# Extensions to existing solutions to the FSSP

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Automata 2007, Fields Institute, Toronto

# The Problem
## (Firing Squad Synchronization Problem)

* Given a firing squad, how to make all soldiers fire at the same time?
  - Difficulty: any screamed order by general need different times to reach different soldiers
  - Difficulty: soldiers cannot count but up to a very low limit

# The Problem

## as a Cellular Automata Problem

* Find a CA such that given any line of n cells, and starting from a configuration such as:

**Gqqqqqqqq**

starting configuration: time 0

the evolution leads to:

**FFFFFFFFF**

firing configuration: time T(n)

and avoids configuration such as:

**. . F . . . . . .**

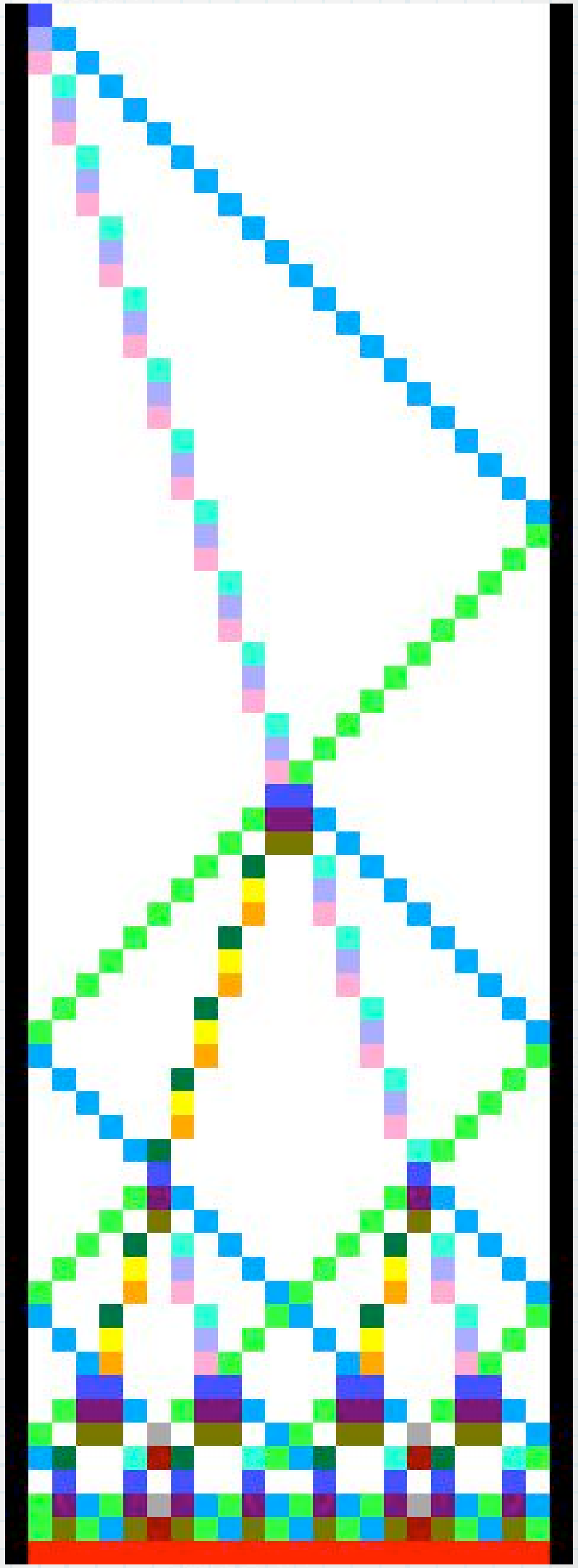
forbidden configuration

# A Solution

* Minsky/McCarthy (1967) strategy:

  divide & conquer

* 15 states
* T(n)≈3n
* it is not the minimal possible time



# Variants

* There exists numerous variants of the problem:
  * higher dimensions, generalized graphs;
  * fault-tolerant;
  * living graphs: growing, shrinking;
  * communication: limited bandwidth, delays;
  * arbitrary position of the general;
  * etc.

# Minimality

* for arbitrary n, at least 2n-2 steps to synchronize a line of n soldiers:

$$T(n) \geq 2n-2$$

  - intuitively: necessary time to get acknowledgment from the other end

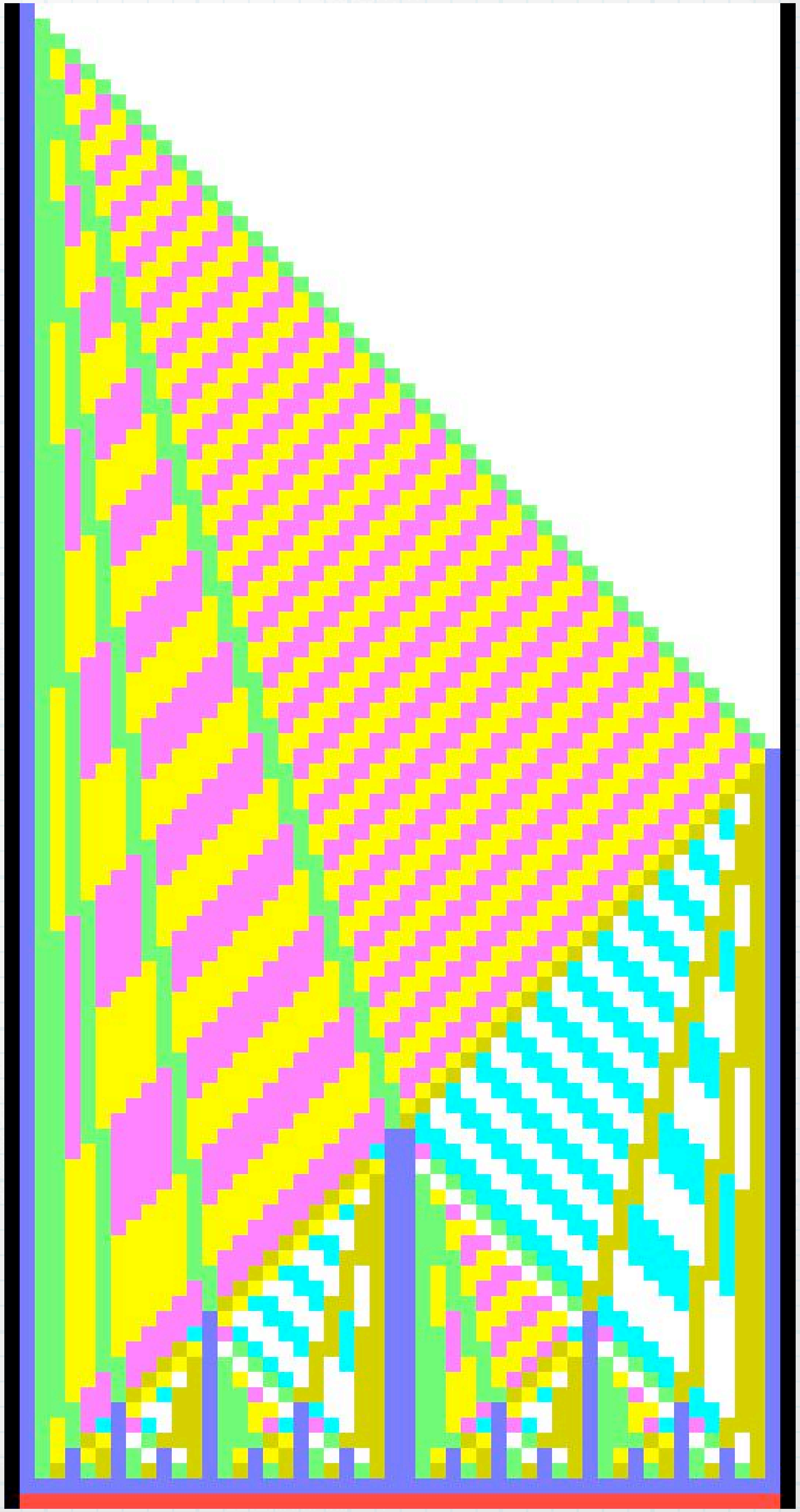
* minimal-time solutions exists: Goto 1962, Waksman 1966, Balzer 1967, Mazoyer 1986, Gerken 1987...

