Information Dissemination in Mobile Wireless Networks

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Mobile Wireless Networks (MWN)

- Nodes in wireless networks for civilian and military applications highly mobile
  - Connections between nodes established and broken intermittently
  - Network topologies frequently-changing
- (Delay-tolerant) information dissemination
  - Single delay-tolerant source initiates message for broadcasting to whole network.
  - Mobility can be exploited to assist in spreading information
  - Especially when network not well connected at any time

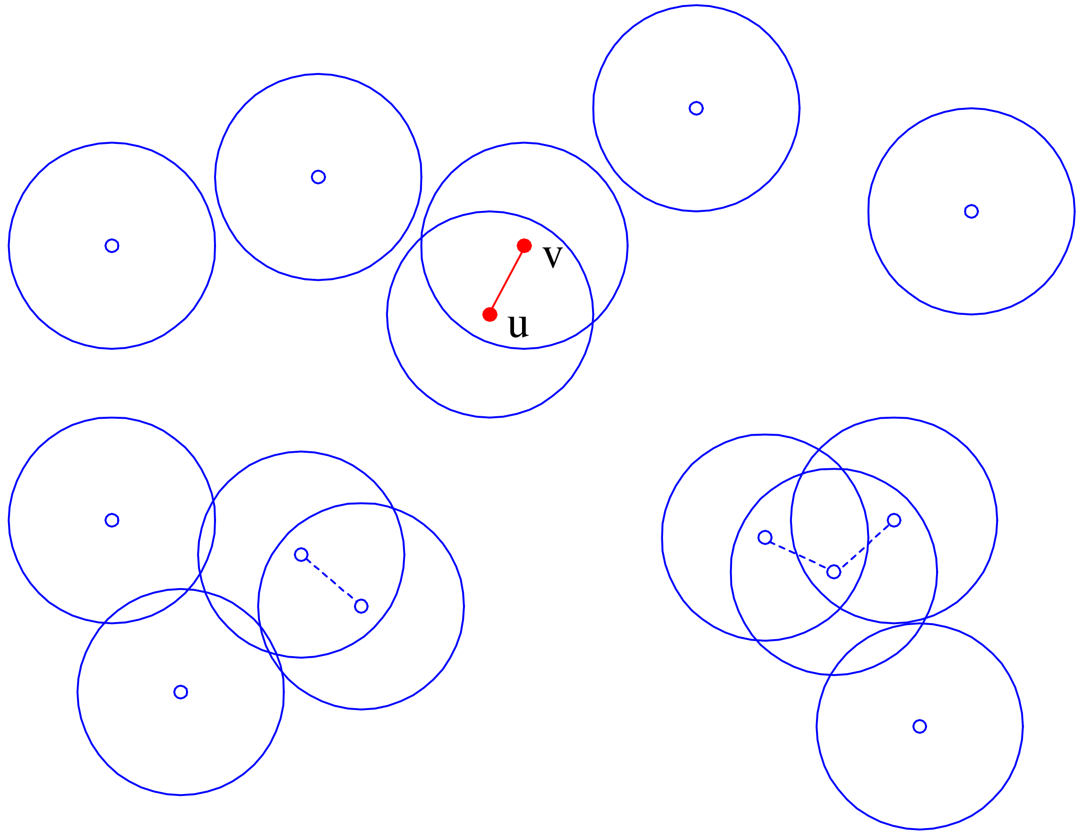
Connectivity in Static Wireless Networks

- Full connectivity required to guarantee every node receives message
  - May be overly restrictive and impossible to achieve.
  - Large-scale wireless networks exposed to severe natural conditions, vulnerable to enemy attacks, natural hazards, or resource depletion
- Percolation-based connectivity
  - Phase transition in macroscopic behavior of wireless networks
  - Network percolated (supercritical)—exists a large component of nodes spanning entire network; large number of nodes receive message
  - Network not percolated (subcritical)—network consists only of small isolated components of nodes; few nodes receive message

Connectivity in Mobile Wireless Networks

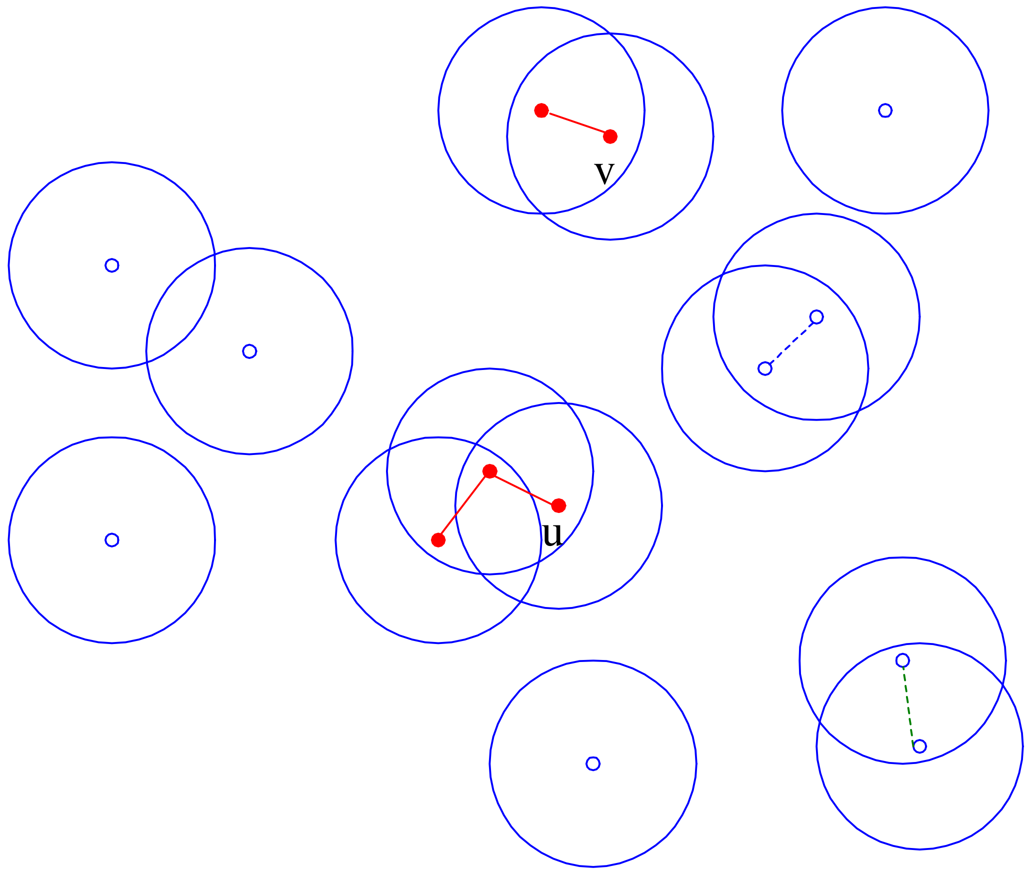
- In moving environments, connectivity should not be assessed at fixed time instant
- Two nodes can exchange information (share a link) if they can decode each other's signals at some point in time
- Different pairs of nodes share links at different points in time
- Connectivity must be analyzed over time

