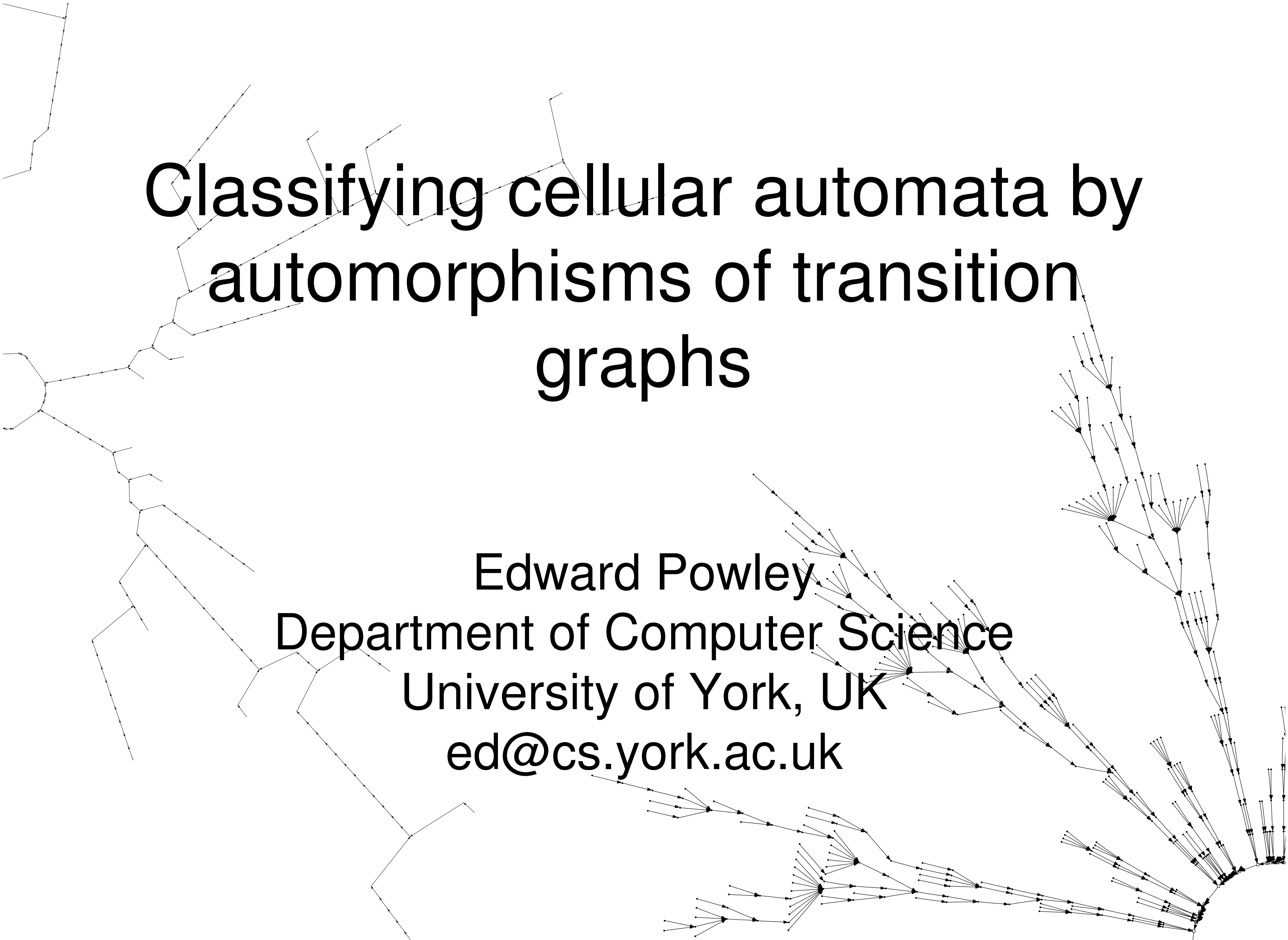

Classifying cellular automata by automorphisms of transition graphs

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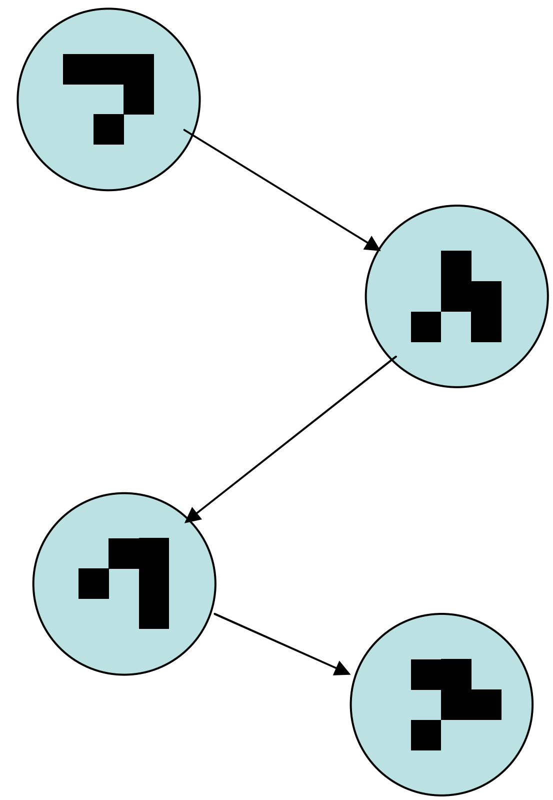
Outline

- Transition graphs and automorphisms
- Symmetries of CAs
- Symmetries and automorphisms are in one-to-one correspondence
- Numbers of syms/auts vs dynamics (Wolfram classes or similar) of CAs
  - “Eyeballing” ECAs
  - Correlation with Langton’s λ parameter

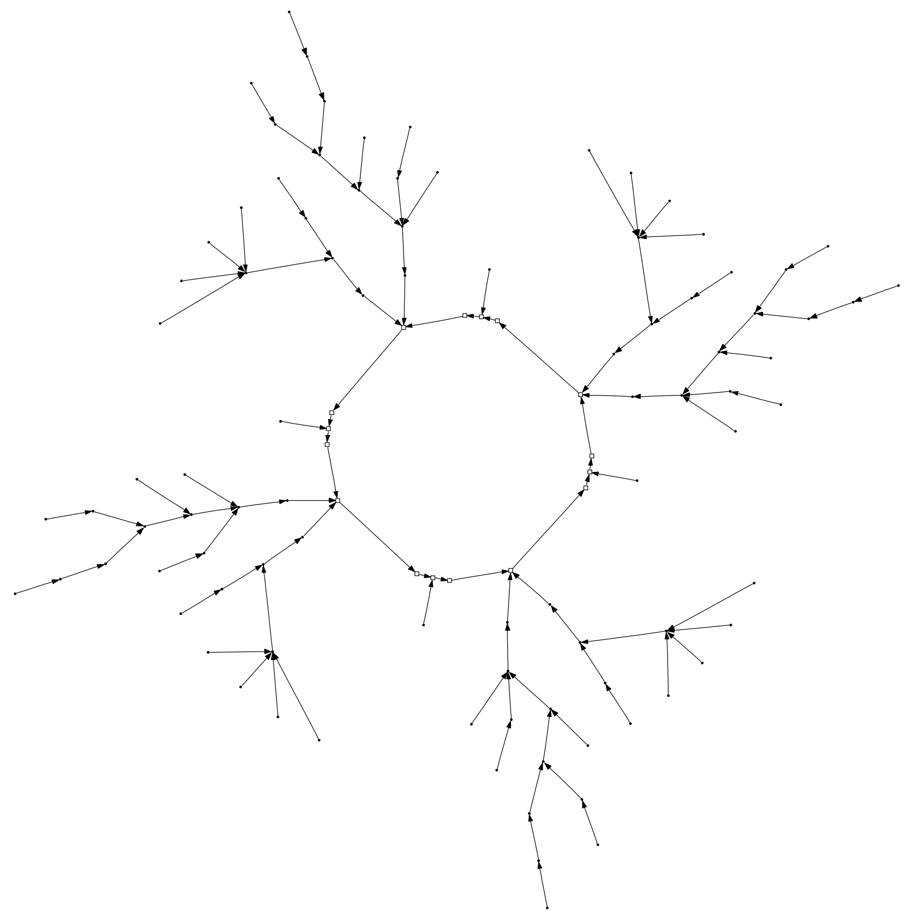
Transition graphs

- A graphical representation of configuration space
- Nodes represent configurations
- Edges represent transitions between configurations