# Minimality

- Balzer 1967 idea:

  mirrored Minsky

- 8 states
- T(n)=2n-2
- hard: infinitely many signals



# Minimality

world record

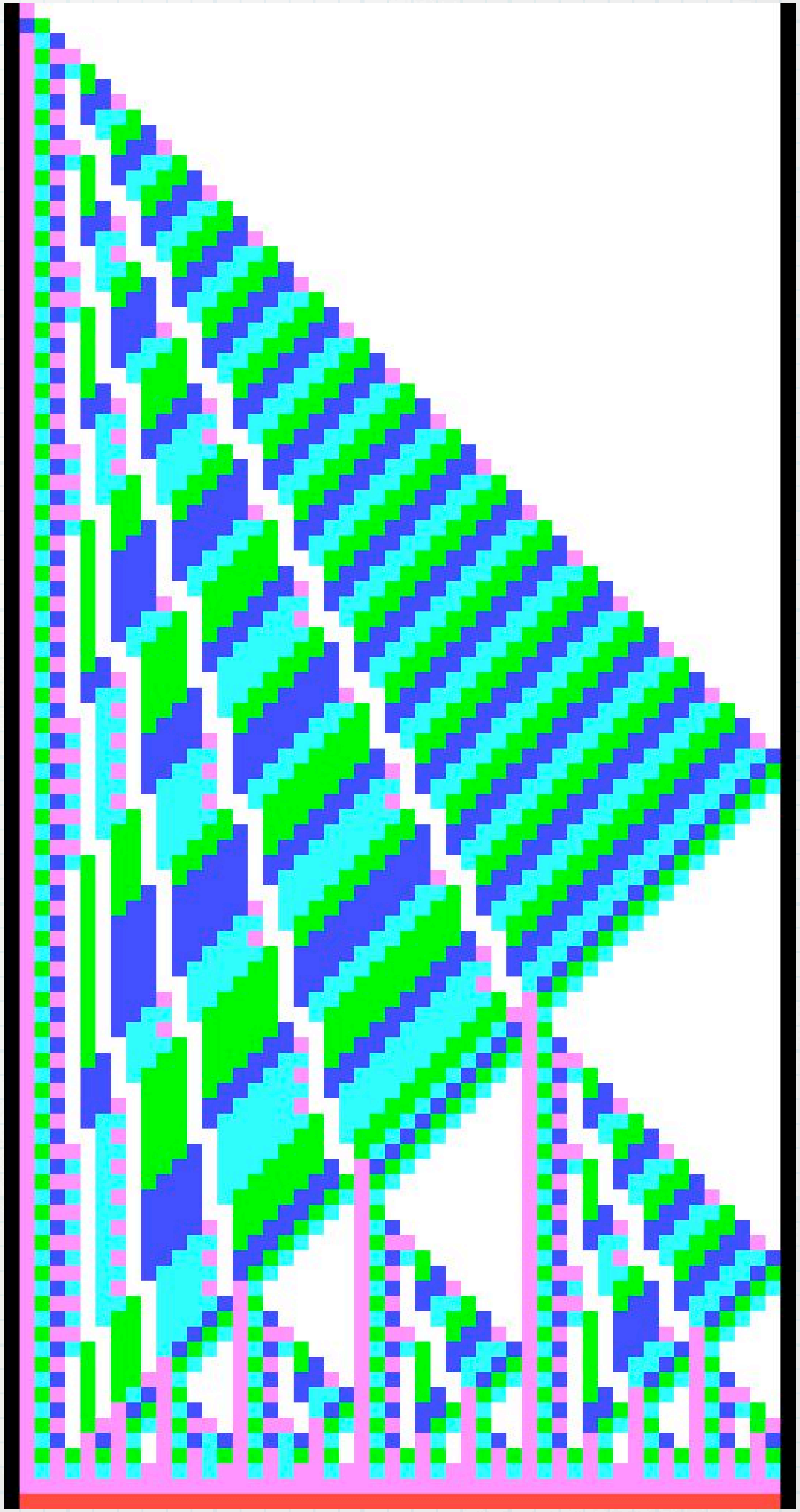
* Mazoyer 1986 idea:

  break the symmetry

* 6 states
* T(n)=2n-2

- hard: very high degree of optimization
- open: 5 states solution?

No 4 states by exhaustion



# Non Minimal Time

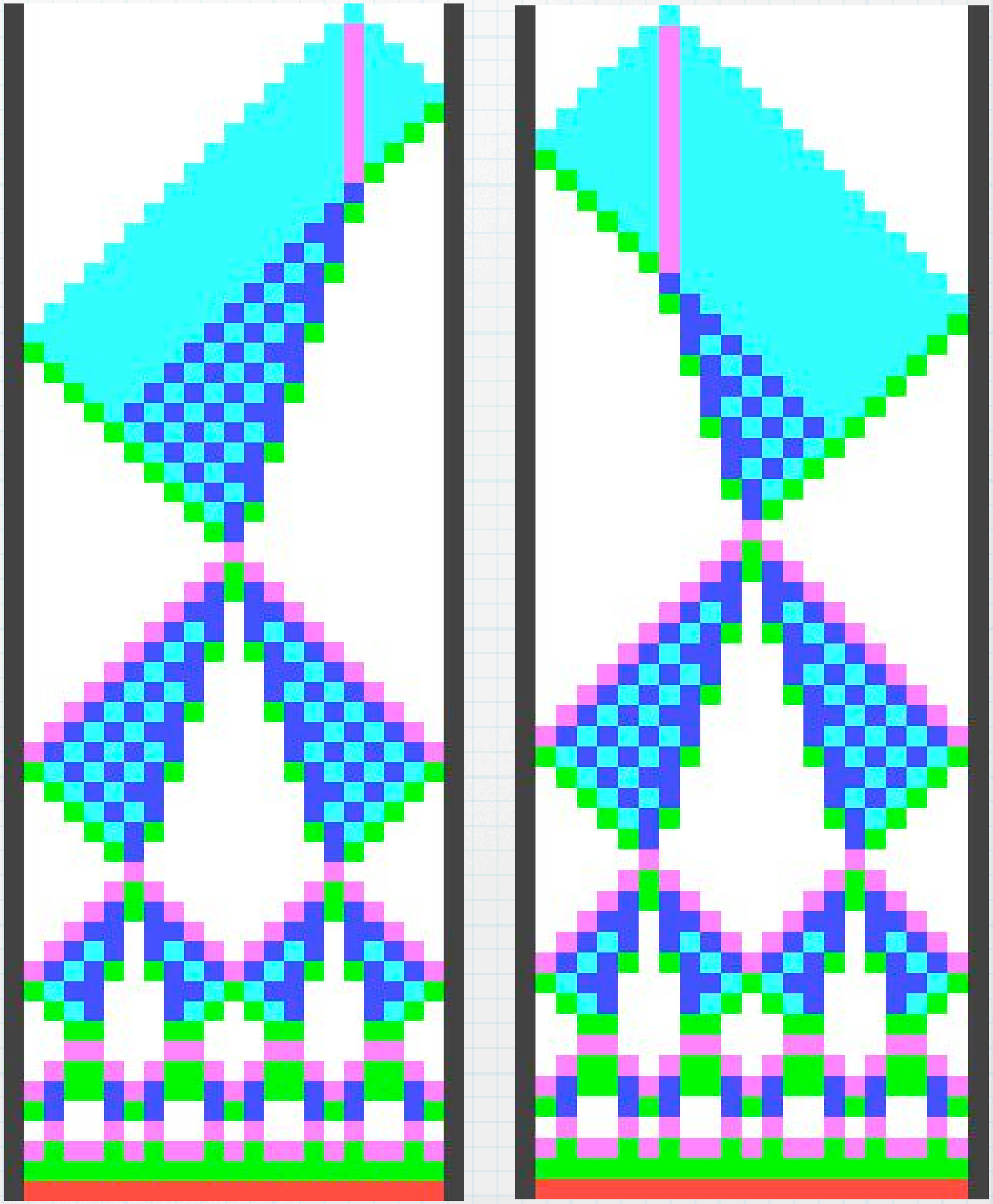
- It has been long believed that:

  minimal state → minimal time

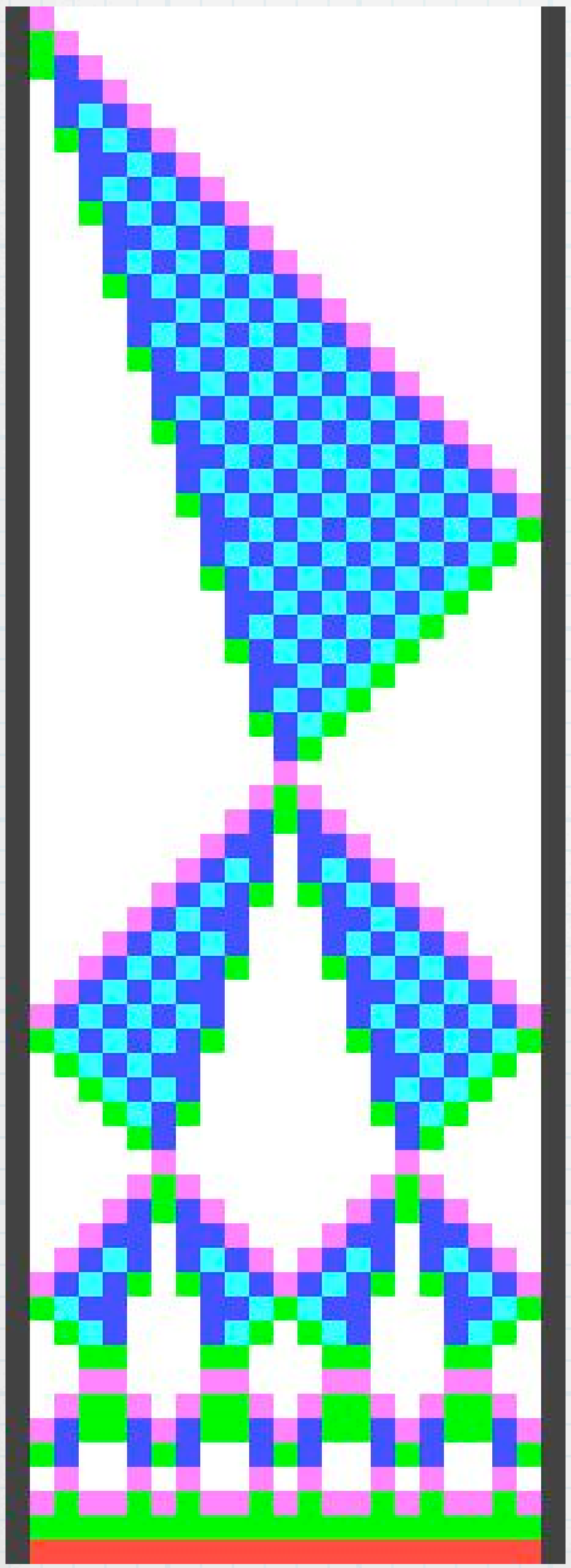
- recent results: NO!
  - Settle & Simon 2002 tricky transf. of Mazoyer
  - Umeo 2006 6-states, $T(n) \approx 3n$, $W(n) \approx O(n^2)$
  - Yunès 2007 6-states, $T(n) \approx 3n$, $W(n) \approx O(n.\log(n))$

# Umeo's solution

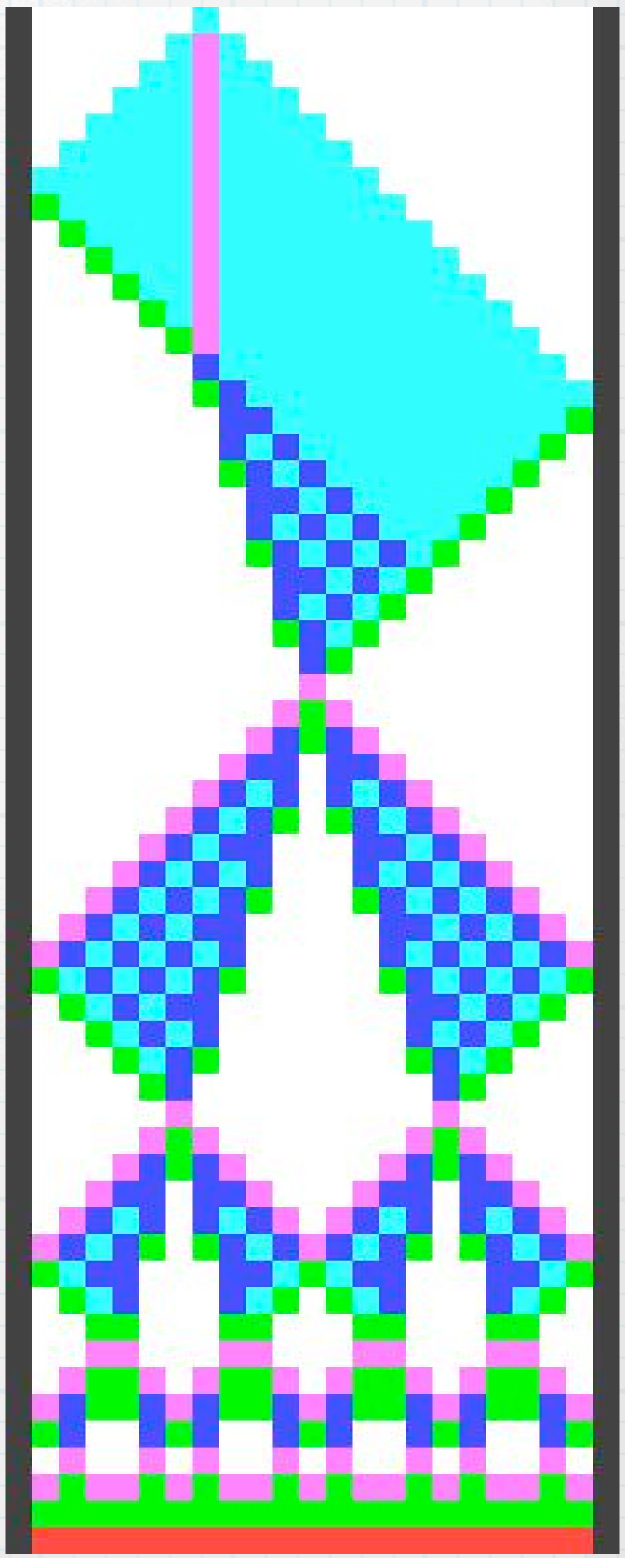
* Surprisingly, Umeo was able to modify his solution such that it synchronizes lines whatever be the position of the officer



# Umeo's 6-states



* Idea: use unused transitions rules of its original solution
* another state used to initiate the process



# Question

* What kind of functionality can we add to existing solutions, extending the automaton?

