1, a_2, \delta_i)$

- $\frac{a_1}{a_2}$, $\prod \tanh\delta_i$ fixed in terms of m, n by b)
- $M/a_1 a_2$ fixed by a)
- M related to Q_1, Q_5, R by c)



(non-SUSY solutions)

Further regularity checks:

- $\tilde{H}_1 > 0$, $\tilde{H}_5 > 0 \ \forall r^2 > r_+^2$
- $g_{\rho y} \to 0$ at $r^2 = r_+^2$
- No horizons: $(\det h)_{r=\text{const}} \propto -g(r)$
- No CTCs: t is a global time function,

$$g^{\mu\nu} \nabla_\mu t \, \nabla_\nu t < 0 .$$

Further integer k introduced by considering orbifolds by

$$y \sim y + 2\pi R/k .$$

- Freely acting if k rel prime to m, n.

$\Rightarrow$ For given Q_1, Q_5, R, family of solitons labeled by (m, n, k).

Completely smooth non-BPS solitons.

NB: no vacuum solutions

Relation to CFT:

Imagine $Q_1, Q_5 >> M, a_1^2, a_2^2$

- near-core AdS region: Cvetic + Larsen

$$ds^2 = -\left(\frac{\rho^2}{\ell^2} + \frac{1}{k^2}\right) d\tau^2 + \left(\frac{\rho^2}{\ell^2} + \frac{1}{k^2}\right)^{-1} d\rho^2 + \rho^2 d\varphi^2$$

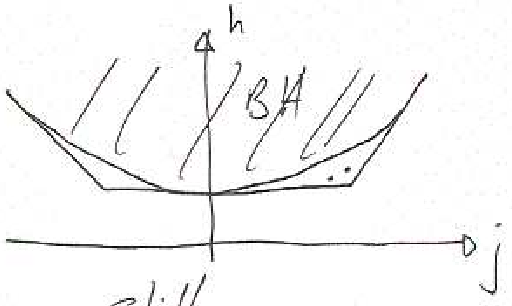
$$+ \ell^2 \left[d\theta^2 + \sin^2\theta \left(d\phi + \frac{m}{k} d\varphi - \frac{n}{k\ell} d\tau\right)^2 + \cos^2\theta \left(d\psi - \frac{n}{k} d\varphi + \frac{m}{k\ell} d\tau\right)^2 \right]$$

Read off CFT charges from asymptotics:

$$h = \frac{c}{24}\left(1 + \frac{(m+n)^2 - 1}{k^2}\right) \qquad j = \frac{c}{12}\frac{(m+n)}{k}$$

$$\bar{h} = \frac{c}{24}\left(1 + \frac{(m-n)^2 - 1}{k^2}\right) \qquad \bar{j} = \frac{c}{12}\frac{(m-n)}{k}$$

R ground state on one side if $m^{*} - n^{*} = 1$



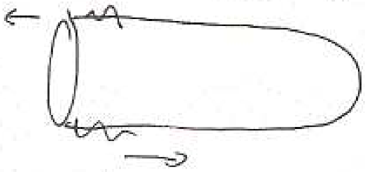
In general, not even spectral flow of R ground state!

- still a very special family of states: $U(1) \times U(1)$ invt.

CFT states related to full AF geometry?

Wave equation + ergoregion:

Probe geometry by scattering plane wave:



$$\Delta t_{SUGRA} = \pi R k \left(\frac{\cosh^2\delta_1 \cosh^2\delta_5 \cosh^2\delta_p - \sinh^2\delta_1 \sinh^2\delta_5}{\sinh\delta_1 \cosh\delta_1 \sinh\delta_5 \cosh\delta_5} \right.$$

[Valid when $\omega^2 - \lambda^2 \gg \frac{R^2}{(r_+^2 - r_-^2)}$, $\Delta t \gg \frac{R}{\sqrt{\omega^2 - \lambda^2}}$]

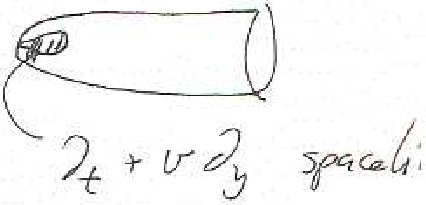
Naive CFT picture:

$\Delta t_{CFT} = \pi R k$

Discrepancy? - redshift in some cases
- generally small when there's a large AdS region
- needs to be understood better.

Ergoregion:

- No everywhere timelike KV



- No superradiance for free fields.

Conclusions

- ∃ non-SUSY solitons in D1-D5 system
- Map to CFT extends to these states

Future directions:

- Classical stability
- Time-dep string backgrounds?
- CFT states in more detail
- Further tests of matching of CFT states to geometries.