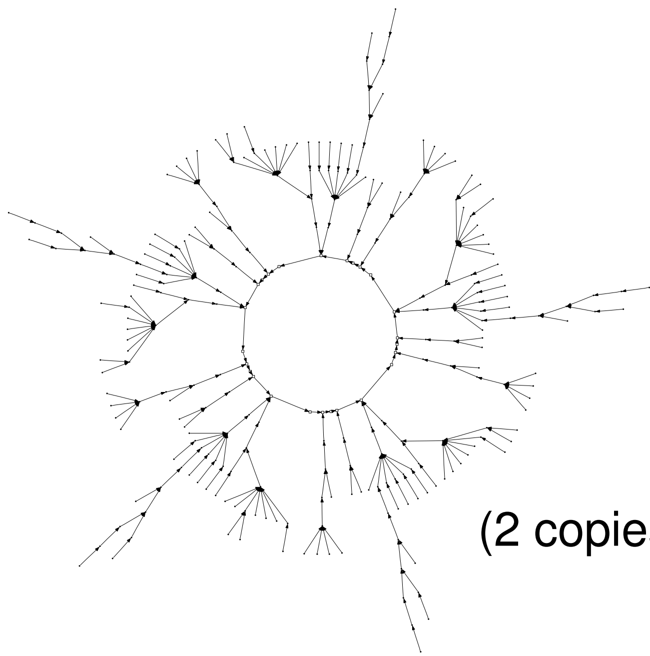


Topology of transition graphs

- Every node has out-degree 1
- “Circles of trees”



Example: rule 110, lattice size 10



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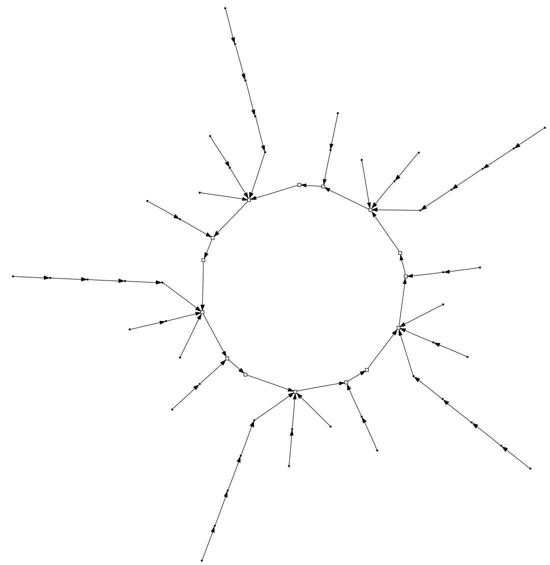

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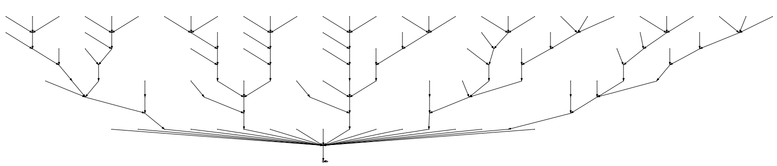

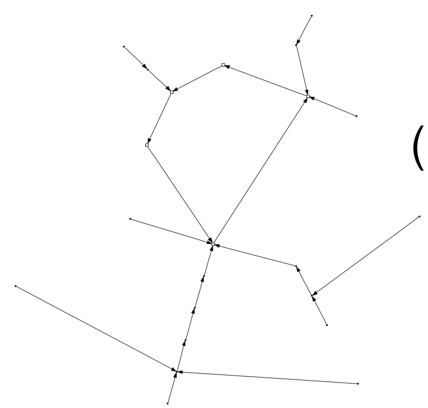

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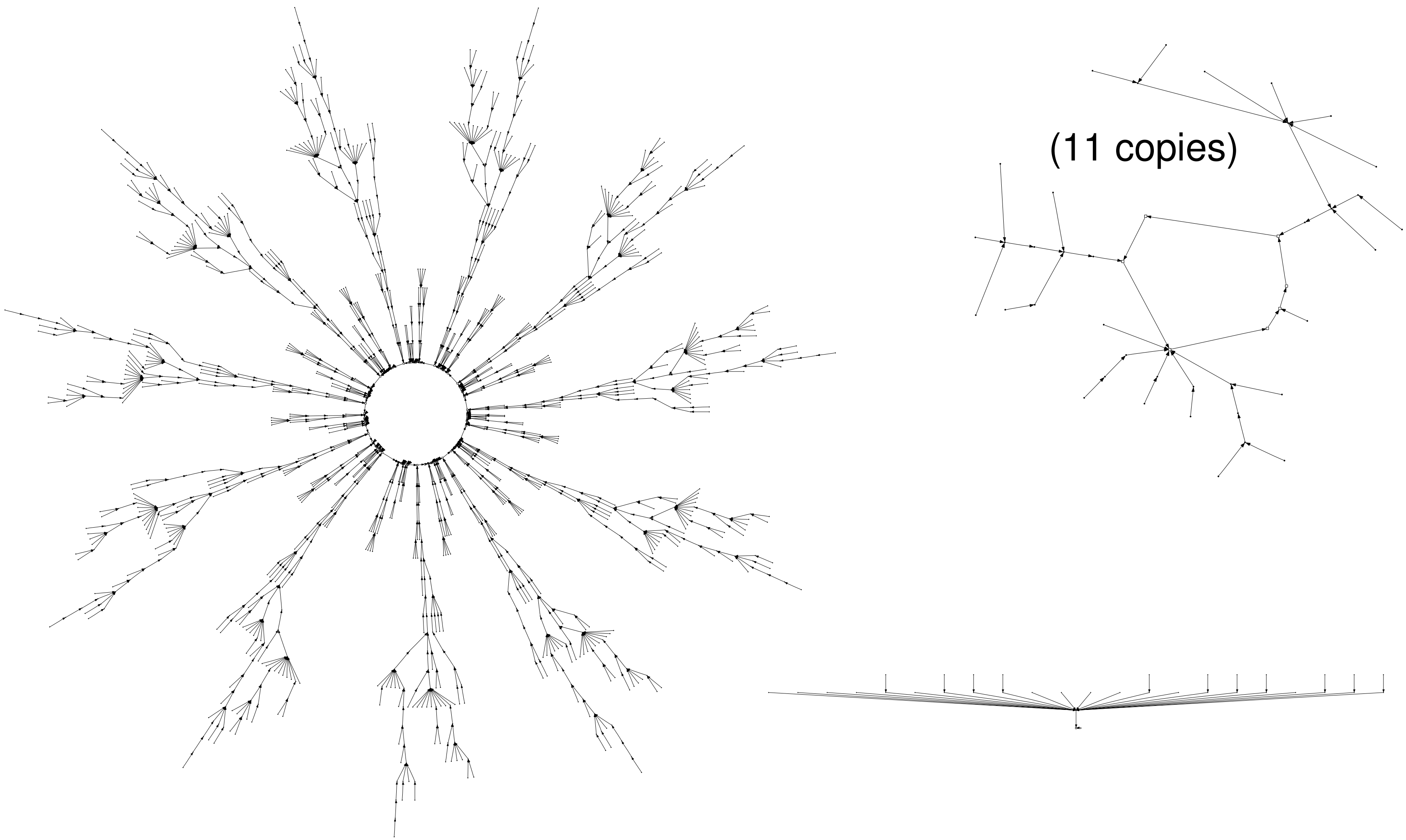
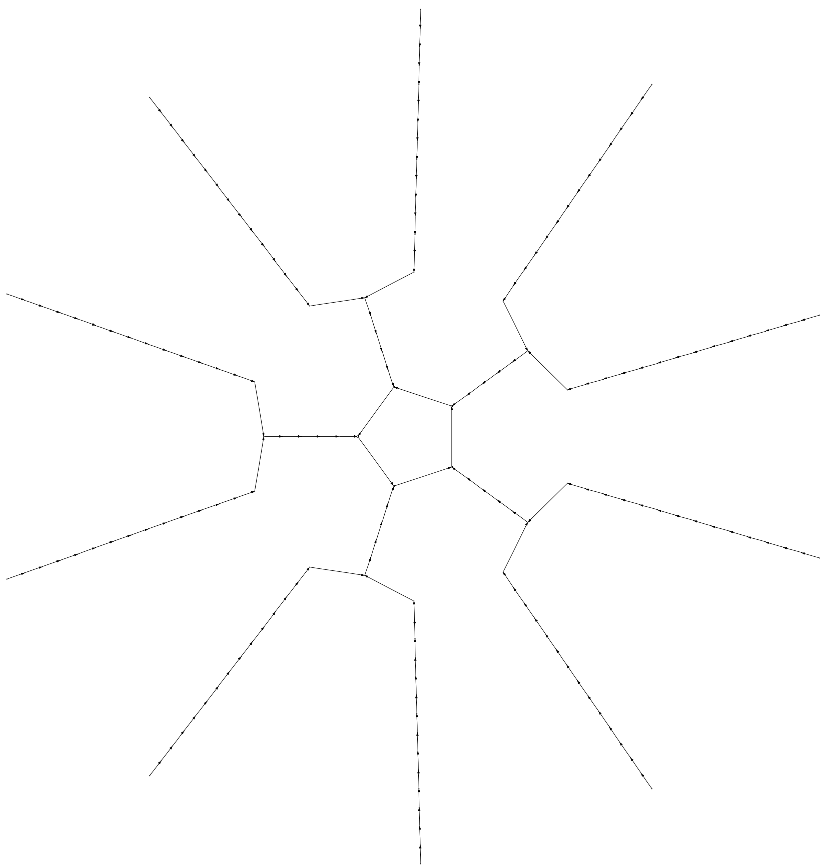
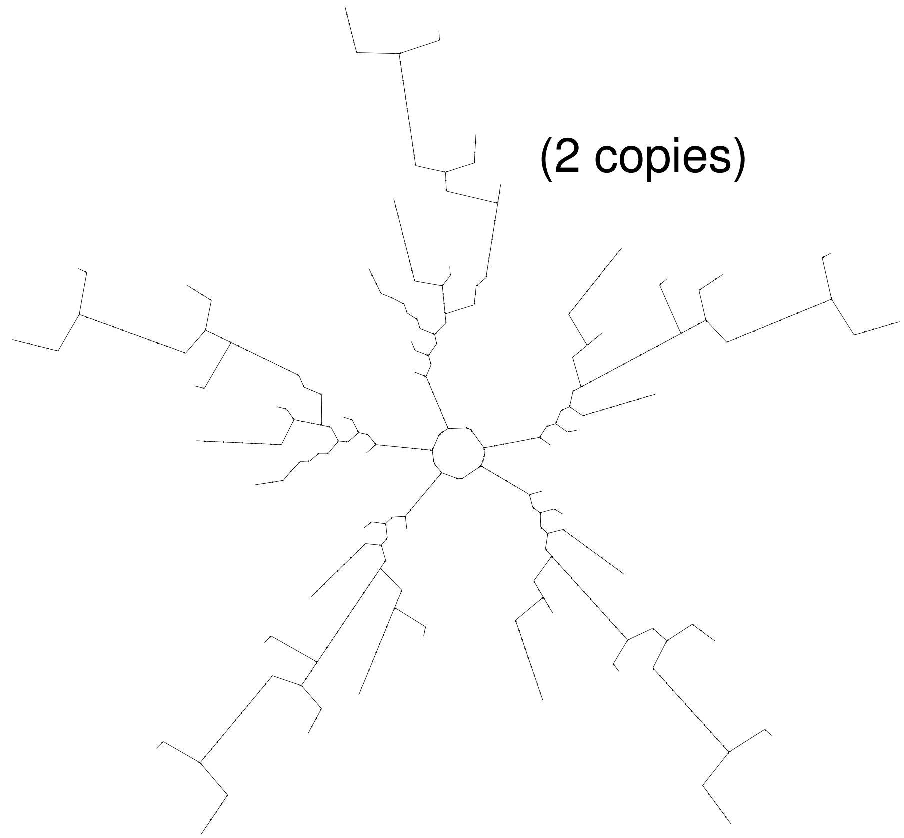
Example: rule 110, lattice size 11



