Asset Allocation with Annuities for Retirement Income Management

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Agenda

- The retirement income management problem
- A model without annuities
- Adding single premium immediate annuities to the model
- Refinements to the model
- Summary

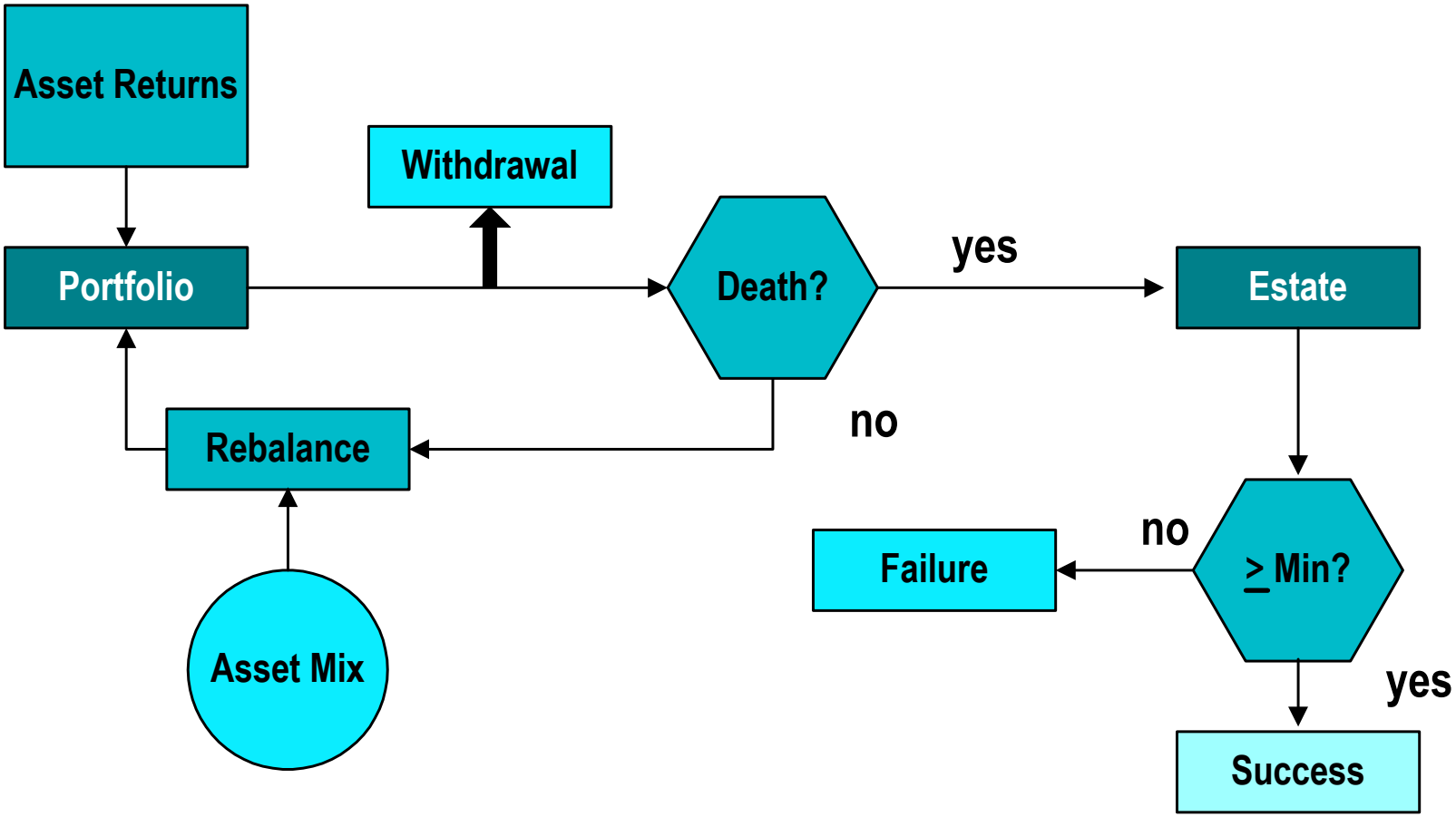
The Retirement Income Management Problem

- Generate a steady stream of real income during retirement from
  - Fixed income sources such as Social Security & DB plans
  - Payments from annuities purchased with retirement savings
  - Withdrawals from remaining invested funds
- If income goals cannot be met with fixed sources and annuities, must not exhaust funds before death
- May want to leave an estate
- Market risk & longevity risk must both be managed

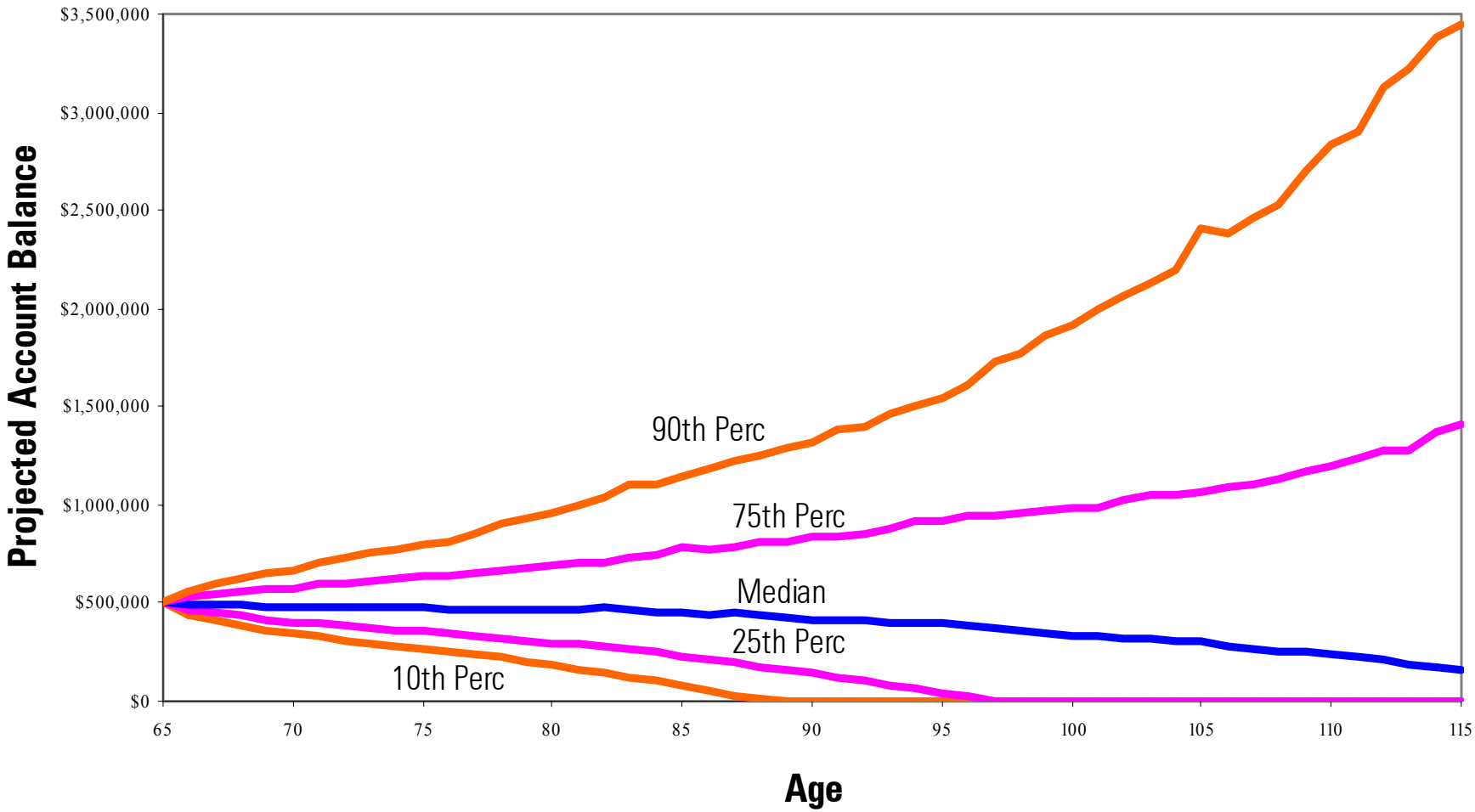
Retirement Income Management Decisions

- How much income to generate
- How much of nest egg to put in annuities
- How to invest remaining nest egg
- How much to leave for heirs