- amazing results

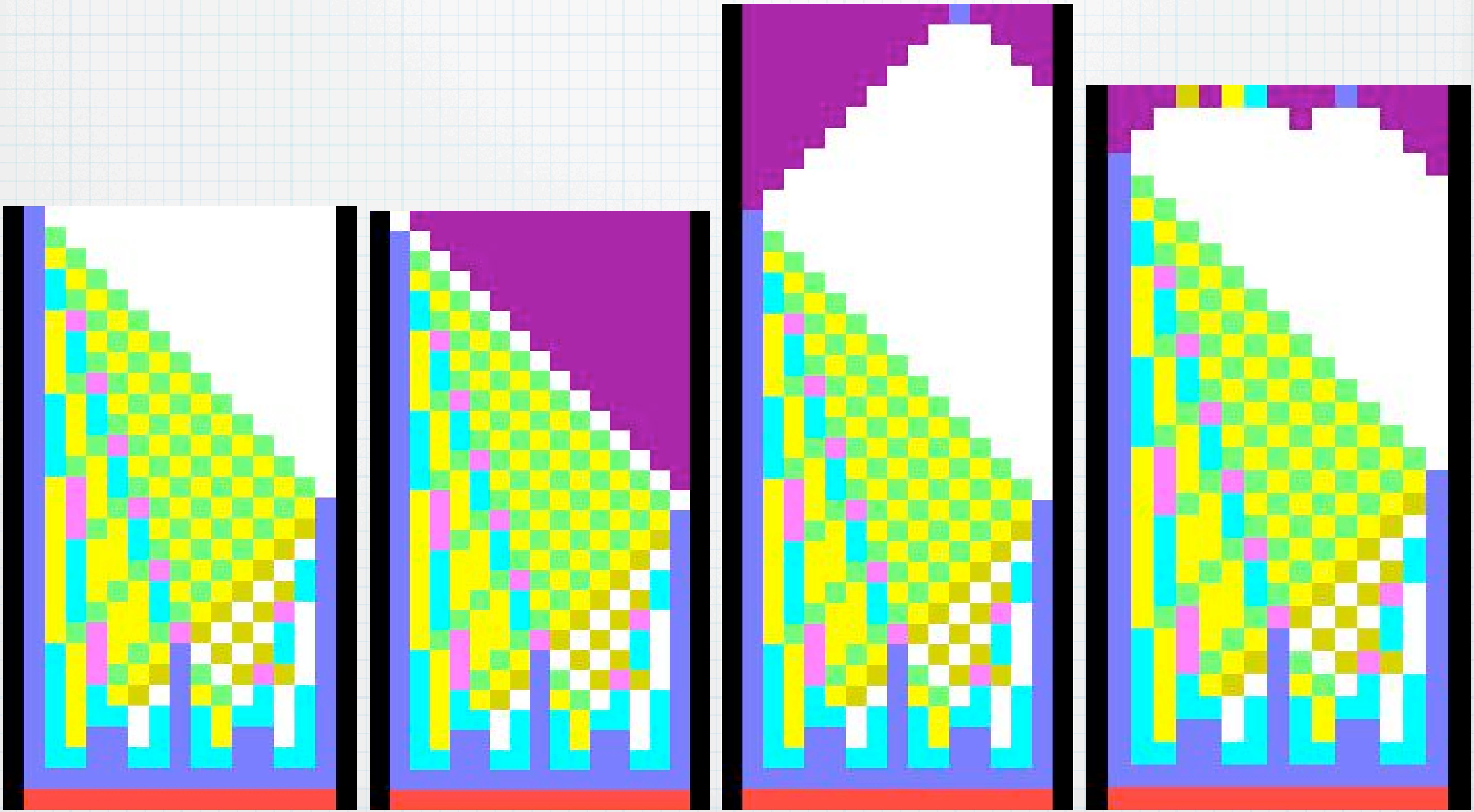
# One more state

- What can be done adding only ONE state to the transition function?
  - There exists an universal simple transformation that is able:

given an s-states solution to the original problem produce an s+1-states solution in which any state can be used to initiate the process anywhere on the line (and also solve the A-MG-FSSP)

Yunès 2007

# A generic extension: s → s+1



# No more state

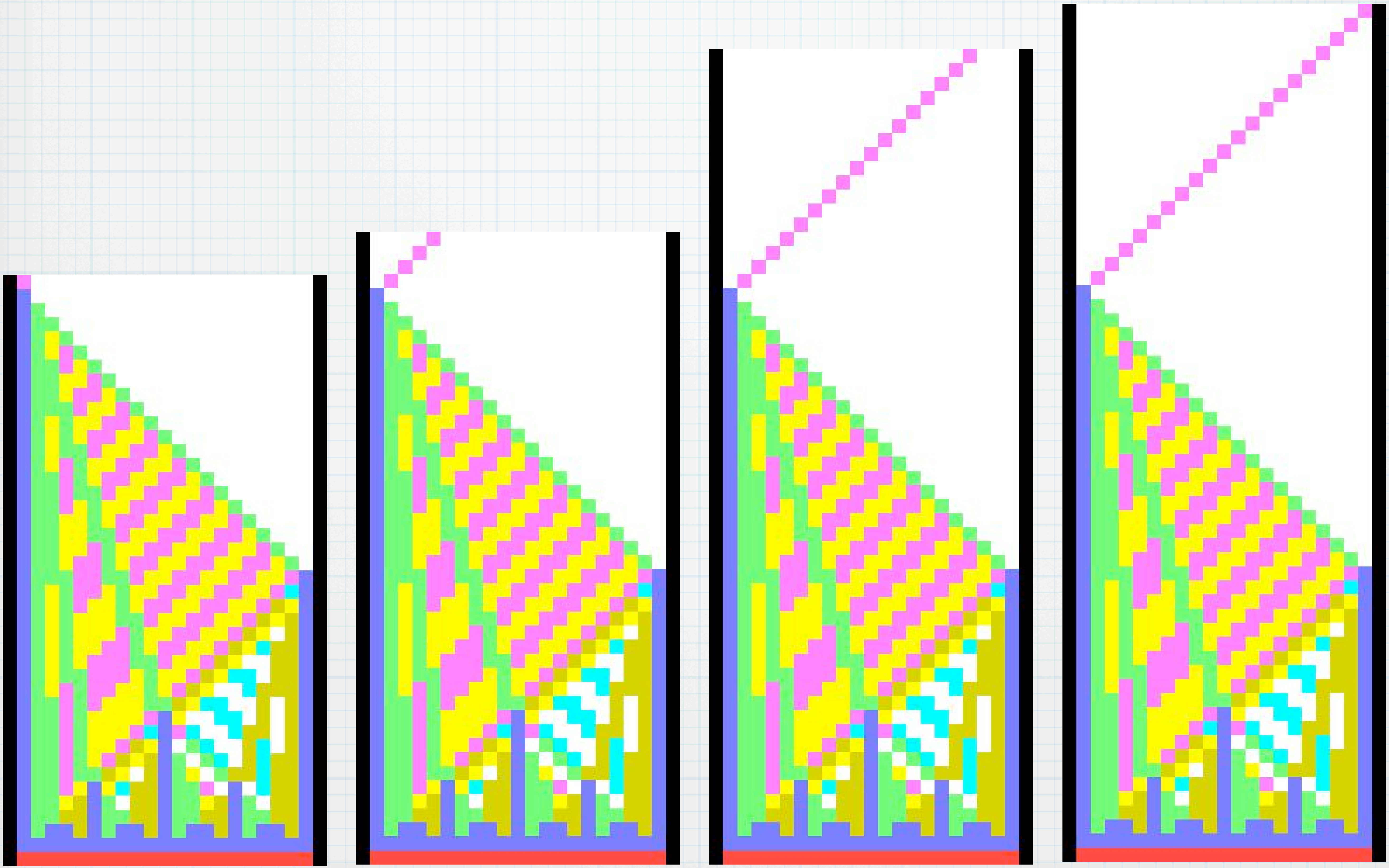
* Is it possible to extend the functionality of existing solutions adding only new transitions to the transition function?
  - YES! We consider two cases: position of the general and state used for the general.

# Position independency

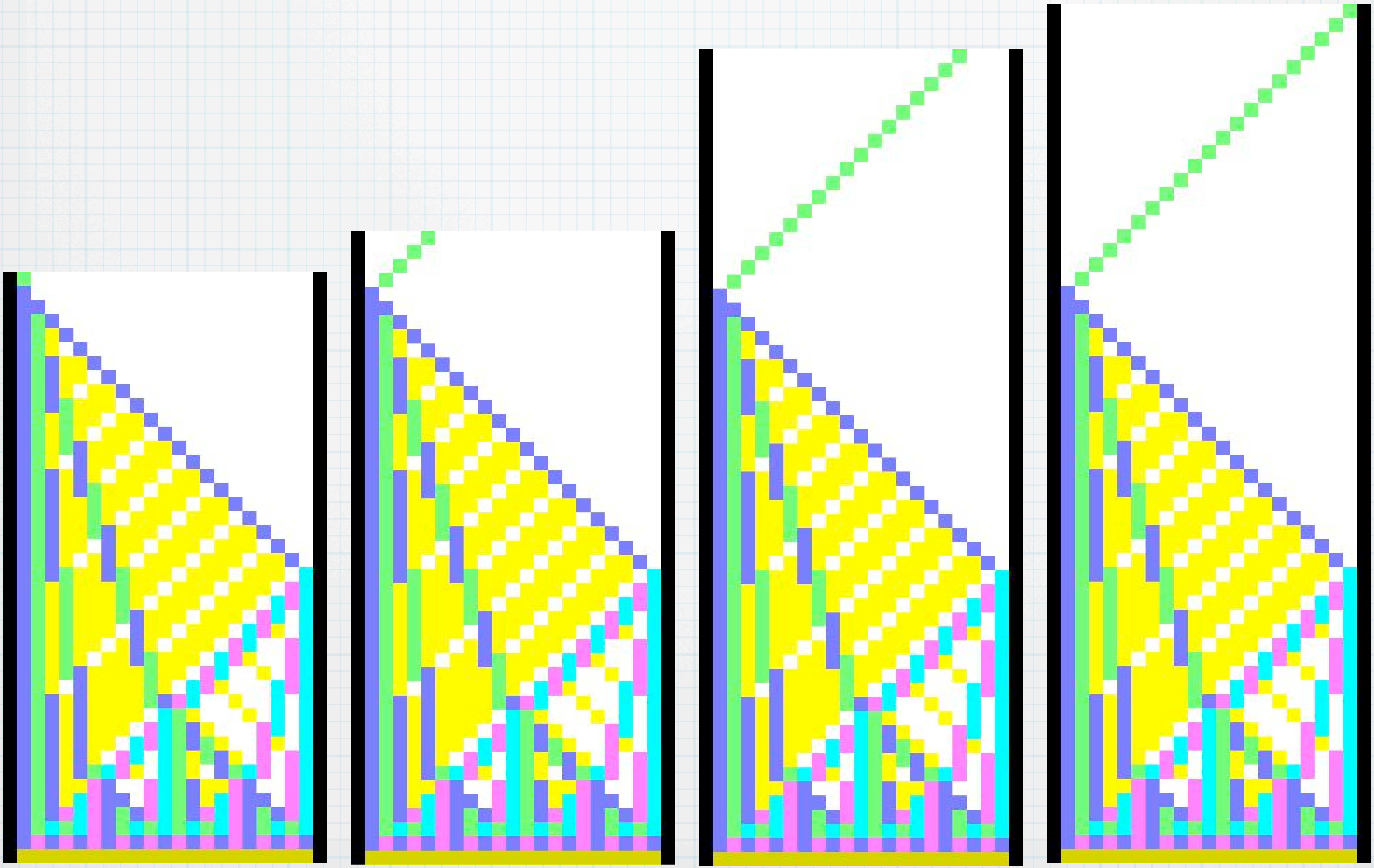
* Is it possible to extend the transition function of any existing solution in order to synchronize whatever is the position of the general?