Example: rule 30, lattice size 10





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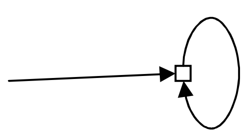

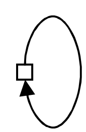

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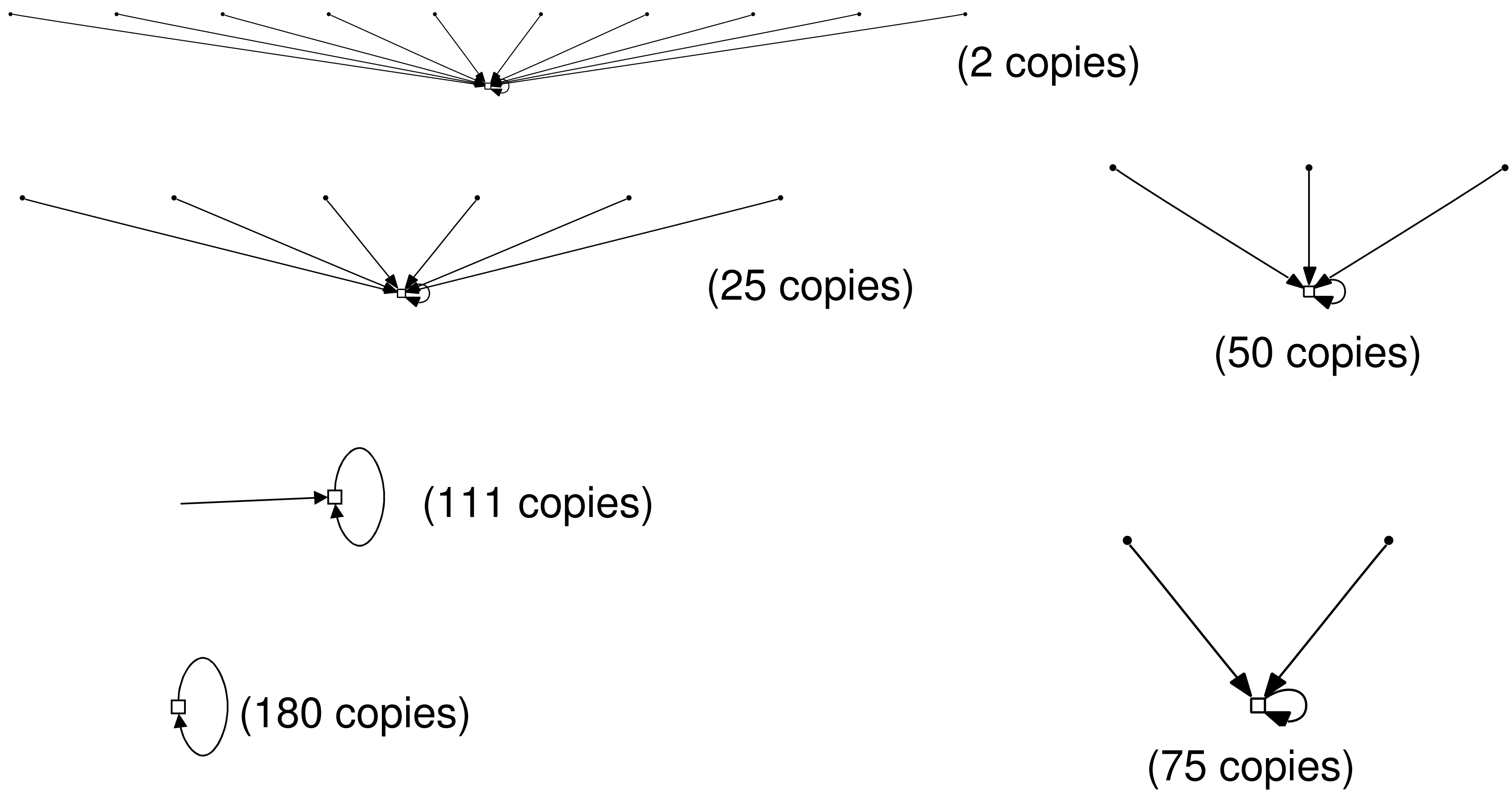
Example: rule 76, lattice size 10



Observation

- There seems to be a lot of symmetry in these graphs…

Automorphisms

- An **automorphism** is an isomorphism of a graph onto itself
- Edge-preserving permutation of the nodes
- “Symmetry” of the graph