Simulation Using Withdrawal Experiments



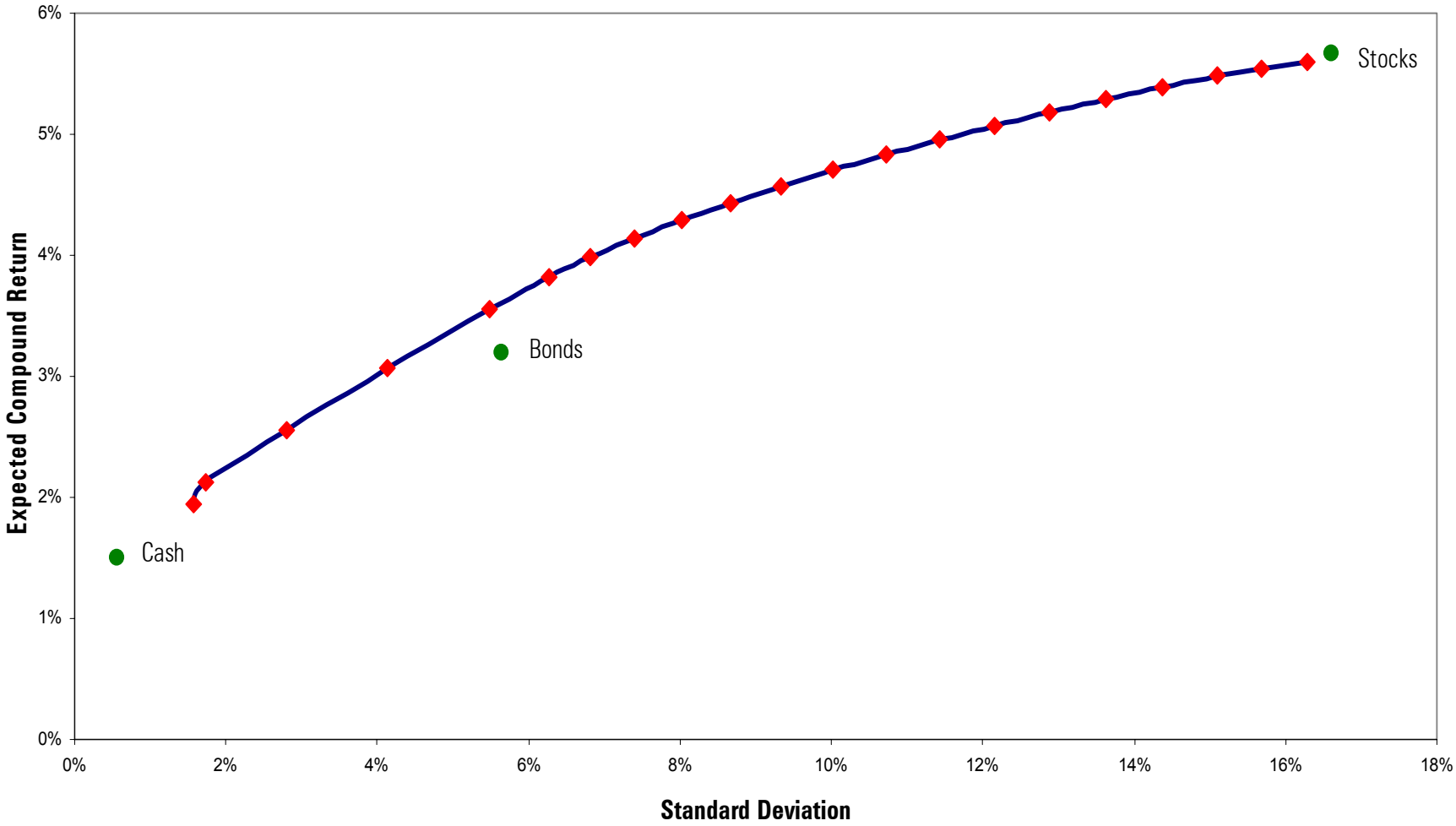
Percentiles of Simulated Balances at Various Ages



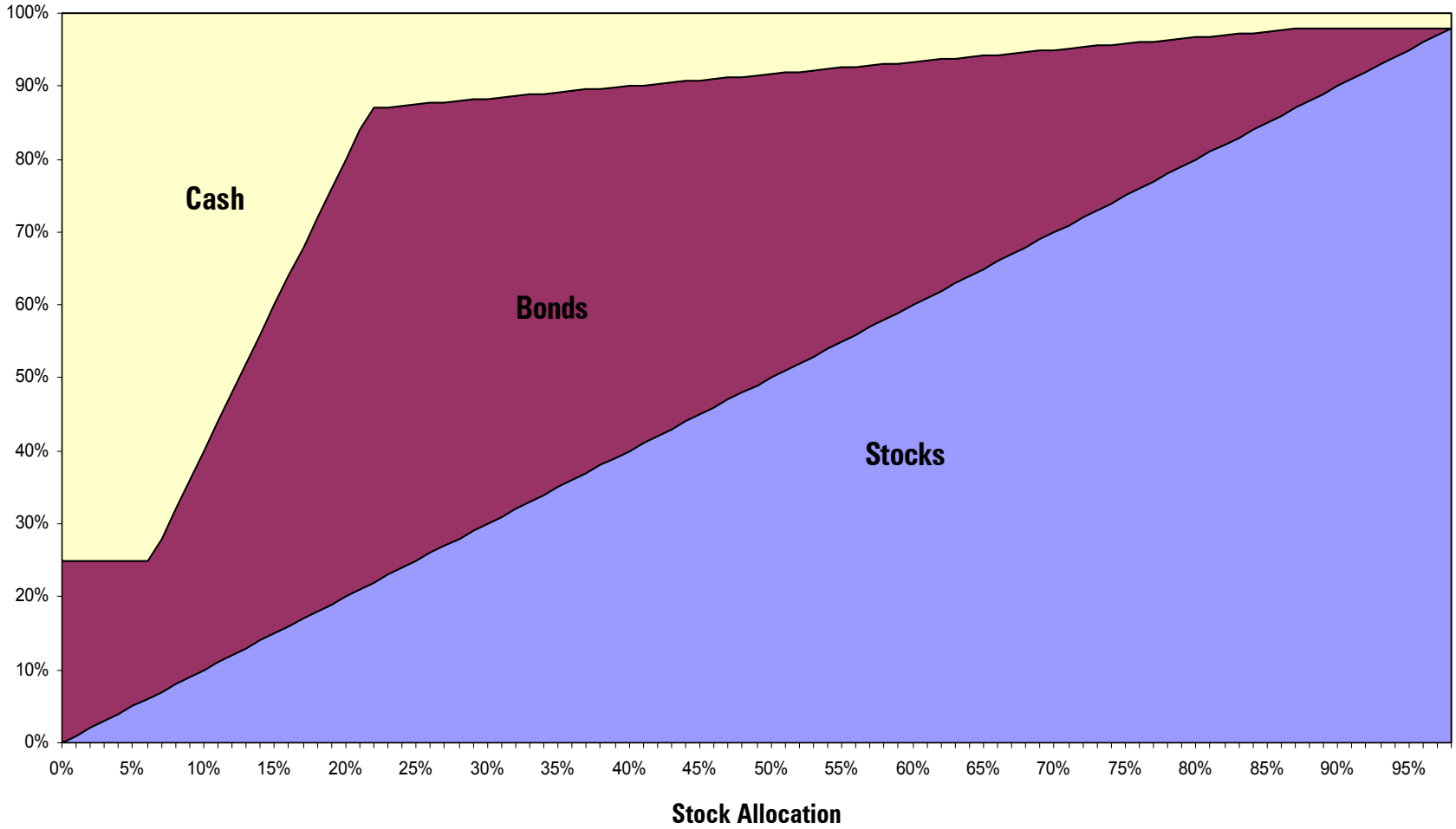
A Model Without Annuities

- Parametric model of real asset returns
- Preset asset mixes
- Constant real income level
- No taxes
- Probabilistic time horizon
- Find relationship between withdrawal rate and success probability