- YES!

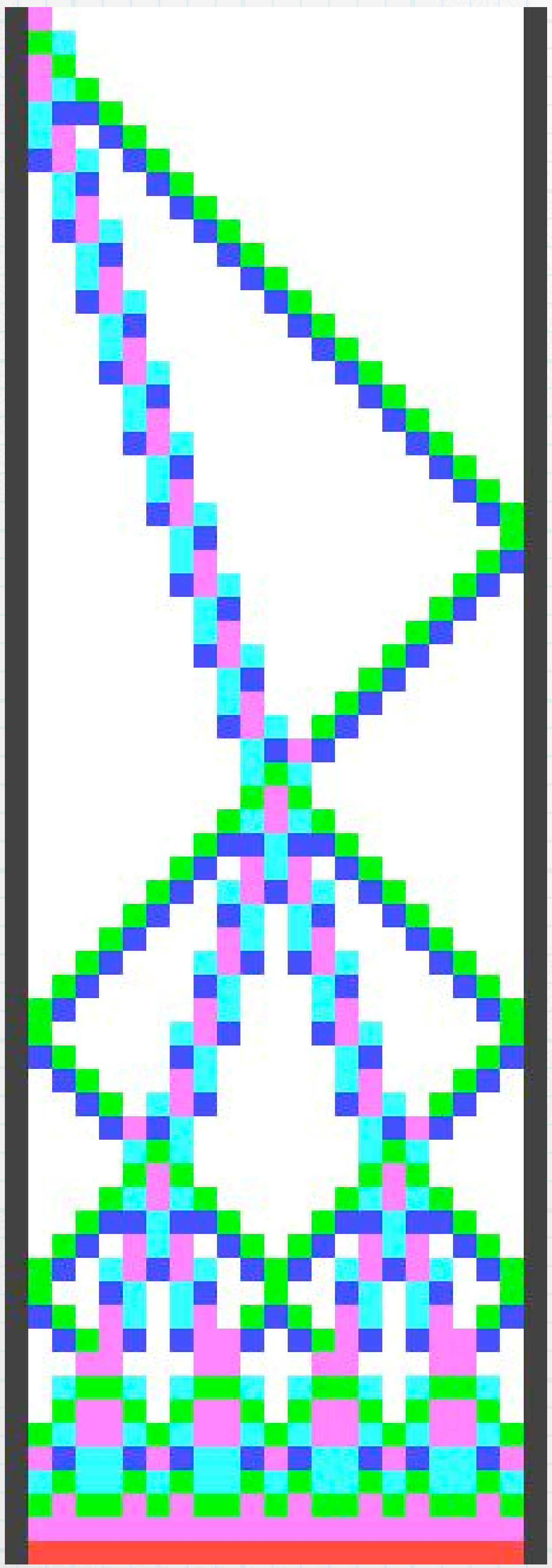
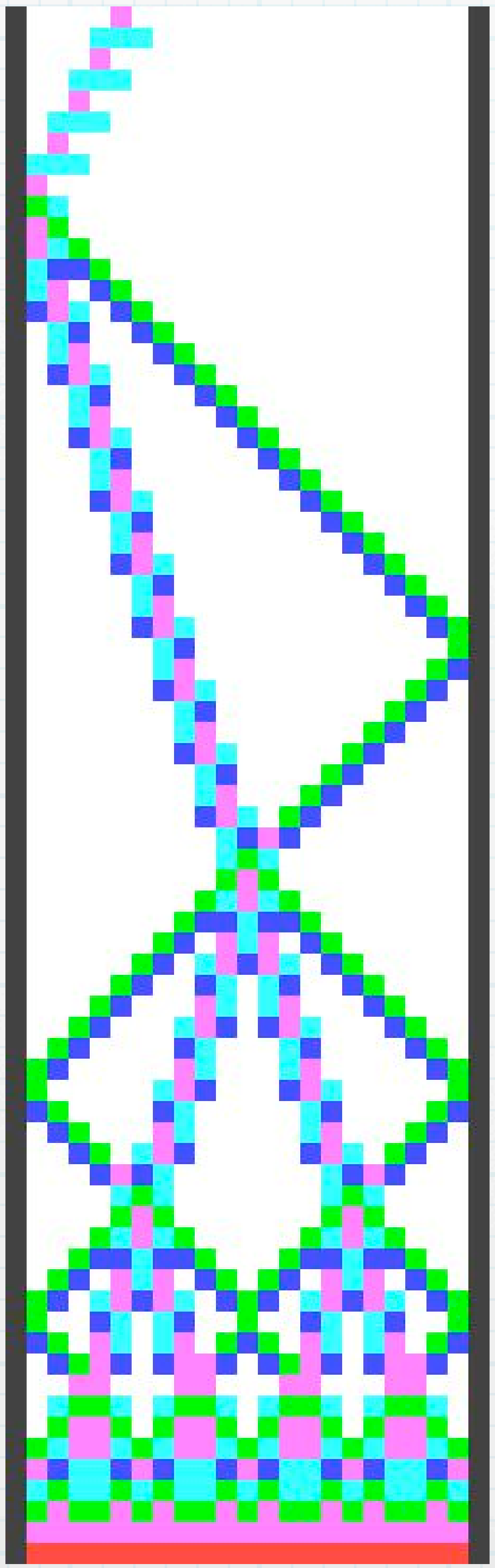
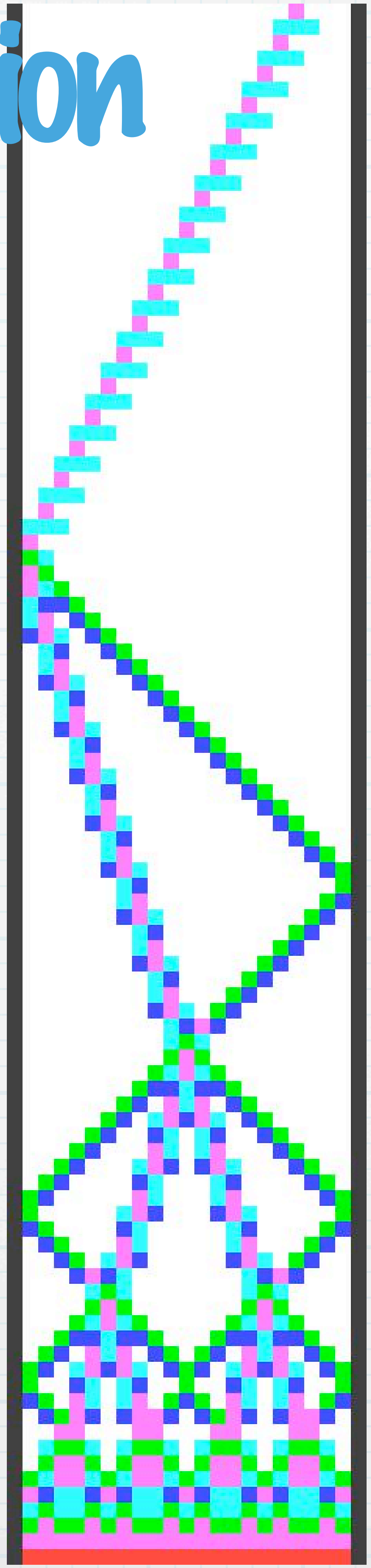
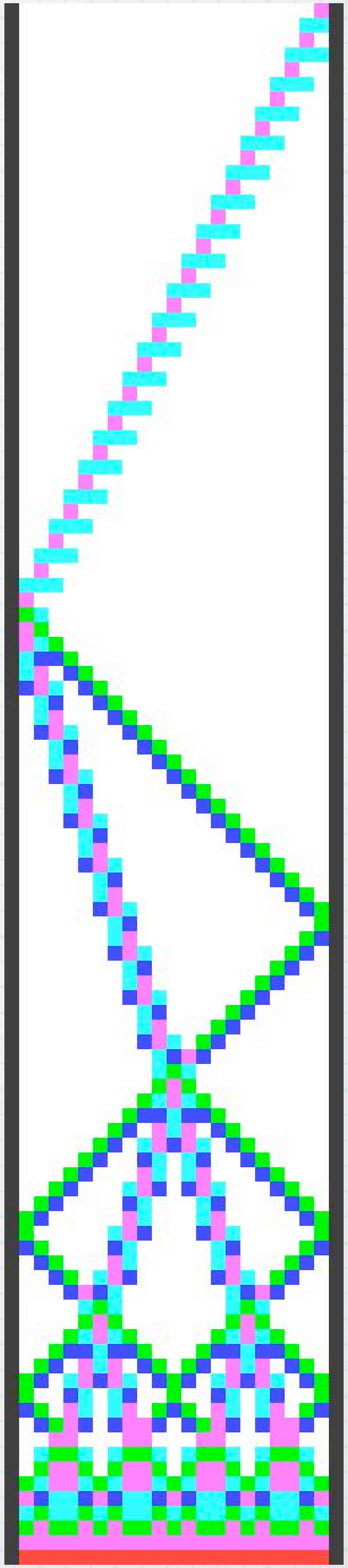
# Extension of Balzer's solution