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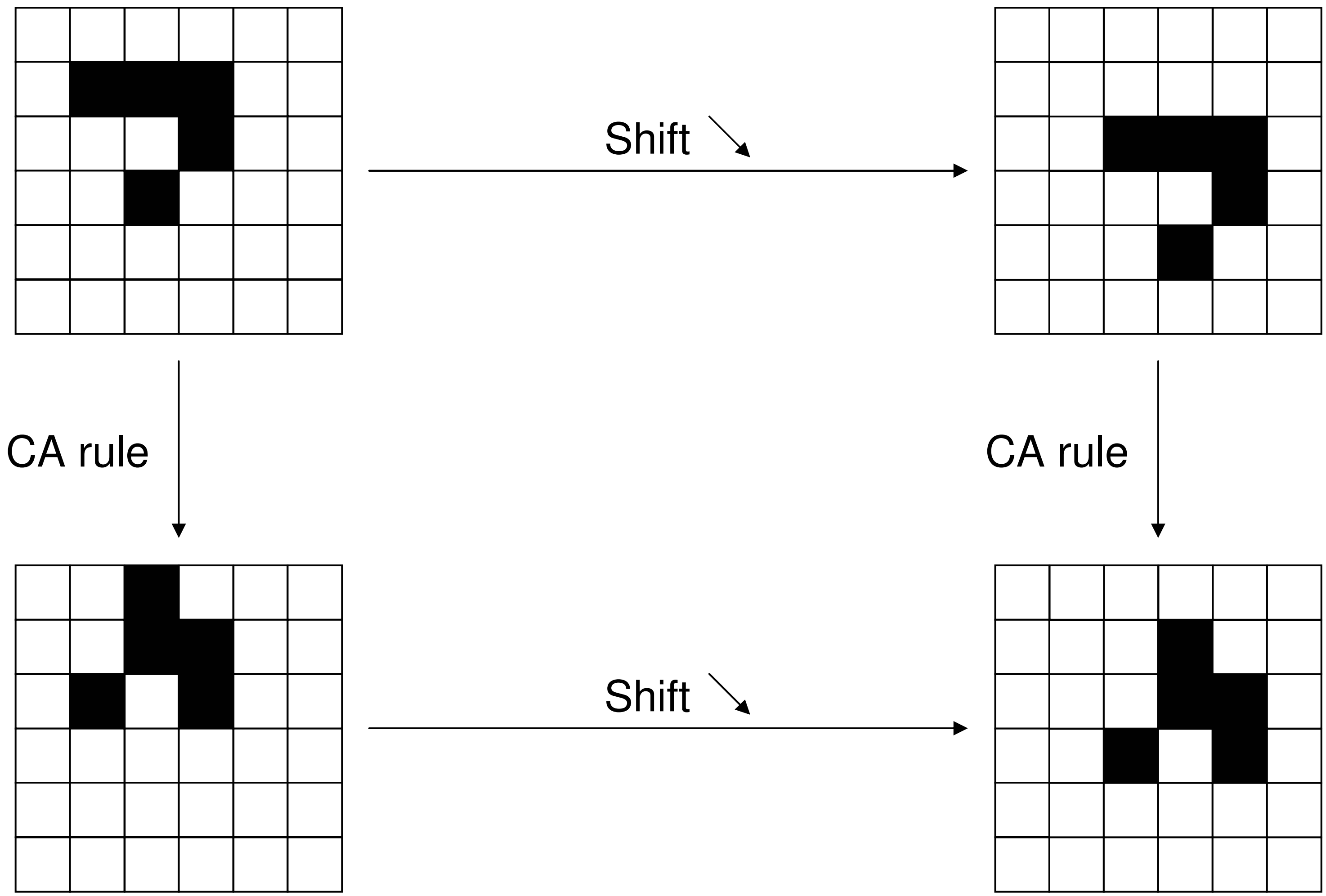
- Numbers of syms/auts vs dynamics (complexity classes) of CAs
  - ECAs
  - Correlation with Langton's λ parameter

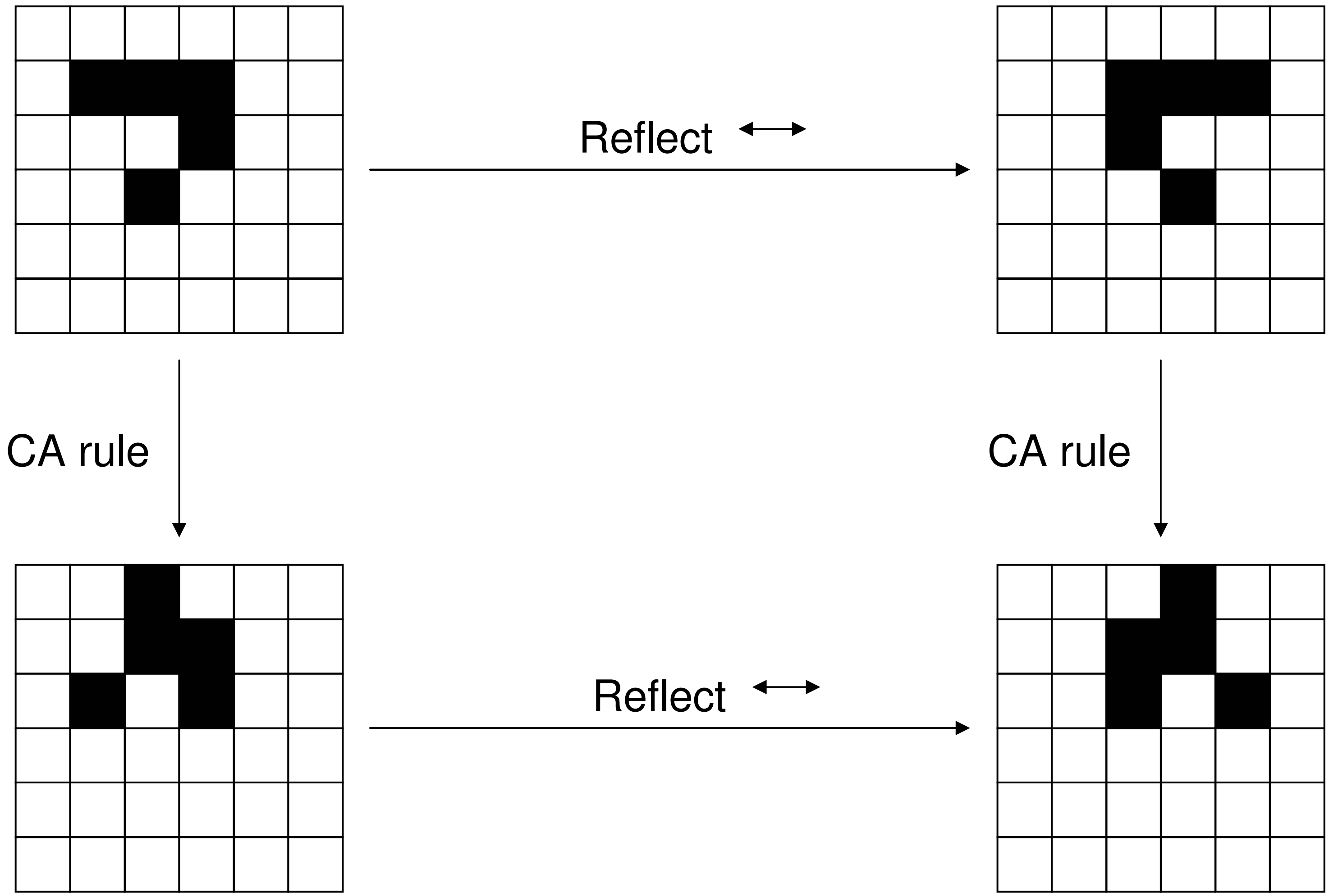
Definition

- A **symmetry** of a CA is a bijection which commutes with the global update rule

