# Time-like Linear Dilaton and Open-Closed Duality

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hep-th/0411019 T.T.
hep-th/0503184 Seiji Terashima（Rutgers U.) and T.T.
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# ①Motivation

One of the simplest time-dependent backgrounds in string theory will be a time-dependent linear dilaton background. → Solvable CFT

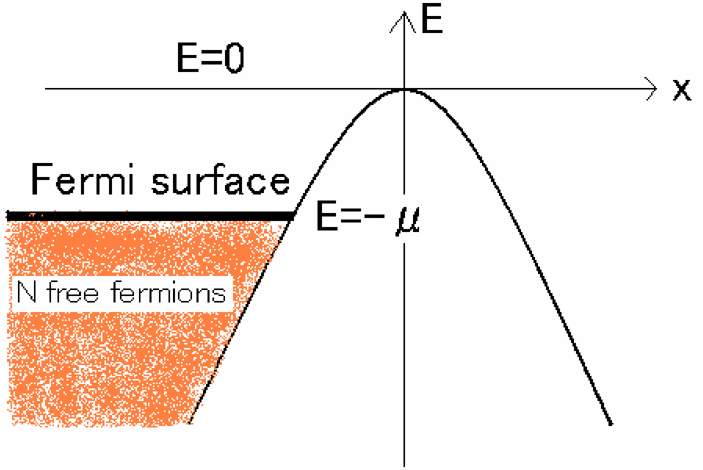
$$g_s = e^{qX^0}.$$

However, it inevitably includes a strongly coupled region, where non-perturbative effects become important.

Thus we need a non-perturbative description of string theory.

For this purpose, a good laboratory will be 2D string theory because it has the matrix model dual as a non-perturbative formulation via the open-closed duality.

2D string theory
= Infinitely many fermions
in inverse harmonic potential
(i.e. c=1 matrix model)




In this context, such an example is given by the c<1 string whose matter part is the time-like linear dilaton CFT.

cf. Minimal c<1 string

## ② c<1 String with Time-like Linear Dilaton Matter

Definition on the world-sheet

Time (matter) $X^0$ : time-like linear dilaton CFT

$c_m = 1 - 6q^2$

Space (Liouville) $\phi$ : space-like linear dilaton CFT

$c_L = 1 + 6Q^2$

String coupling: $g_s = e^{qX^0 + Q\phi}$

Liouville term : $\mu \int dz^2 \, e^{2b\phi}$

$c_m + c_L = 26 \rightarrow Q = b + \frac{1}{b}, \quad q = \frac{1}{b} - b \qquad (0 < b \leq 1)$

# Description as a background in 2D string

We can regard the c<1 string as a time-dependent background in the c=1 string, i.e. 2D string theory.

To see this, we consider the Lorentz boost

$$\tilde{X}^0 = \frac{Q}{2} X^0 + \frac{q}{2}\phi \ , \quad \tilde{\phi} = \frac{q}{2} X^0 + \frac{Q}{2}\phi \ .$$

Then we find the static string coupling $g_s = e^{2\tilde{\phi}}$.

But, the Liouville term becomes time-dependent

$$\mu \int dz^2 \, e^{(b^2-1)\tilde{X}^0 + (1+b^2)\tilde{\phi}} .$$

↑

Rolling closed string tachyon

In this way, we have obtained a series of time-dependent backgrounds in 2D string.

This theory is already solvable on the world-sheet.

The next task is to construct its matrix model dual.

# ③Matrix Model Dual and Holography

## (3-1) Direct Construction

Remember the open-closed duality in 2D string

c=1 matrix model = open string theory of
N unstable D0-branes

[Mcgreevy-Verlinde, Klebanov-Maldacena-Seiberg, Sen,…]

Roughly, we can identify the N × N Hermitian matrix $\Phi$ with the open-string tachyon field T on the D0-branes.