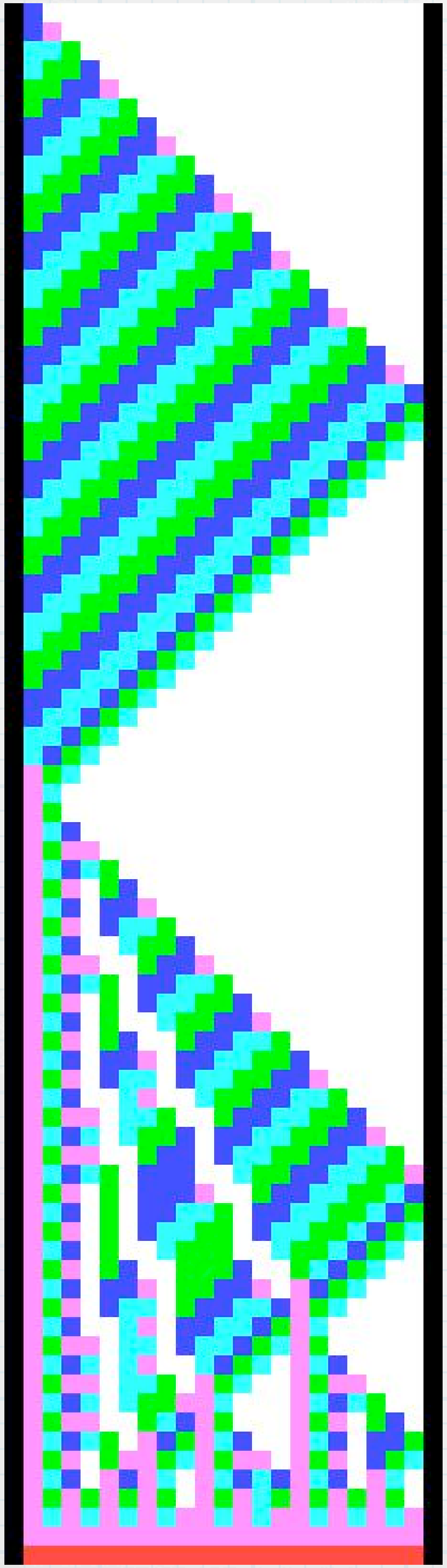
# Extension of Gerken's solution

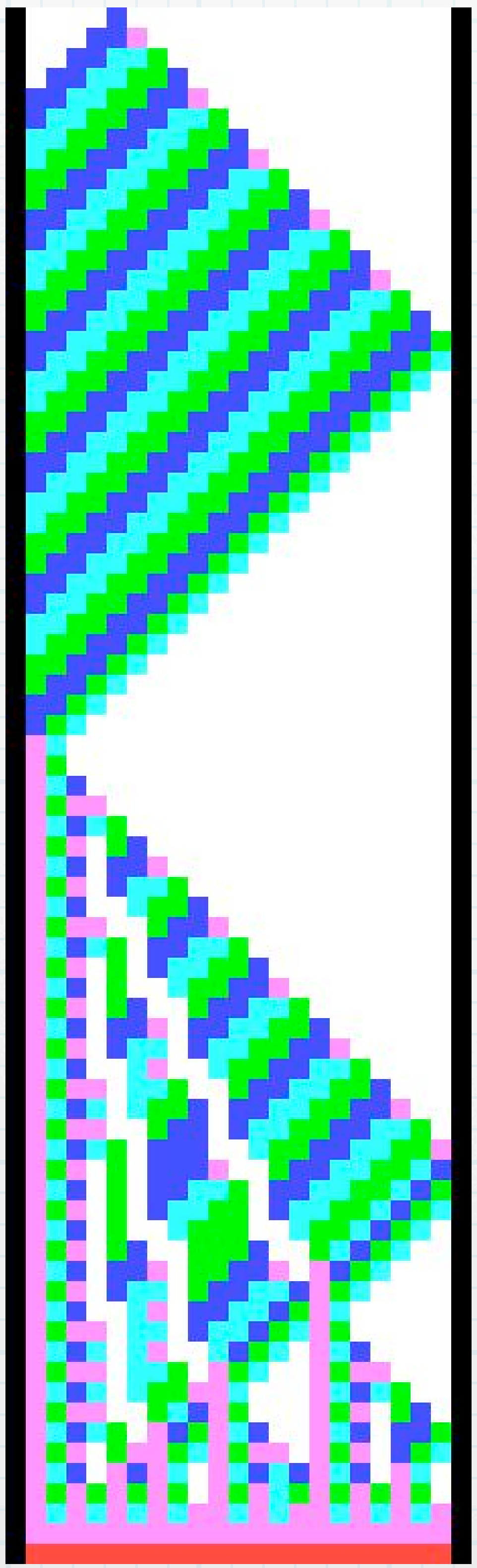


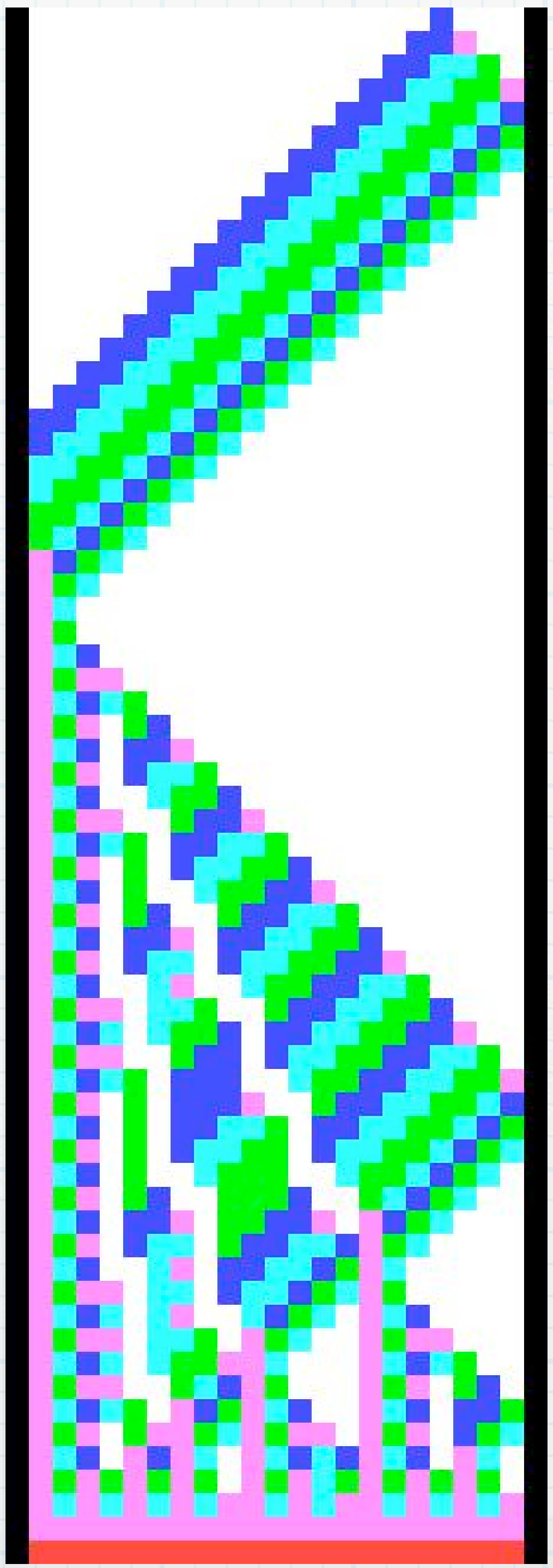
# Extension of Yunès' solution

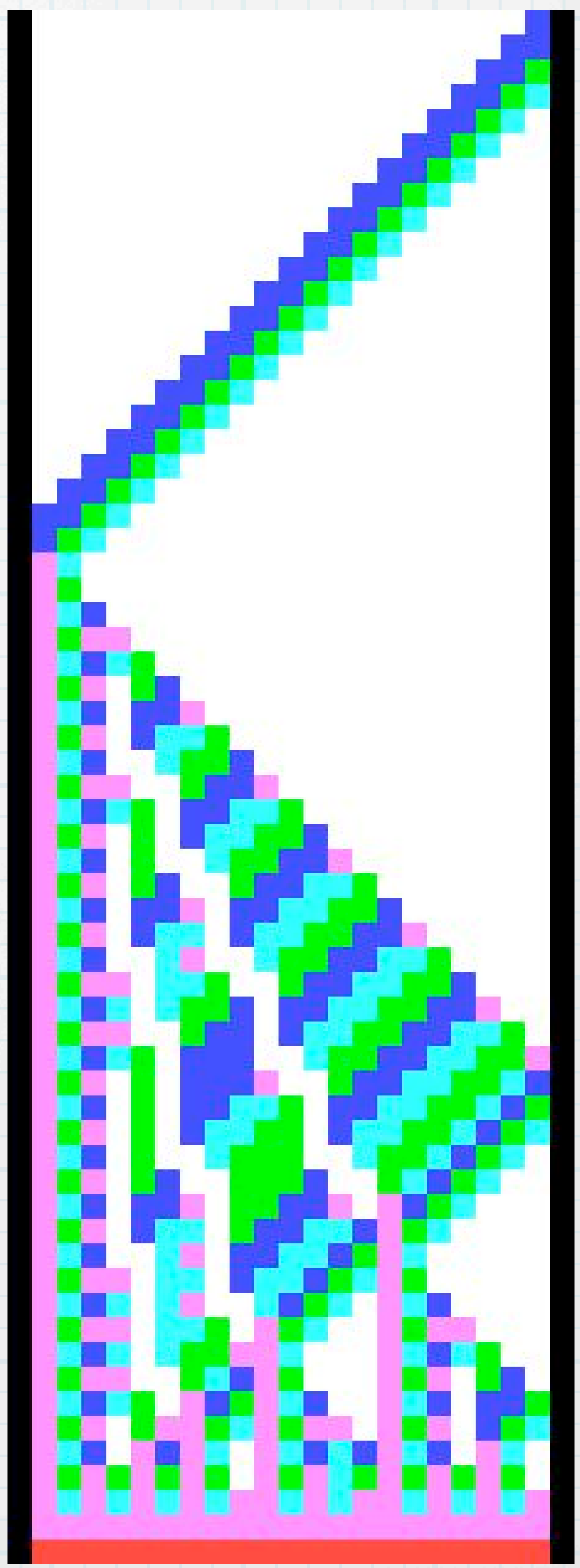





# Extension of Mazoyer's solution





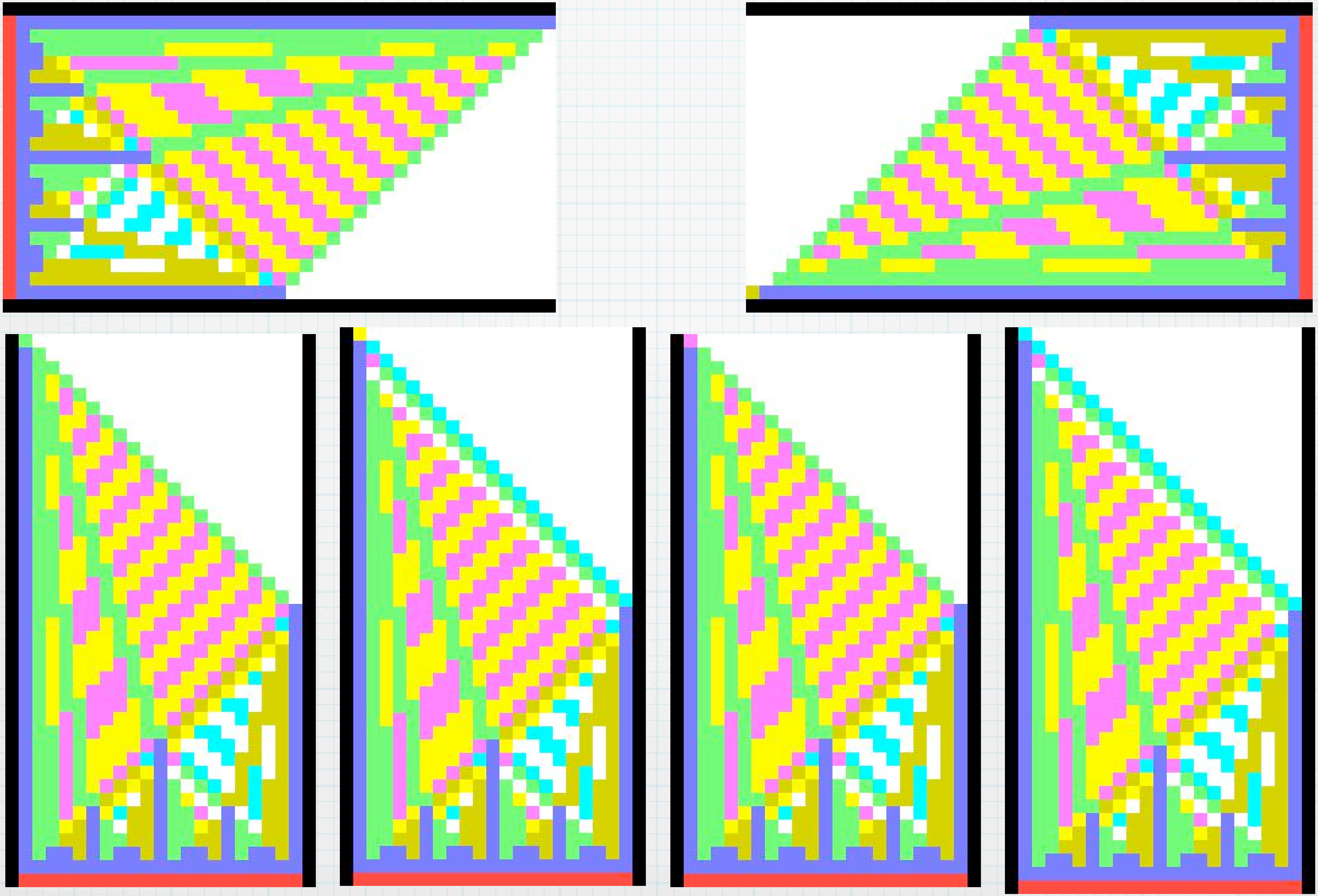




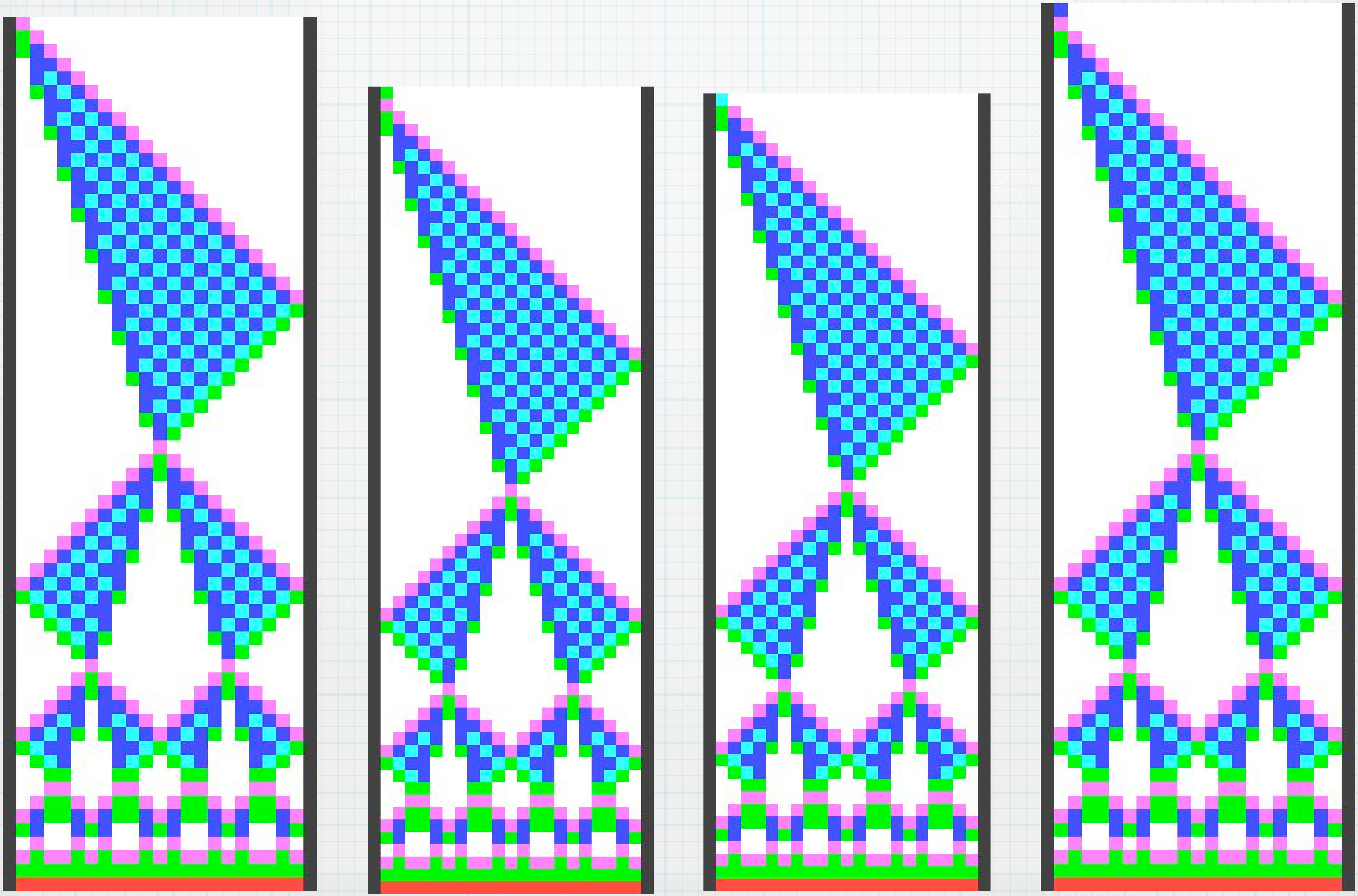