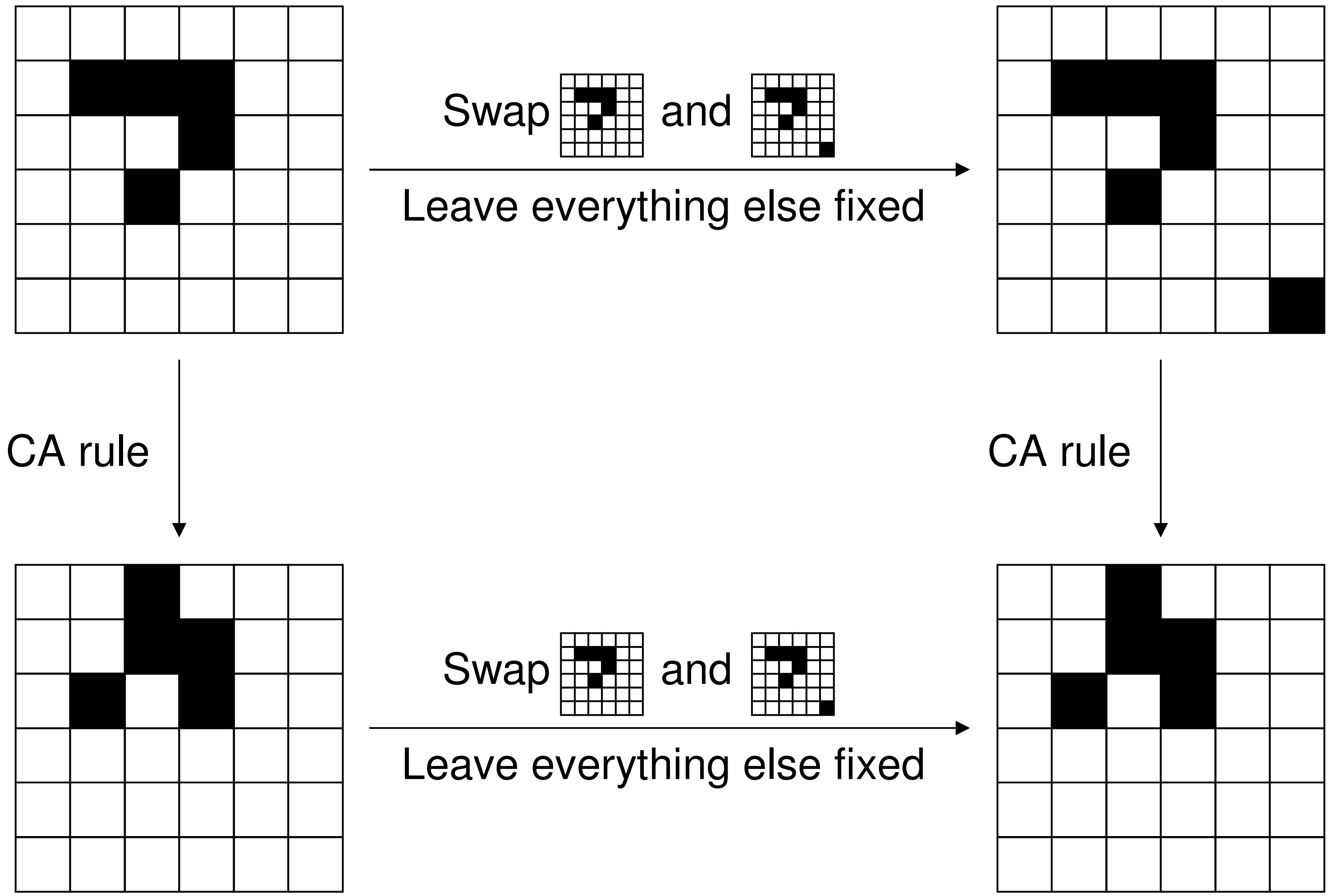
Symmetry: Example 1



Symmetry: Example 2



Symmetry: Example 3



Theorem

- For a given CA, the following are equivalent:
  - α is an automorphism of the CA's transition graph
  - α is a symmetry of the CA
- So "symmetry" in the transition graph corresponds to symmetries of the CA, and vice versa

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Counting automorphisms

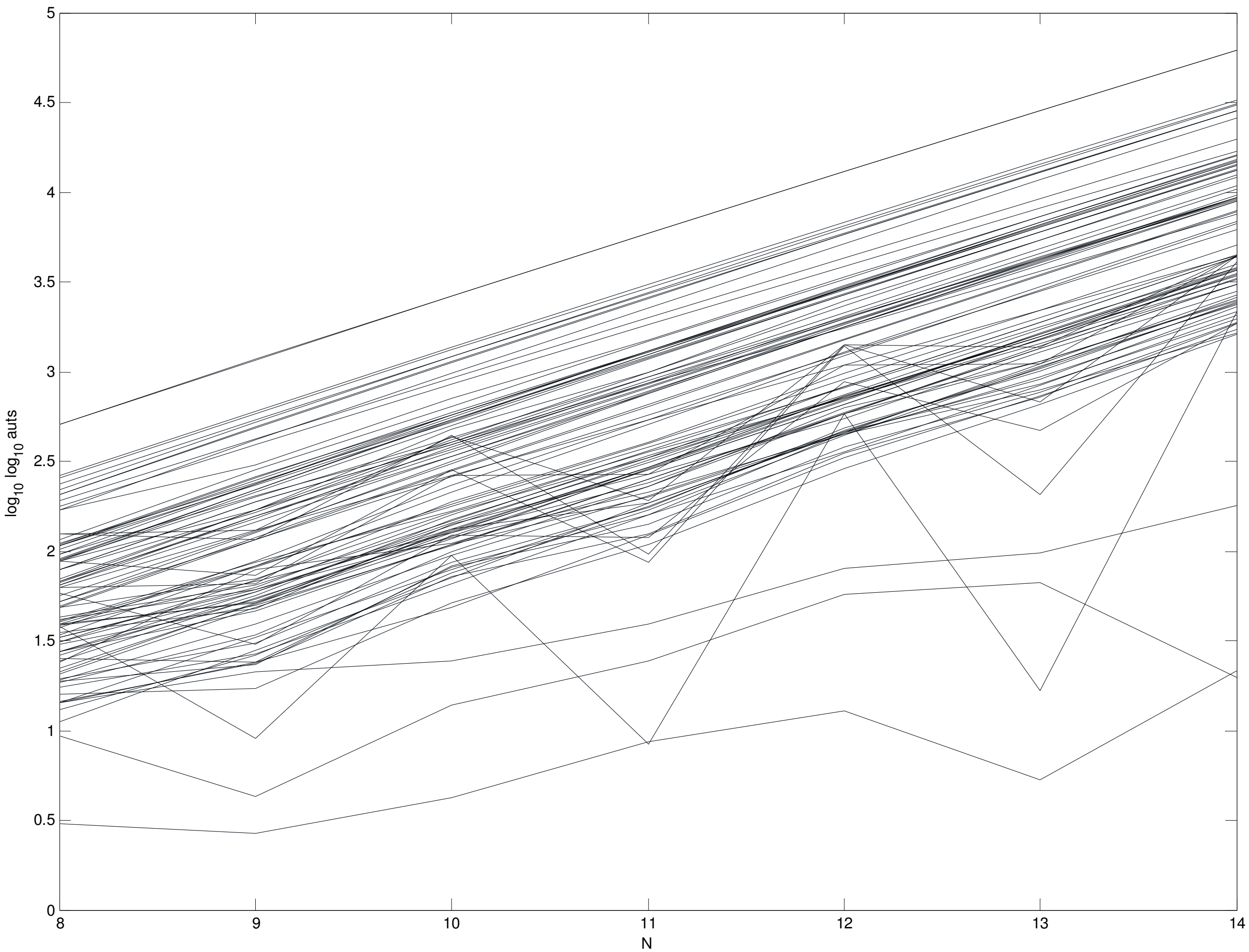
- “Simple” combinatorics…

$$A_{\mathrm{all}}(G) = \prod_{I \in \{H_j\}/\cong} |I|!\ A_{\mathrm{circ}}(H \in I)^{|I|}$$

$$A_{\mathrm{circ}}(H) = \frac{k}{p} \prod_{i=1}^{p} A_{\mathrm{tree}}(r_i)^{\frac{k}{p}}$$