⟶ The double scaled c=1 matrix model action

$$S_{c=1} = \int dt\, \mathrm{Tr}[(D_0\Phi)^2 + \Phi^2]$$

In our c<1 string, consider N unstable D0-branes again.

$$\text{Boundary State}: \ |D0\rangle = |Neumann\rangle_{X^0} \otimes |ZZ\rangle_{\phi}$$

As in the c=1 string case, the naïve guess leads to

$$S_{c<1} = \int dt \, e^{-qt} \, \mathrm{Tr}[(D_0\Phi)^2 + \Phi^2]$$

↑

(string coupling)$^{-1}$ on the D-branes $g_s = e^{qt}$ [TT]

However, one may be puzzled because open string field theory actions generally include many complicated interaction terms.

Clearly, the same issue also arises in the c=1 case.

Actually, we can check that such a drastic simplification of open string theory indeed occurs in 2D string theory by computing the open string tachyon scatterings.

On-shell vertex: $V_+ = e^{X^0/b}, \qquad V_- = e^{-bX^0}$

Actually, the momentum conservation is very restrictive.

For the amplitudes like $\left\langle (V_+)^{N_+} (V_-)^{N_-} \right\rangle$ ,
we find the following constraint

$$-N_- b + \frac{N_+}{b} = \left( \frac{1}{b} - b \right) \chi \quad .$$

Then it turns out that all non-trivial S-matrices are vanishing when $b^2$ is irrational because

$$N_+ - \chi = (N_- - \chi)b^2 \quad \to \quad N_\pm = \chi \leq 1 \ .$$

Continuity w.r.t. $b^2$ ➡ This should be true for any b.

Thus we find that the action is the free quadratic one as we argued.

⟶ c=1 matrix model action should also be quadratic.

b=1 limit

Note: when $b^2=p/q$ (rational), we have another c<1 string, i.e. (p,q) minimal string. In this case, it is natural to have higher order interactions in open string theory.

To interpret the matrix model itself in terms of open strings, we need an open string field theory.

For this purpose, the convenient one will be the Background Independent or Boundary SFT [Witten, Gerasimov-Shatashvili].
Kutasov-Marino-Moore

We argue that BSFT for 2D string is equivalent to the c=1 matrix model up to a smooth field redefinition (at least classically).

$$S_{c=1}(\Phi) \cong S_{BSFT}(T)$$ [TT-Terashima]

Note: This shows that the c=1 matrix model after the double scaling limit is equivalent to open string theory.

# (3-2) Description in c=1 Matrix Model

On the other hand, the c<1 string can be regarded as a time-dependent background in 2D string theory.

Generally speaking, a time-dependent background in 2D string corresponds to a time-dependent Fermi surface in c=1 matrix model. [Polchinski, Minic-Polchinski-Yang,… Alexandrov-Kazakov-Kostov, Karczmarek-Strominger,…]

Note: Indeed we can show that the previous c<1 matrix model action is equivalent to c=1 matrix model via a field redefinition.

# Matrix Model Dual of c<1 String

In the end, we find that the matrix model dual is given by the following Fermi surface in the c=1 matrix model

$$(-p-x)^{b^2}(p-x)=\mu\, e^{(b^2-1)t}$$ [TT]

1) At b=1, we reproduce the familiar static vacuum of c=1 string $H=p^2-x^2=-\mu$ .