# Order independency

* Is it possible to extend the transition function of any existing solution in order to synchronize whatever be the state used to initiate the process?
  - ~~NO ABSOLUTELY IMPOSSIBLE!!!~~
  - YES!

# Extension of Balzer's solution



# Extension of Umeo's solution



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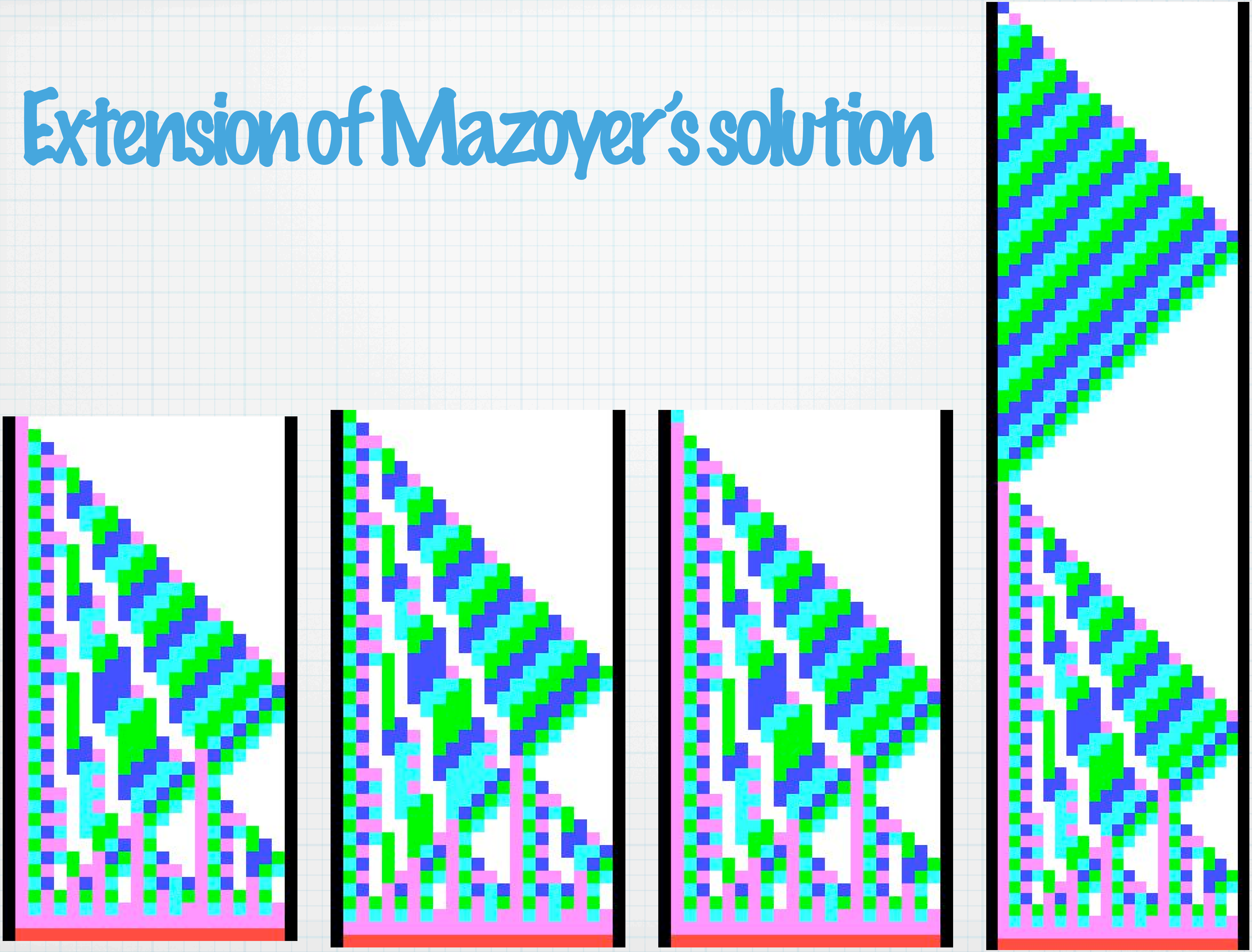

# to sum up

- New 6 states solutions, $T(n) \approx 3n$, $T(n) \approx 4n$
- Is there something general behind this?
- Such transformations were done on every kind of known (to me) solution
- What does this means? Does this correspond to some property of the problem? of the model? of the implementations? what?
- Do we need to reformulate the problem?