Real Return Assumptions on Asset Classes and Mixes



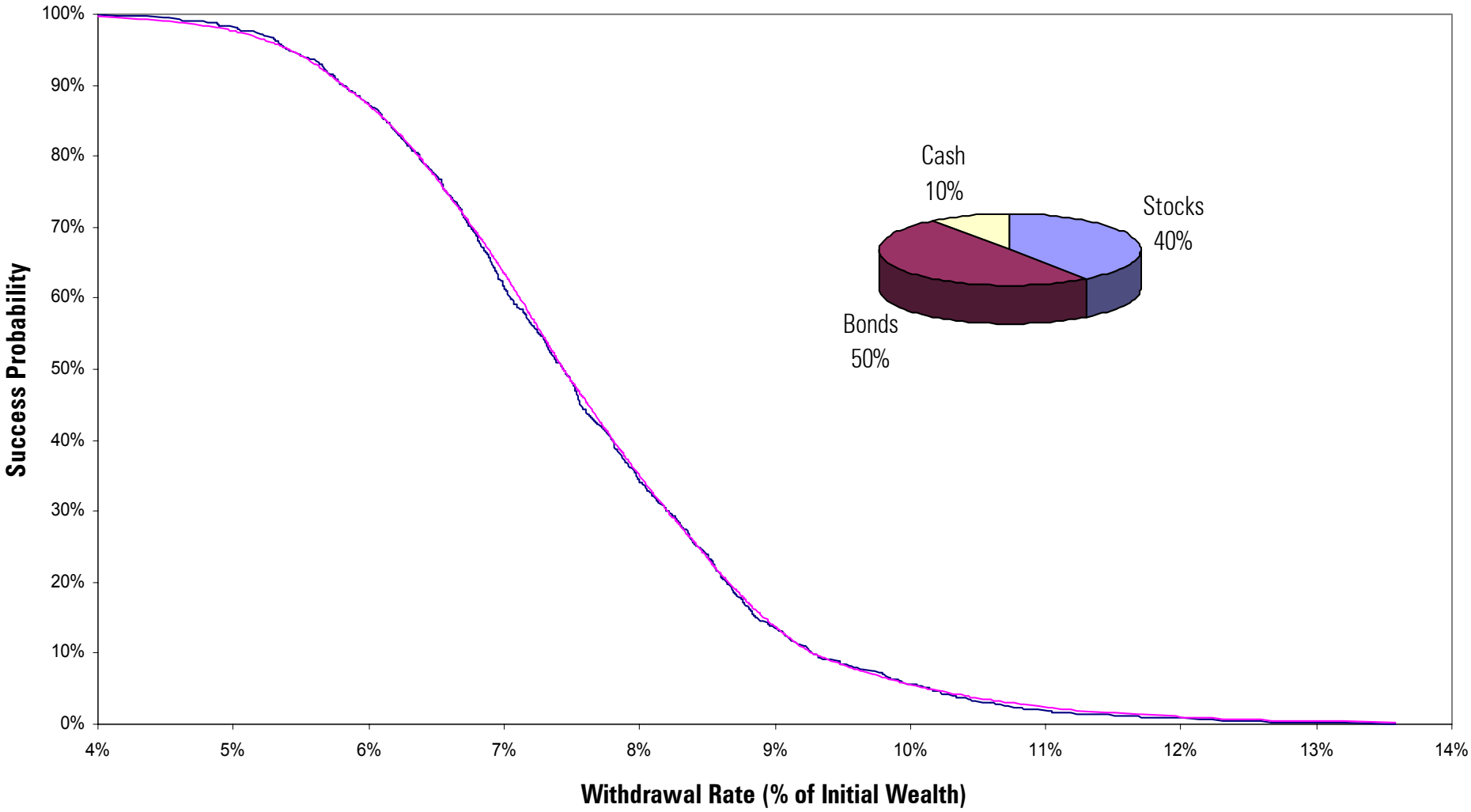
Asset Mixes


100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Cash
Bonds
Stocks
0%
5%
10%
15%
20%
25%
30%
35%
40%
45%
50%
55%
60%
65%
70%
75%
80%
85%
90%
95%
Stock Allocation

Solving the Model for a Given Asset Mix

- 1,000 experiments
- For each experiment at each horizon, solve for withdrawal rate that just works
- At each horizon, sort withdrawal rates across experiments to find relationship to success probability
- Fit smooth curve to withdrawal/probability relationship

Withdrawal Rate vs. Success Probability: 20-Year Horizon



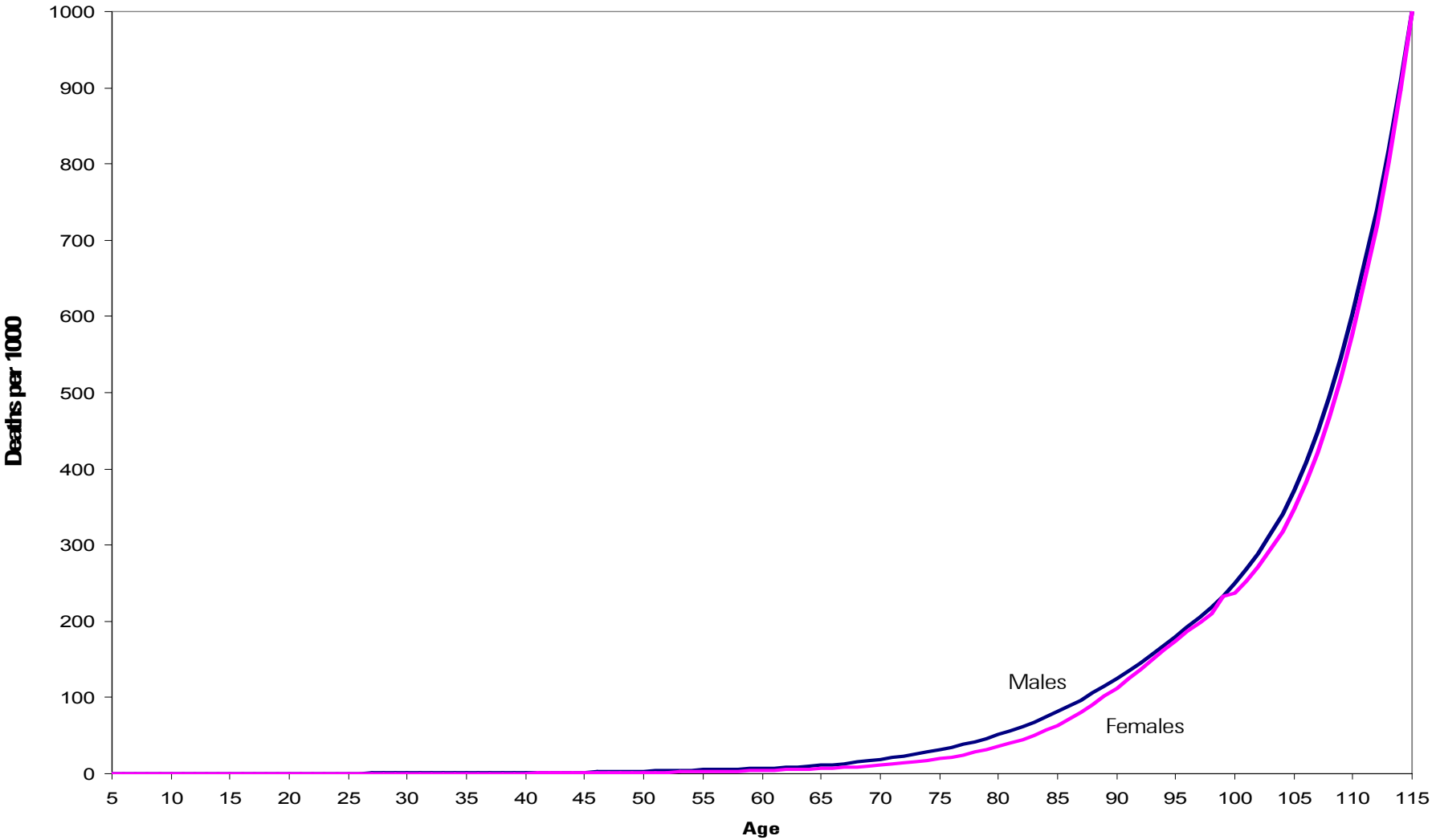
Modeling Uncertain Time Horizon

- Length of retirement period unknown
- Gender-based mortality rates have been estimated and published
- Calculate probability of death for each horizon from mortality rates

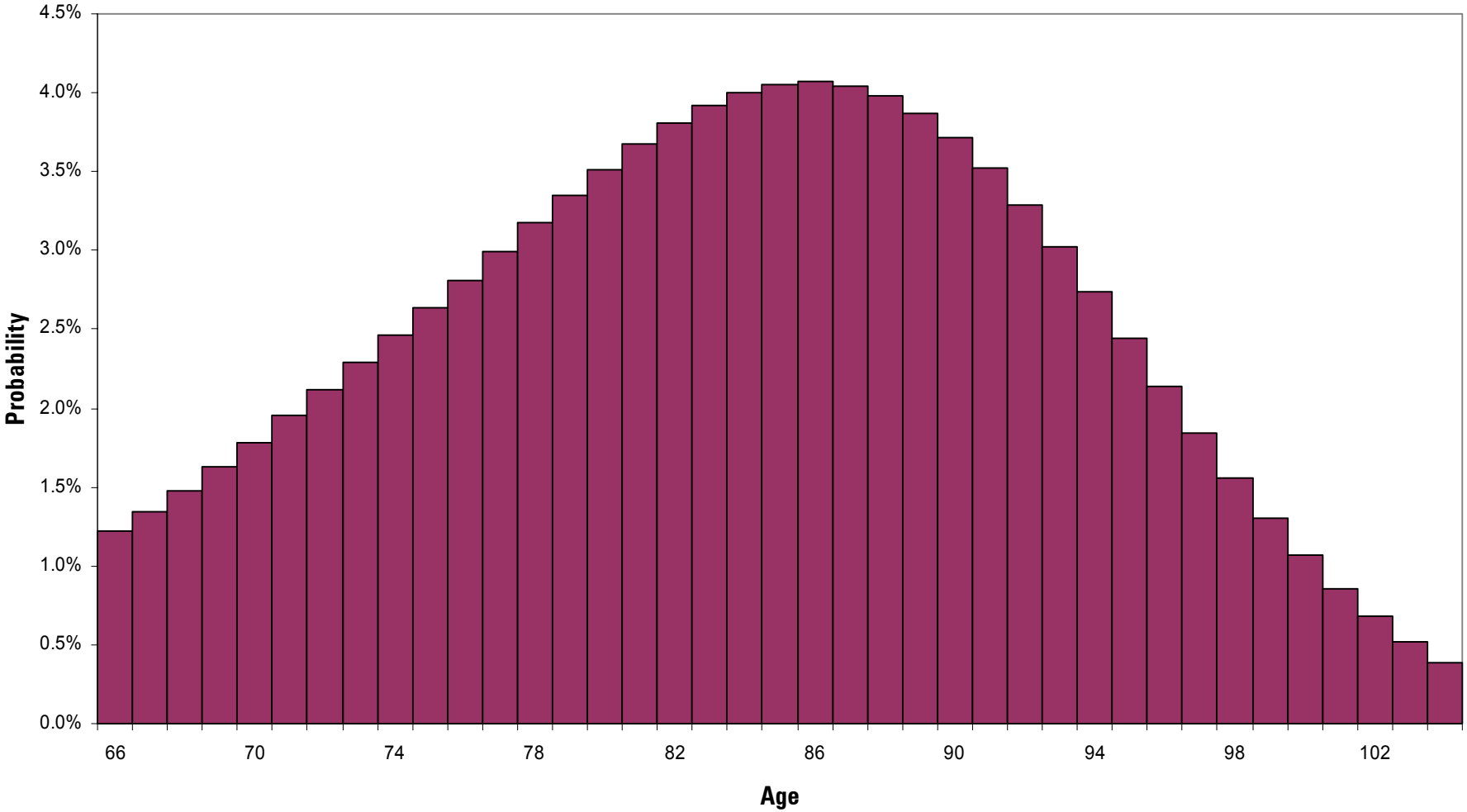
Incorporating Uncertainty of Horizon

- Estimate withdrawal/probability relationship for all horizons
- Calculate probability of death for each horizon
- Weigh success probabilities by death probabilities
- Fit smooth curve to death probability-weighted withdrawal/probability relationship