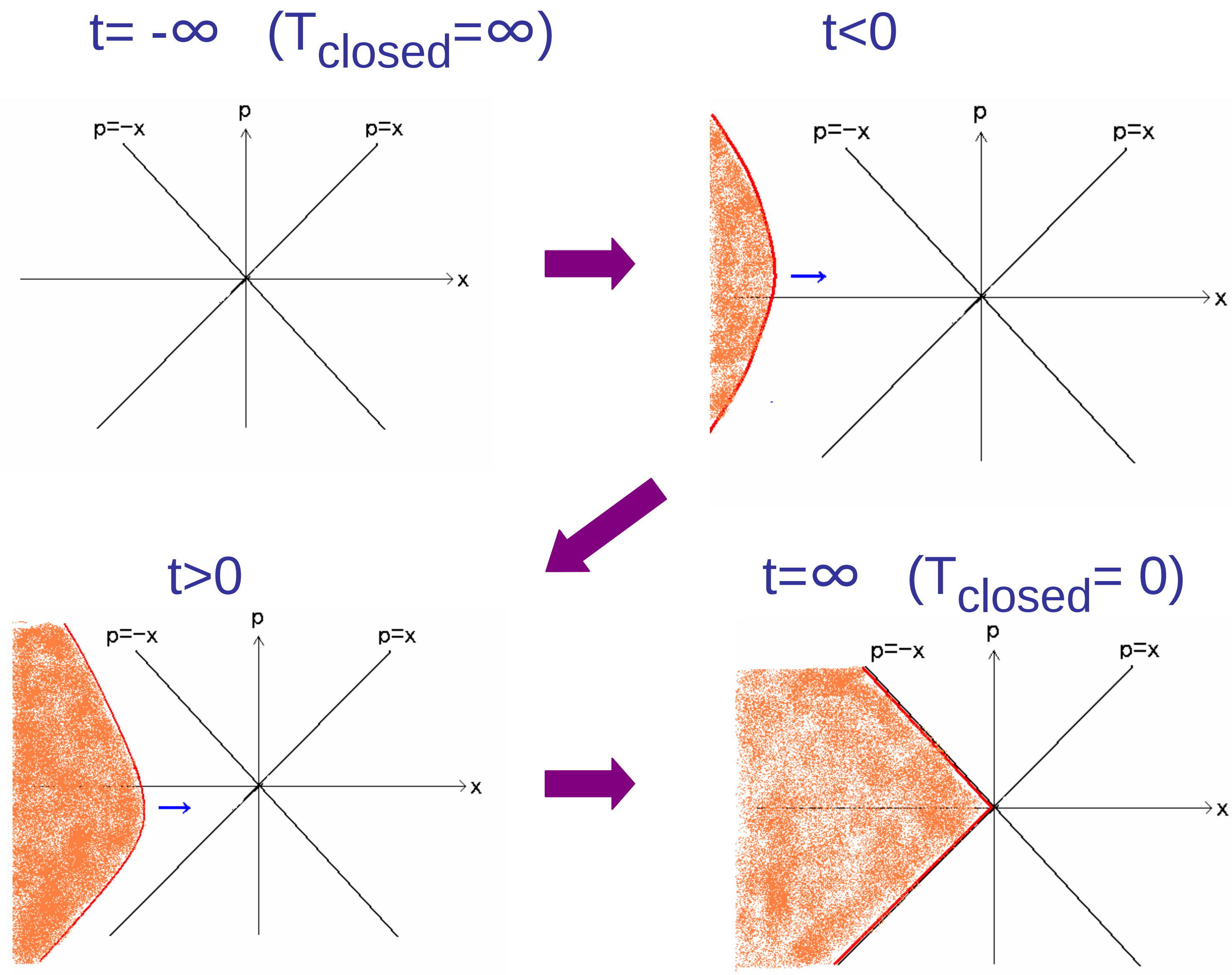
2) For $b^2<0$, the world-sheet theory becomes the time-like Liouville theory: $\mu\int dz^2\, e^{2|b|X^0}$ .

[Da Cunha-Martinec, Strominger-TT, Schomerus]

3) It is straightforward to extend this result to 2D type 0 string case. ⟶ a non-perturbative completion

[Douglas-Klebanov-Kutasov-Maldacena-Martinec-Seiberg, TT-Toumbas]

# Time Evoluion of Fermi Surface



# Comparisons with the World-sheet Computations

## (i) Closed String Tachyon Field

The deformed Fermi surface reproduces the asymptotic behavior of the closed string tachyon field correctly.

$$T_{closed} = \mu\, e^{(b^2-1)\tilde{X}^0 + (1+b^2)\tilde{\phi}} + \text{dual potential.}$$

## (ii) S-Matrices

We can find the S-matrix for the tachyon waves $\eta$ on the Fermi surface [b=1: Polchinski, Moore-Plesser]

$$\partial_+\eta(y) = \sum_{n=1}^{\infty} \frac{(b\mu^{-1/b^2})^{n-1}}{2^{n-1}\cdot n!} \cdot \frac{\Gamma(-b^{-1}\partial_y + 1)}{\Gamma(-b^{-1}\partial_y + 2 - n)} \cdot \left( e^{(n-1)qy} (\partial_-\eta(y))^n \right).$$

$y = t - \phi$ in-coming out-going $y = t + \phi$

This agrees with world-sheet computations.