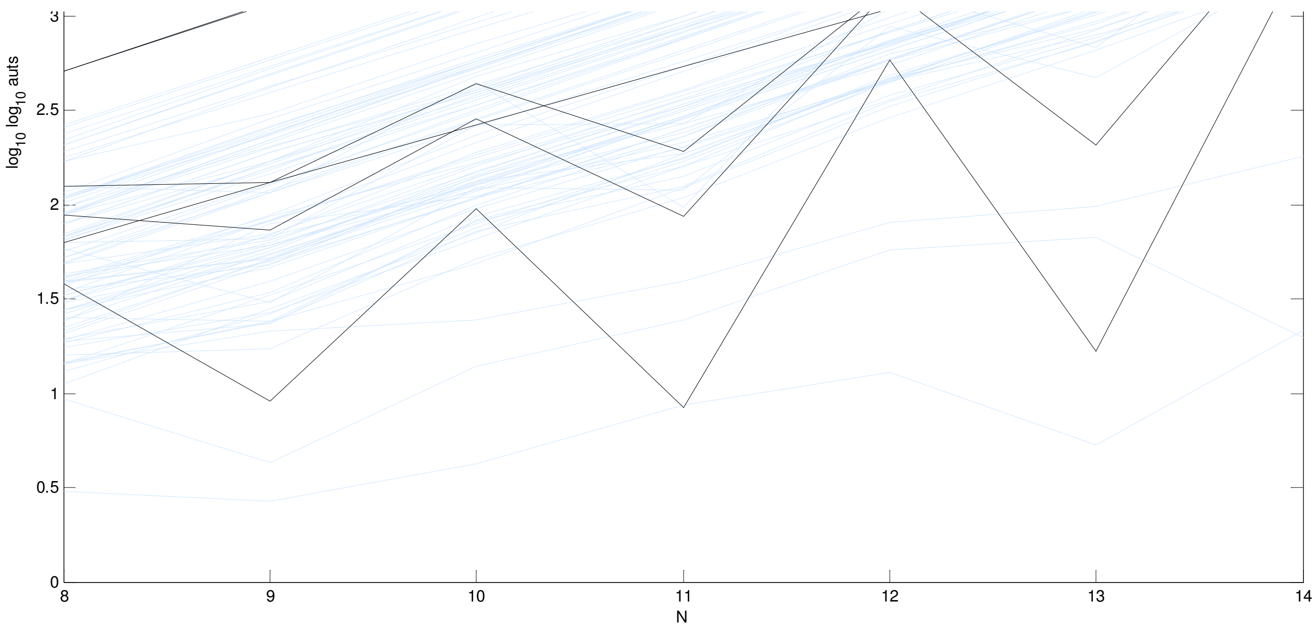
$$A_{\mathrm{tree}}(r) = \prod_{I \in C/\cong} |I|!\ A_{\mathrm{tree}}(c \in I)^{|I|}$$

Results for ECAs

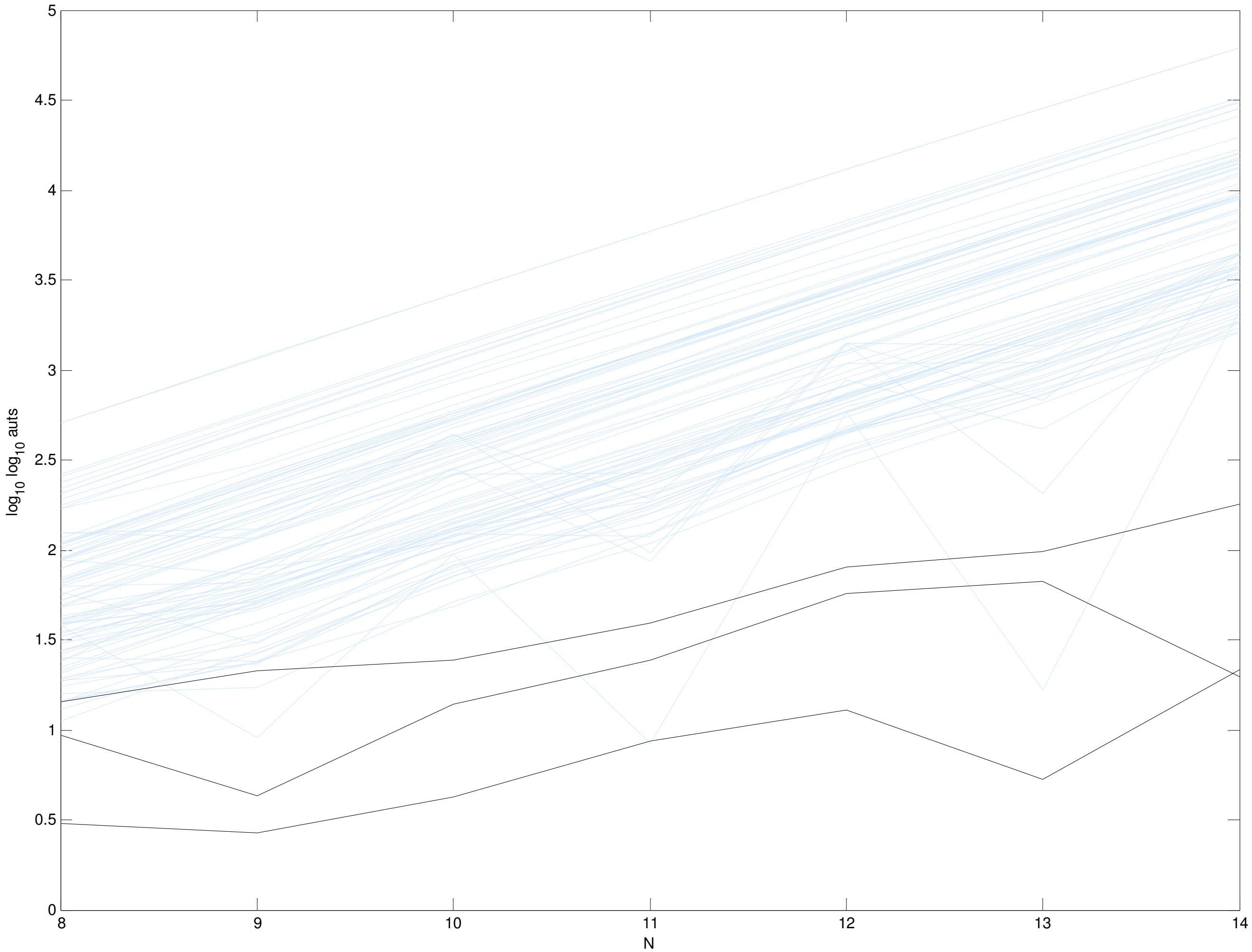


Linear

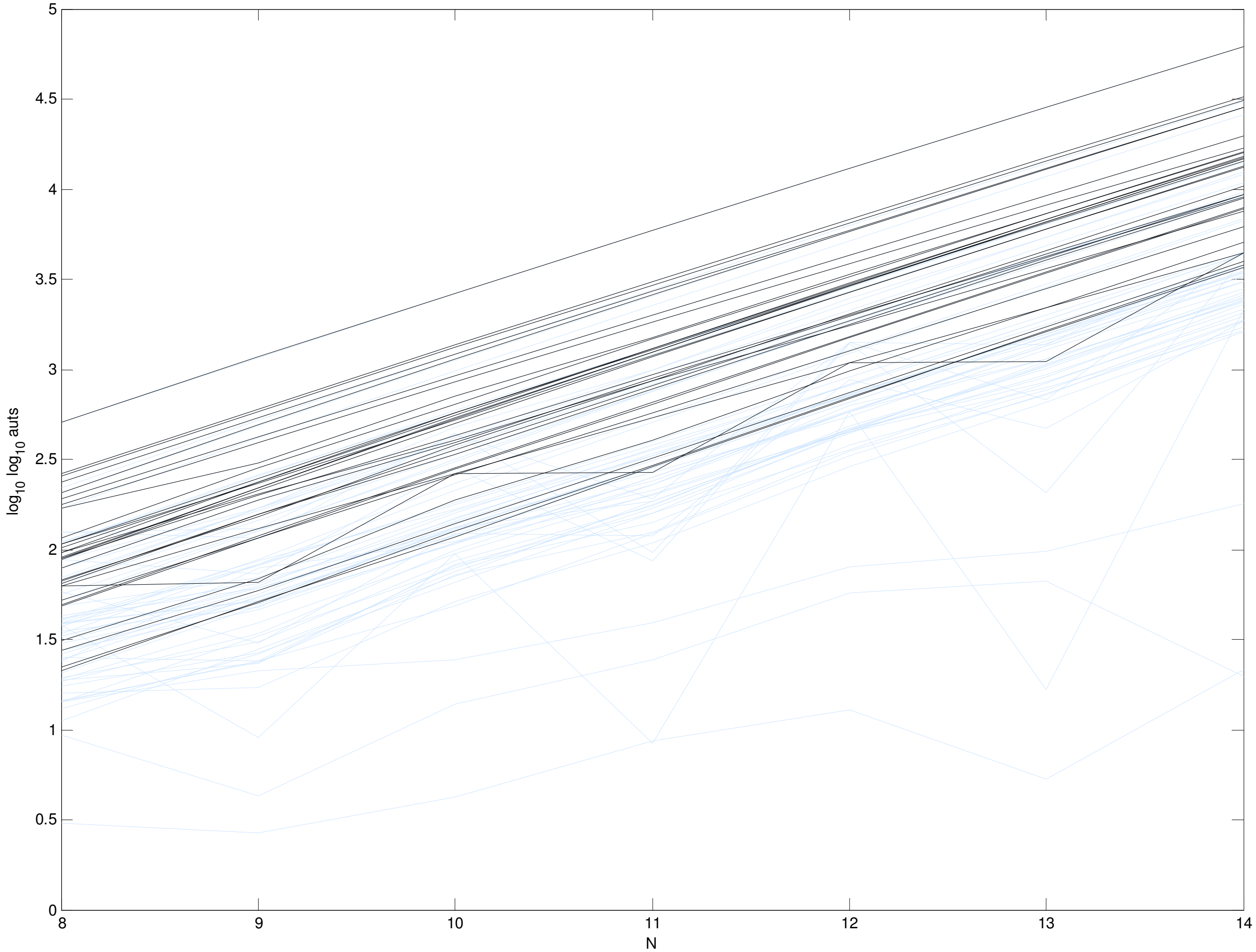
- Some linear rules exhibit different behaviour for N odd/even
  - (e.g. Wolfram, Martin and Odlyzko, 1984)