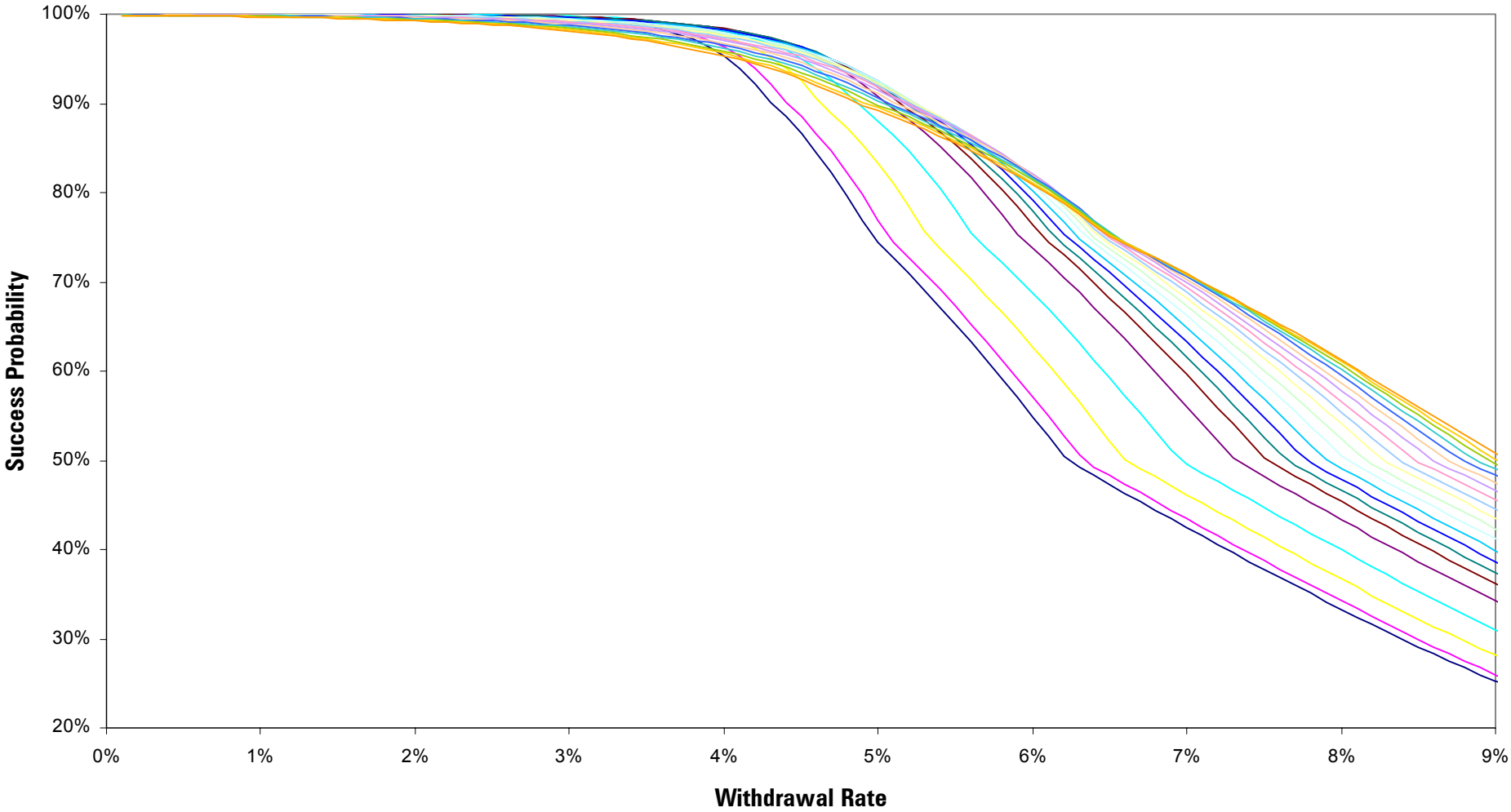
Mortality Rates



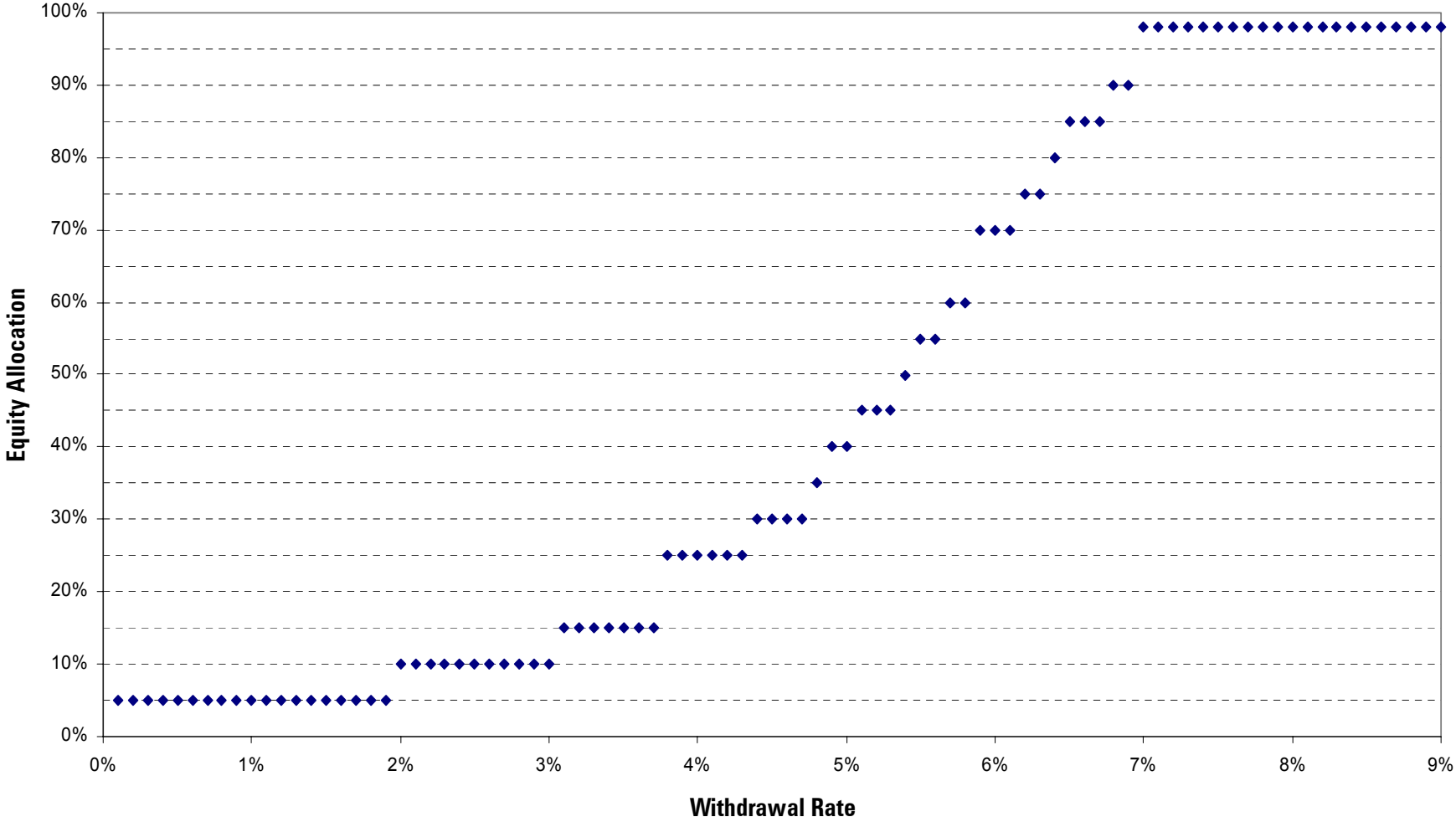
Death Probabilities: 65-Year Old Male



Withdrawal Rate vs. Success Probability: 21 Asset Mixes



Equity Allocations of Mixes with Highest Success Probabilities



Adding Annuities to the Model

- Annuity assumptions
  - Single premium
  - Fixed nominal payout until death
  - No beneficiaries
  - Price based on age & gender
  - Decision irreversible
  - Can only purchase now
- Example: 65-year old male receives $7.50/year for every $100 now

Modeling Inflation

- Need for inflation model
  - Real annuity payout = nominal payout ÷ price index
  - Price index = cumulative value of inflation rate
- Models of inflation rate
  - Serial correlation
  - Mean reverting
  - Inflation surprises correlated with real returns on stocks, bonds, & cash
  - In example, starts at 2.5%, reverts to 2.5%