# ④ Some Applications

## (4-1) Topological String

Is there any topological string model equivalent to our c<1 string as in the c=1 case? ➡ Yes.

$$c = 1 - 6(n-1)^2/n \ \text{String at the radius } R = \sqrt{\alpha' n}$$

$$\cong \ \text{Twisted } N=2 \left( \frac{SL(2,R)_{2+n}}{U(1)} \right) \Big/ Z_n \ \text{Model}$$

2D Black Hole

[TT]

Note: In the particular case n=1, i.e. c=1 string at the self-dual radius , this reproduces the well-known result [Muhki-Vafa].

See also [Aharony-Ganor-Sonnenschein-Yankielowicz-Sochen, Ooguri-Vafa,…] for related discussions.

## (4-2) N=2 String (N=4 Topological String)

It is possible to consider a similar model in N=2 string.

2 + 2 dim.

This defines a $\hat{c} < 1$ non-critical N=2 string.

$$(X^0, X^1) \qquad + \qquad (\phi, Y)$$

$N=2$ Timelike linear dilaton $\qquad$ $N=2$ Liouville Theory

$$\hat{c} = 1 - Q^2 \qquad\qquad \hat{c} = 1 + Q^2$$

$$\text{String coupling}: \quad g_s = e^{Q(X^0 + \phi)}$$

$$N=2 \ \text{Liouville term}: \quad \mu \int \mathrm{d}\theta^2 \, e^{\Phi/Q} + h.c.$$

Finally, we gauge the N=2 Virasoro algebra ($T, G^+, G^-, J$).

The physical state is a massless scalar field plus some possible discrete states. Its vertex operator is given by

$$V = e^{-\phi_L^1 - \phi_L^2 - \phi_R^1 - \phi_R^2} \; e^{(Q+iE_0)X^0 + iE_1 X^1 + (Q+ik_2)\phi + ik_3 Y}$$

The N=2 Virasoro constraint requires $L_0=1$ and $J_0=0$.
The extra condition $J_0=Q(E_1-k_3)=0$ requires $E_1=k_3$ .

Then the physical spacetime is reduced from 2+2 to 3 dimensional (=1 time+ 1 space +1 null).

The next task will be to find its scattering amplitudes.

We can compute the three particle scattering amplitudes, owing to the recent progresses on three point functions in N=2 Liouville theory. [Giveon-Kutasov, Hosomichi]

We find the amplitudes are actually vanishing as opposed to the bosonic and type0 case. This might suggest that the $\hat{c} < 1$ $N = 2$ string is a free theory except the reflection.

Note: When we add a N=2 time-like Liouville term, the amplitudes are non-vanishing. Very roughly, it looks like a square of those in the N=0 or N=1 case.
This model is closely related to the N=2 minimal string.
Also the chiral-primary states ~ (1,p) minimal bosonic string

For recent discussions on N=2 string on ALE (or N=2 minimal string), see [Aharony-Fiol-Kutasov-Sahakyan, Konechny-Parnachev-Sahakyan].

# ⑤ Conclusions

- 2D toy model of a time-dependent linear dilaton background → Non-perturbative formulation
  (e.g. The leg-factor does not change.)

- Open string field theory on D0-branes in 2D string
  → The double scaled matrix model

- It is known that 2D bosonic (N=0) and type 0 (N=1) string have similar perturbative properties.
  But, N=2 string case seems to be different.
  → Is there any matrix model dual or open-closed duality in N=2 non-critical string??