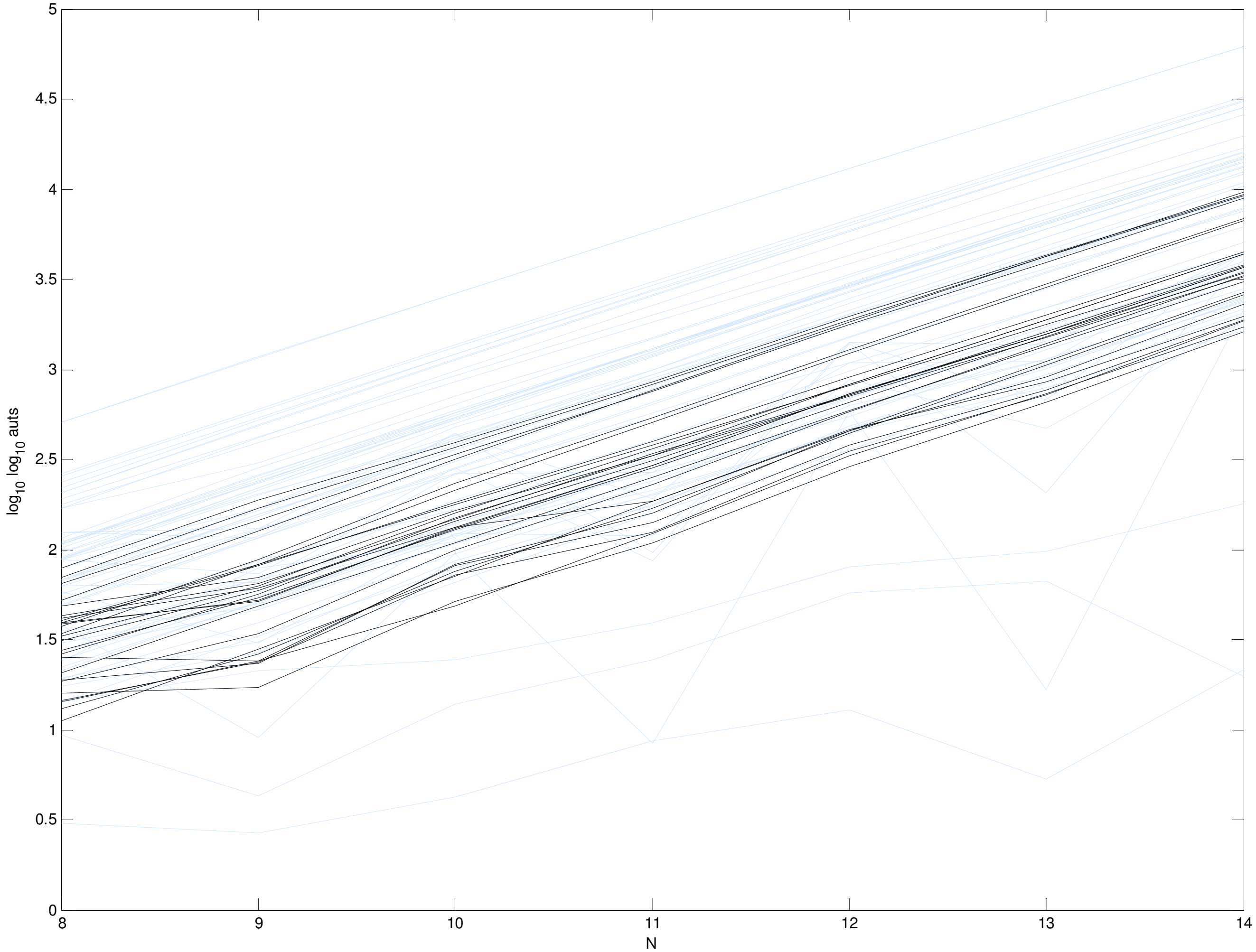
Chaotic
(Wolfram's Class 3)



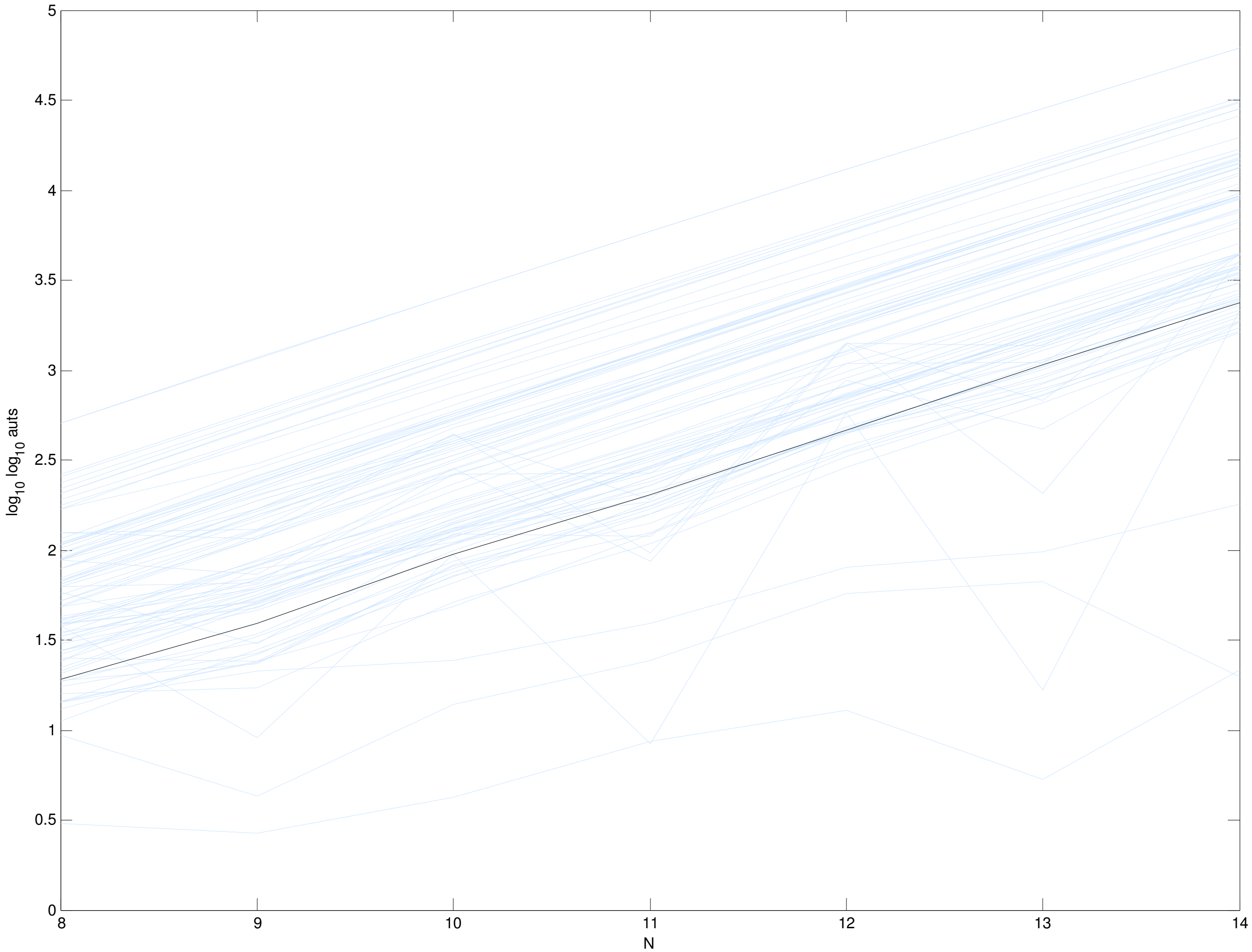
Periodic (short transients) (Wolfram's Class 2)