Three Models of Inflation

- Constant inflation rate

  $\pi_t = \pi_M$

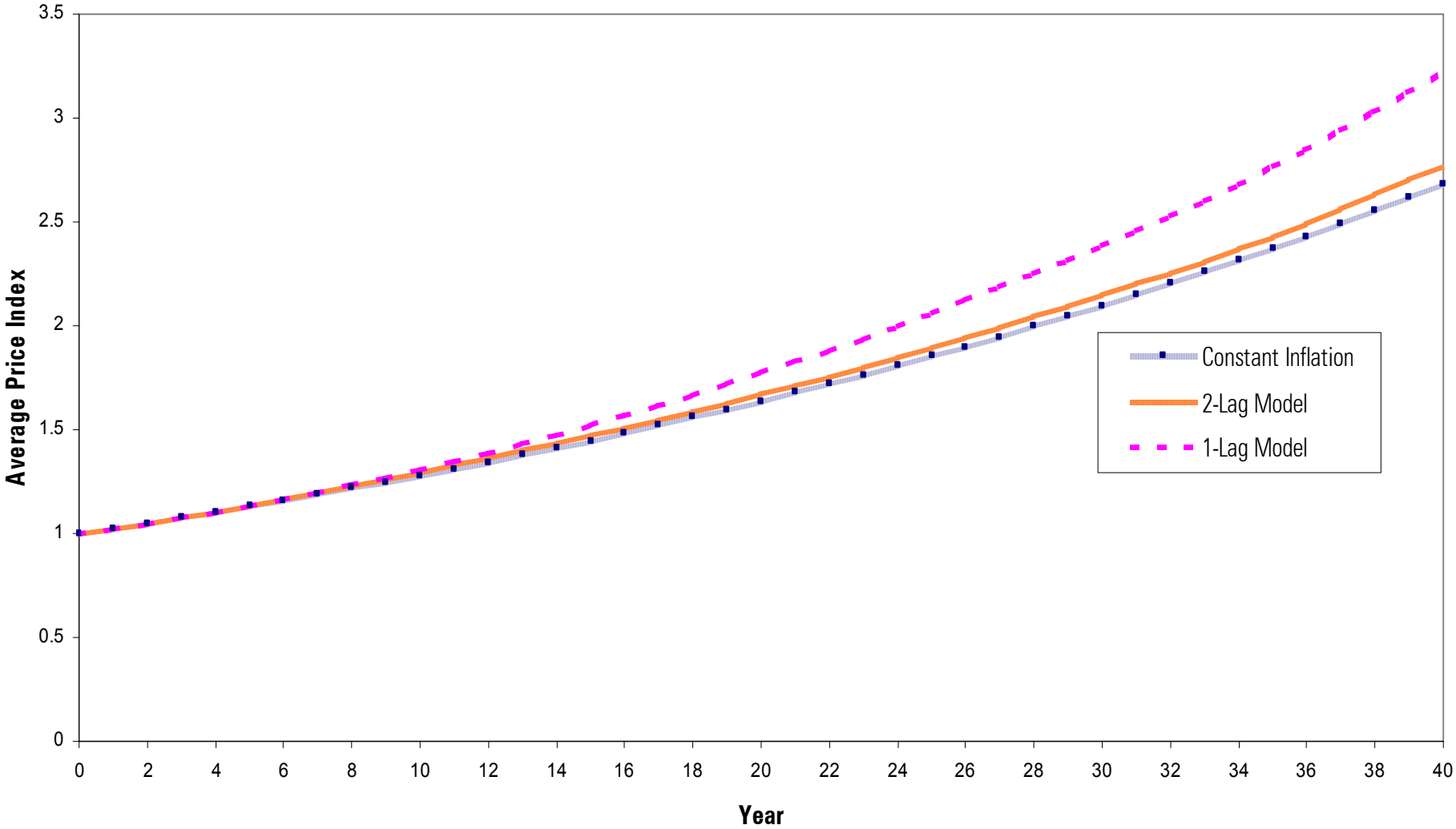
- One-lag autoregressive

  $\pi_t = 0.1145\pi_M + 0.8855\pi_{t-1} + 0.0128z_t$

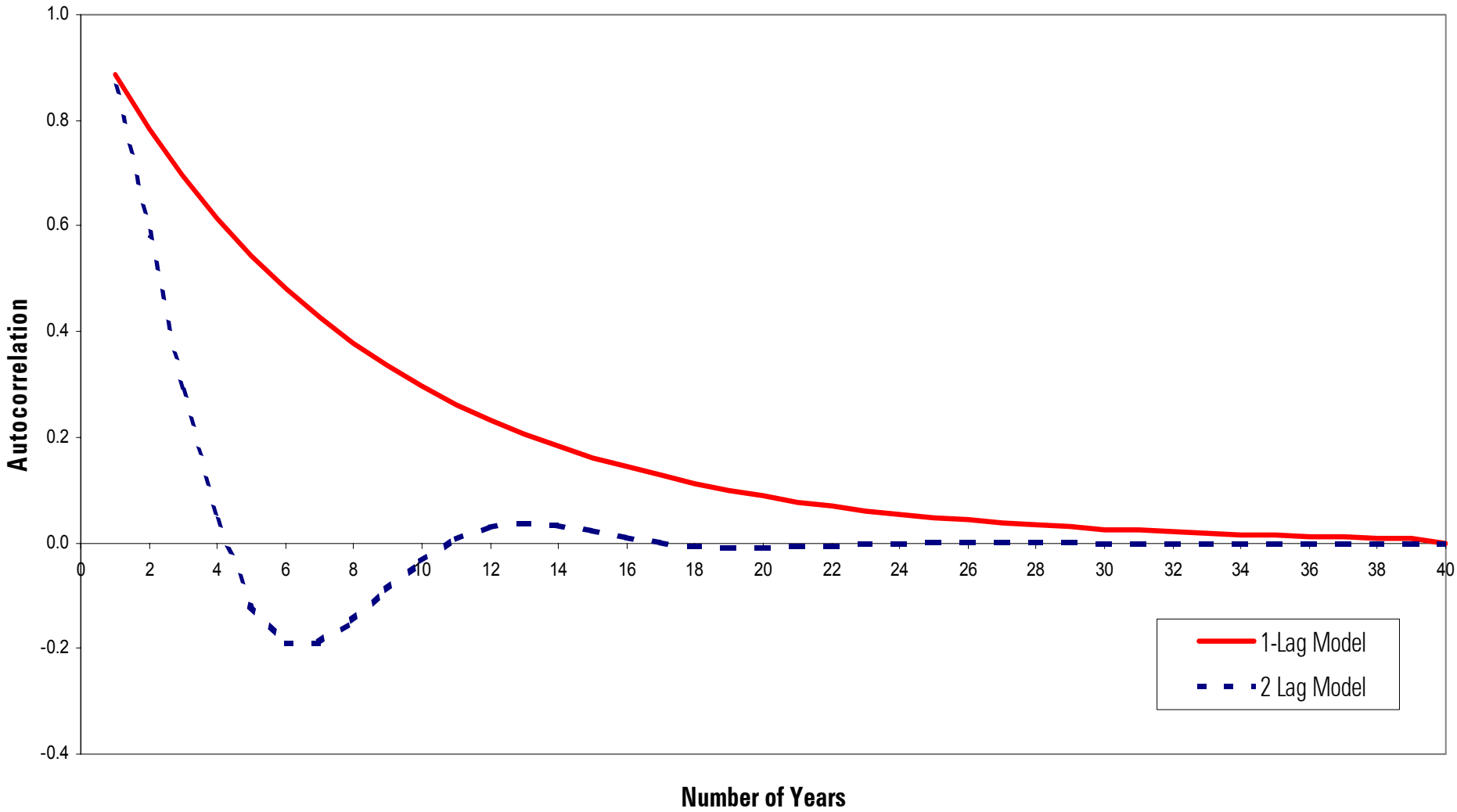
- Two-lag autoregressive

  $\pi_t = 0.2222\pi_M + 1.3876\pi_{t-1} - 0.6098\pi_{t-2} + 0.0086z_t$

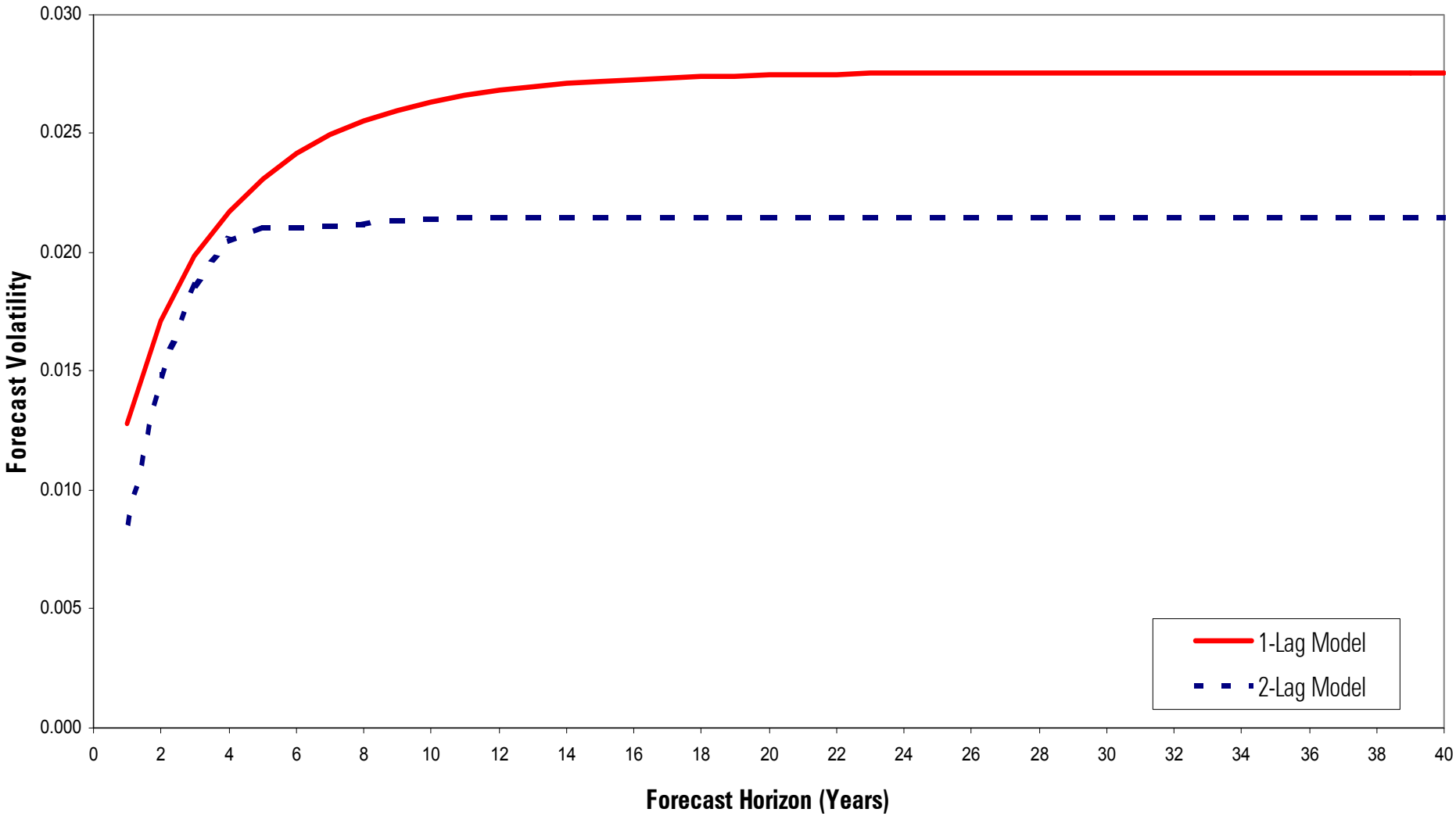
Average Price Indexes with Alternative Inflation Models



Autocorrelation Functions of Alternative Inflation Models