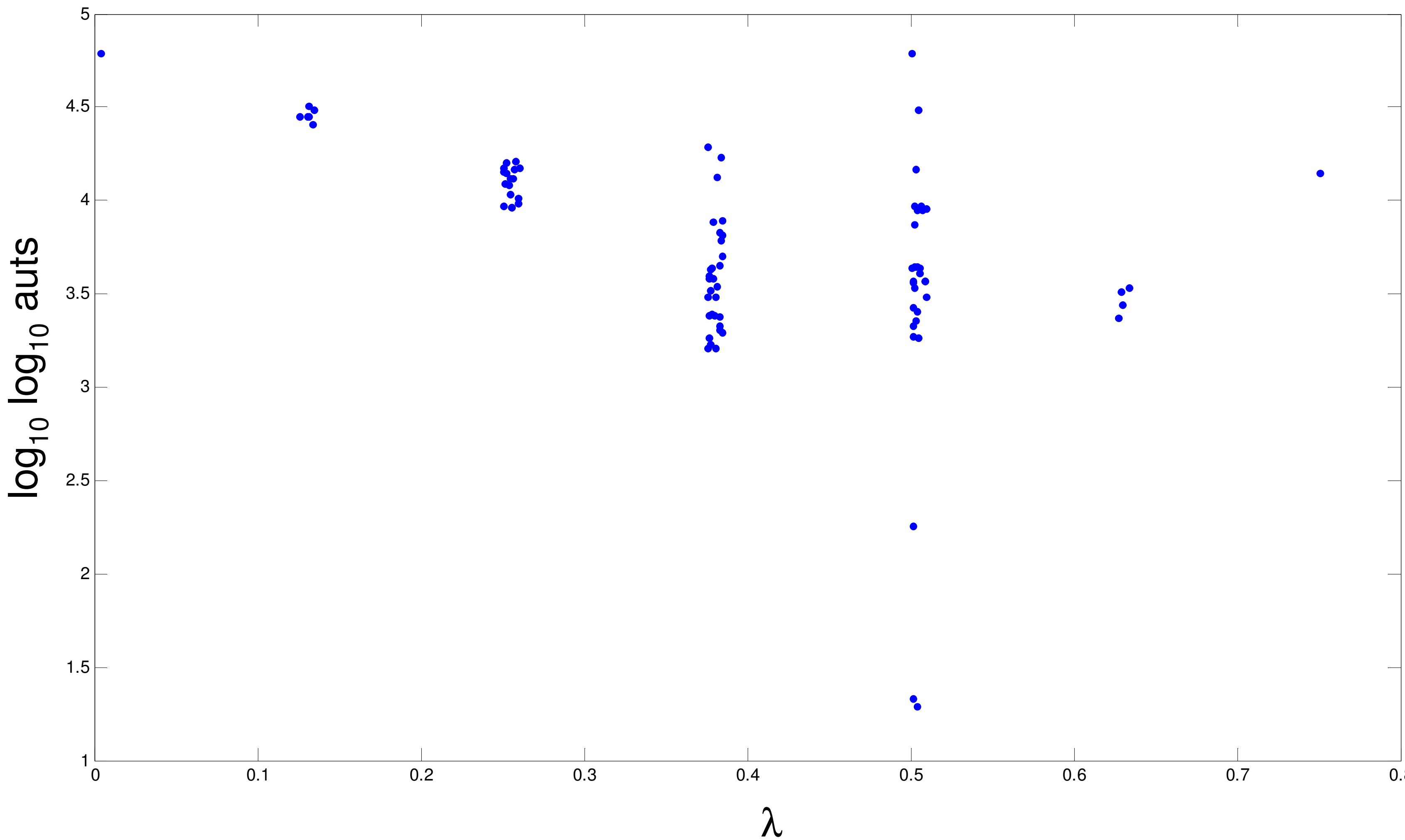
Eventually periodic (long transients) (Wolfram's Class 2)



Rule 110
(Wolfram's Class 4)



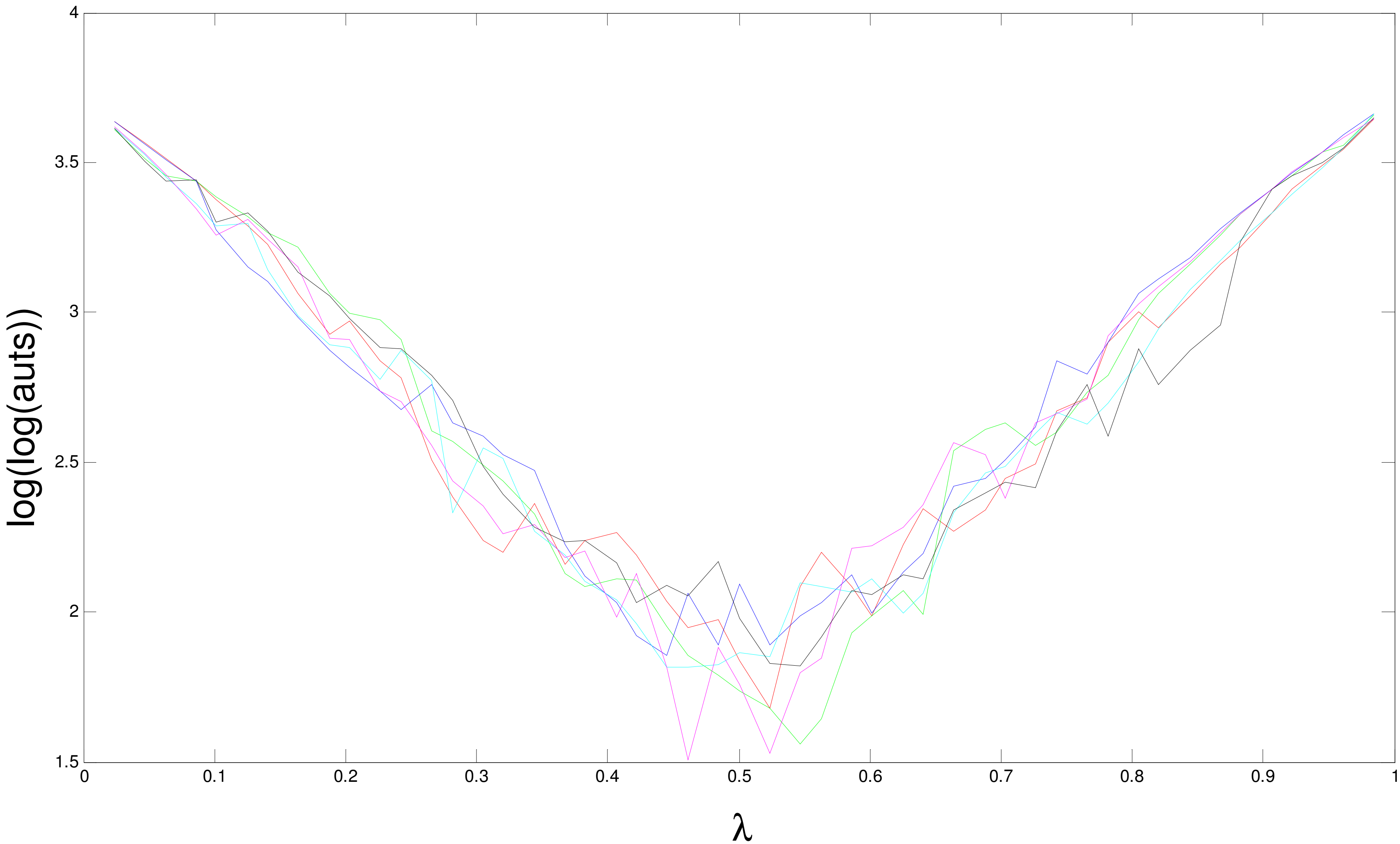
Automorphisms and Langton's λ: ECAs



Automorphisms and Langton's λ

- Langton himself admits that λ is "only roughly correlated" with dynamics of ECAs
- Try a class of CA where the correlation is a little better…
  - State set {0,1}
  - Neighbourhood radius 3
  - Start with "rule 0"
  - Change random 0s in the rule table into 1s until the desired λ value is reached

Automorphisms and Langton's λ



Conclusion

- We can study the “symmetry” inherent in a CA by studying automorphisms on the transition graph
- There seems to be a correlation between the total number of symmetries and the qualitative dynamics (“Wolfram class”) of the CA