Forecast Volatilities for Alternative Inflation Models

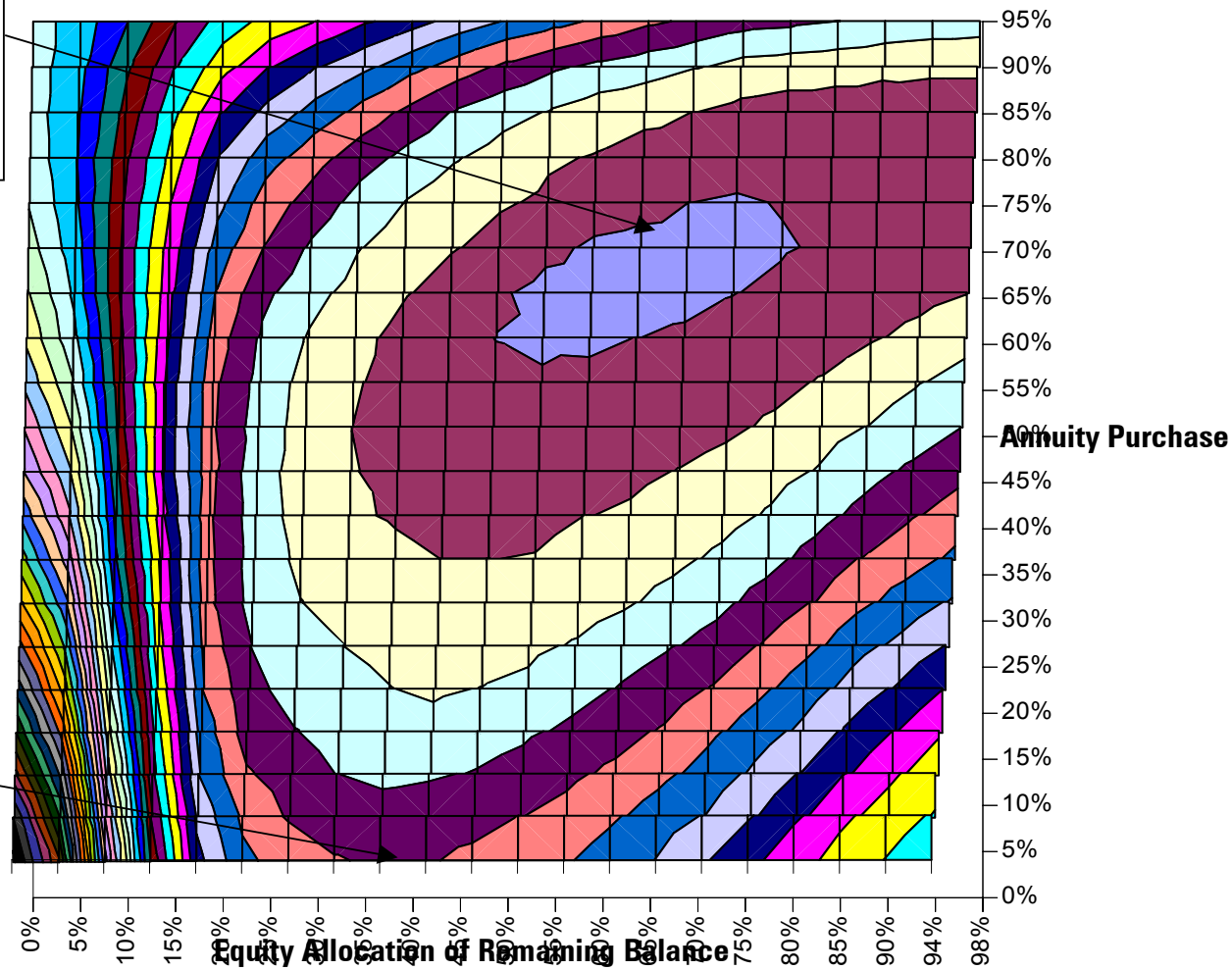


Combining Annuities with Asset Mixes

- 420 combinations
  - 21 asset mixes
  - 20 levels of annuitization (0%, 5%, …, 95%)
- Calculate income/probability relationship for each combination
- For each income level, find combination with highest success probability

Success Probabilities for 5% Income Level*

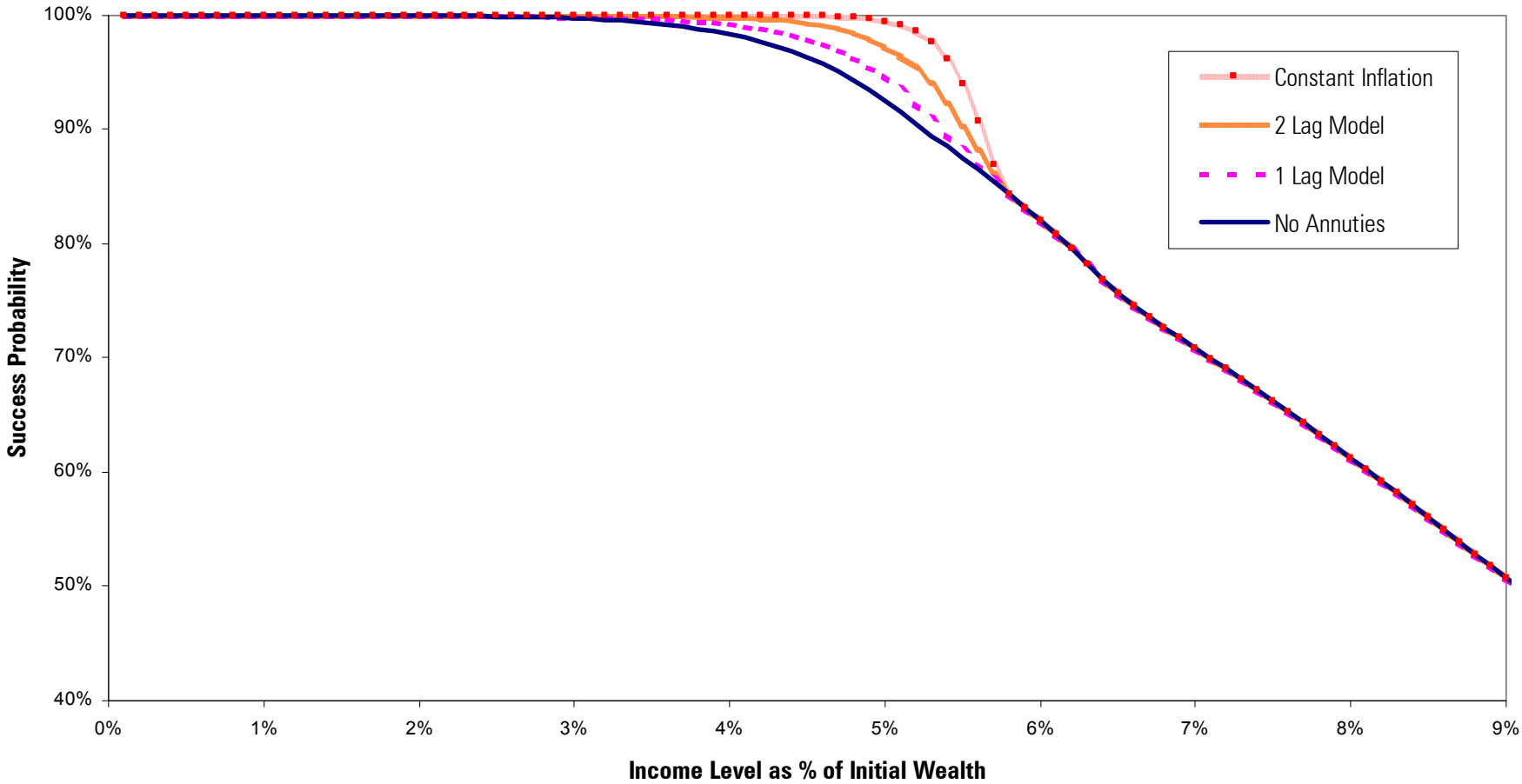
Highest Probability = 94.7%
70% Annuities
70% of remaining balance in equities



Highest Probability without Annuities = 92.6%
40% of balance in equities

*Inflation generated by the one-lag model.

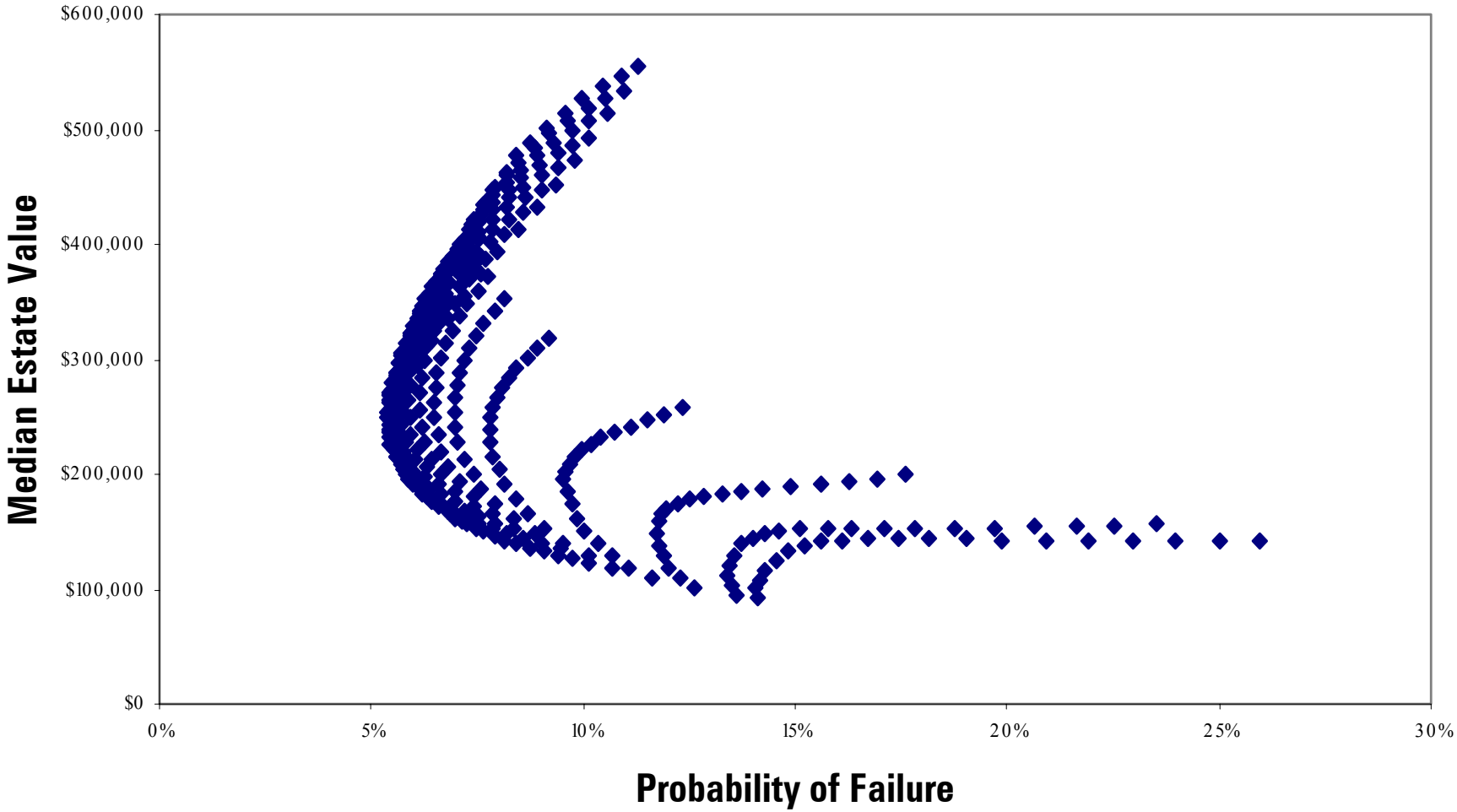
Highest Success Probabilities with & without Annuities, & with Alternative Inflation Models



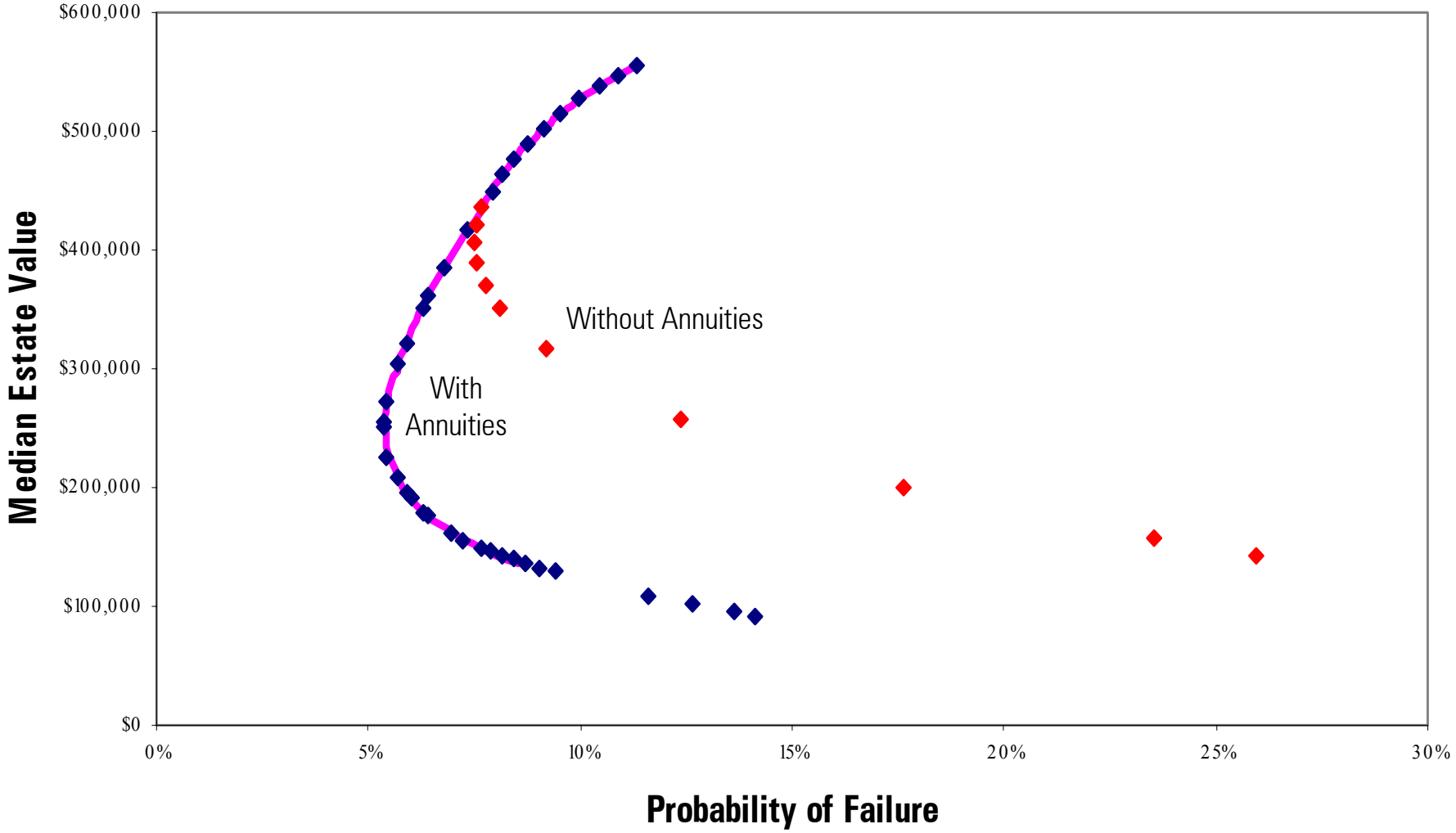
The Estate Dimension

- Annuities may only have a limited impact on success probability
- Annuitization decision largely trade-off between success probability &
  - Availability of funds for additional spending in the future
  - Potential size of estate
- Can measure this dimension with median estate value

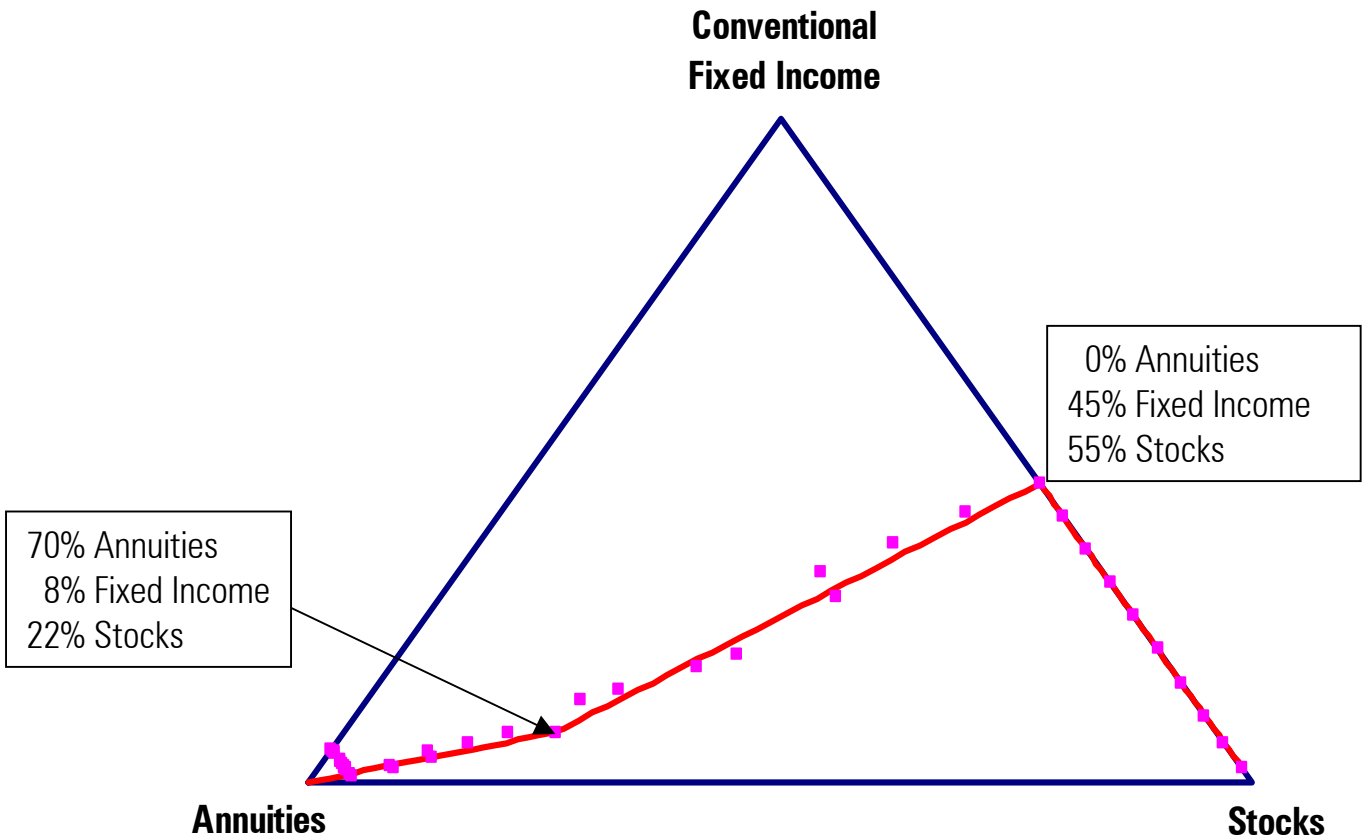
Failure Probability vs. Median Estate Value (5% Income Level & One-Lag Inflation Model)



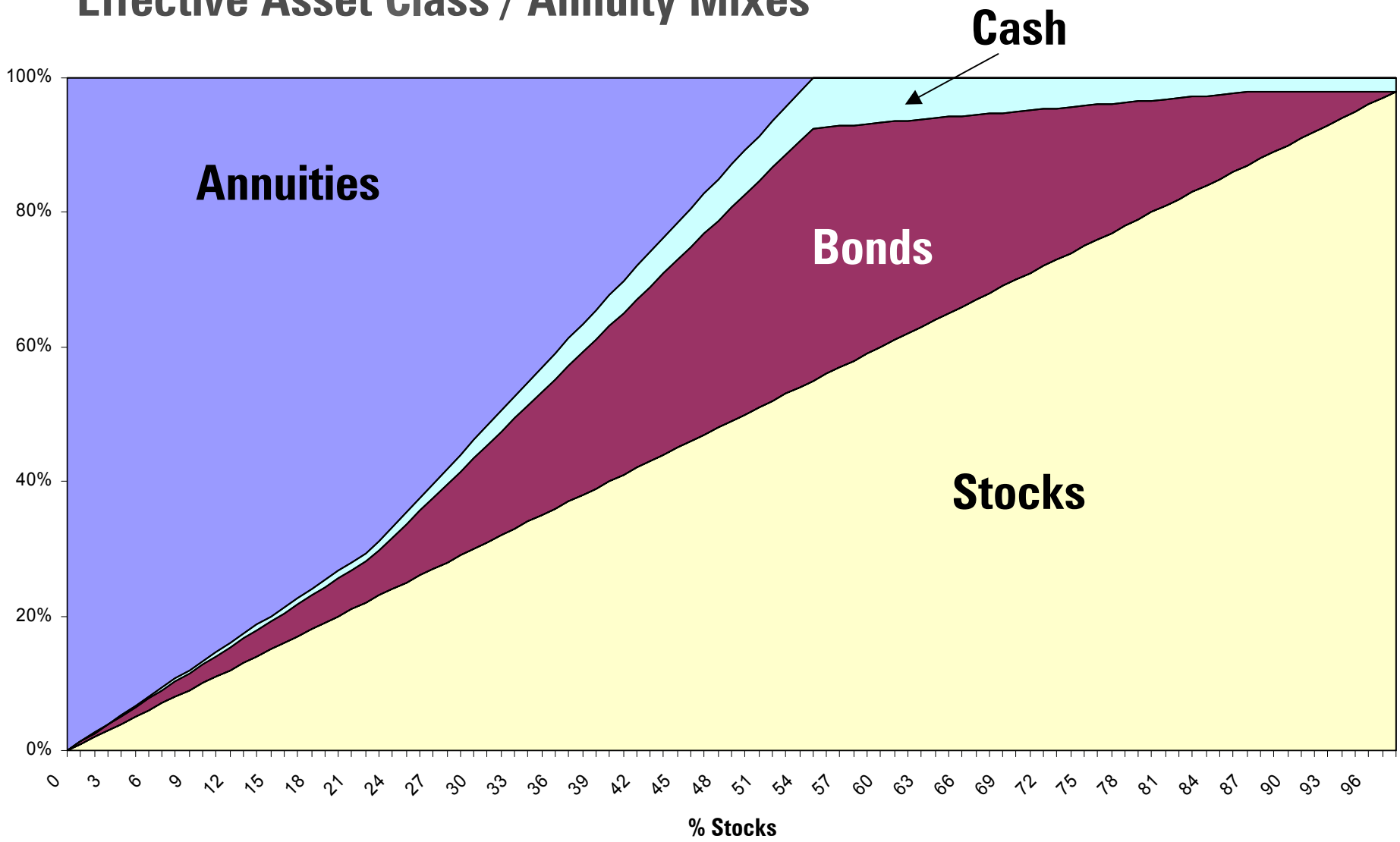
Failure Probability vs. Median Estate Efficient Frontier



Effective Asset Class / Annuity Combinations



Effective Asset Class / Annuity Mixes



Refinements to the Model

- Two-person mortality
- Non-portfolio income
- One-time income & spending events
- Changes in spending level
- Short-term risk aversion
- Required minimum distributions on tax-deferred accounts

Two-Person Mortality

- Portfolio supports two people
- Spending level changes when one person dies
- Estate left when both die
- Model using mortality tables

Non-Portfolio Income

- Social Security
- Defined benefit plans
- Part-time jobs
- Real estate investments

One-Time Income & Spending Events

- One-time income events
  - Downsize home
  - Inheritance
  - Life insurance
- One-time spending events
  - Special vacations
  - Purchase of luxury items
  - Tuition for grandchildren

Changes In Spending Level

- Life style changes

- Medical expenses

Summary

- During retirement, both market & longevity risk must be managed by making choices regarding
  - Income generation
  - Annuitization
  - Investments
  - Estate planning
- Simulation can be used to portray trade-offs between
  - Income level
  - Portfolio volatility
  - Success probability
  - Estate size

Summary

- Since fixed annuities pay a constant nominal amount, we need to model inflation to model their real payouts.
- Choice of inflation model significantly affects results.
- When we consider estate value, there is an efficient trade-off between failure probability & median estate value.
- Points along the efficient frontier have different combinations of equities, conventional fixed income, & fixed annuities.
- The model can be extended to consider two-person mortality, fixed cash flows, short-term risk aversion, & other factors.