Information Dissemination in MWN



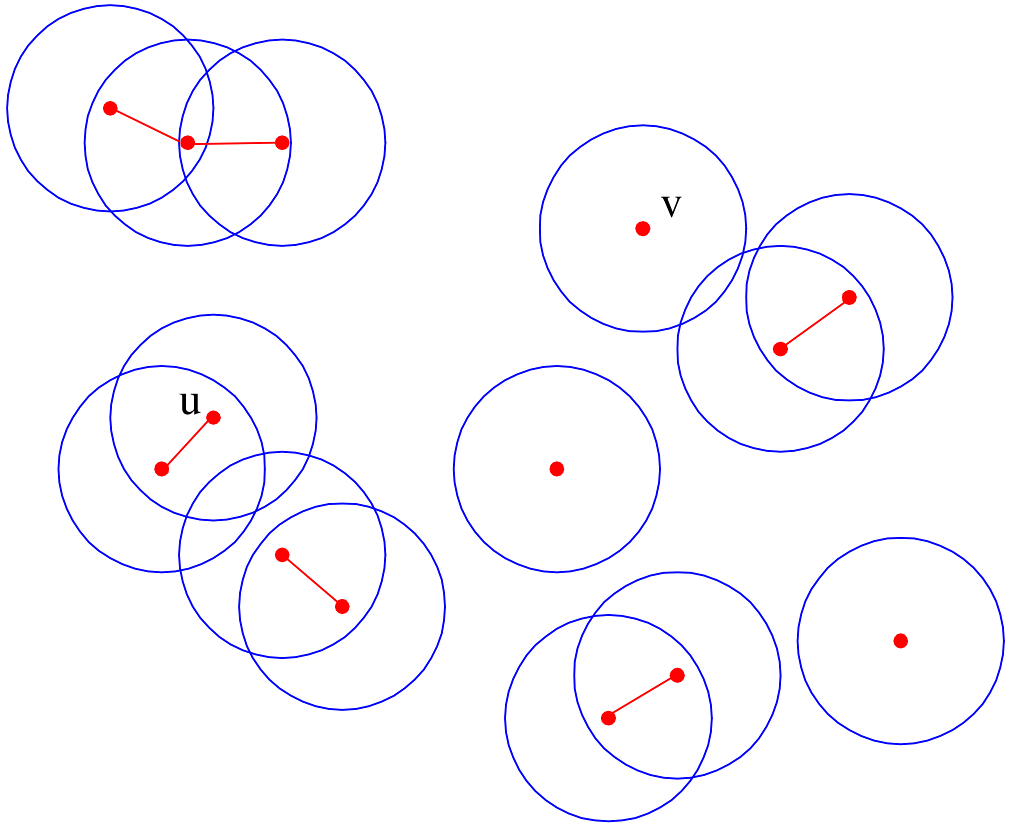
- $t = 0$: network not percolated, source node u broadcasts message
- Ignoring propagation delays, all nodes in component of u receive message instantaneously—small fraction

Information Dissemination in MWN



- As t increases, nodes move, message passed from message-carrying nodes to new nodes whenever they are within communication range
- Information is disseminated throughout network

Information Dissemination in MWN



- When network is well connected in "mobility" sense, as information dissemination process continues, eventually, a large fraction of the network, or even the whole network is informed of the message

Main Questions for Information Dissemination

- We seek answers to following questions:
  - What fraction of network eventually receives message?
  - How long does it take for this to be accomplished?

Main Results

- Answer to 1st question:
  - Under constrained i.i.d. mobility model, when network density satisfies a certain condition, constant fraction of nodes can receive source message eventually w.h.p.
  - Under some other mobility models, e.g., Brownian motion, all nodes can receive the message eventually w.h.p.

- Answer to 2nd question:
  - Network is subcritical (non-percolated): latency for information dissemination scales linearly with initial Euclidean distance between sender and receiver
  - Network is supercritical (percolated): latency for information dissemination scales sublinearly

Previous Work

- Throughput-delay trade-off: Grossglauser and Tse 02, Neely and Modiano 05, El Gammal *et al* 06, Lin *et al* 06, Sharma *et al* 06.
- Information dissemination in mobile networks: Groenevelt 05, Gunnarsson *et al* 06.

Stationary Random Geometric Graphs (RGG)

- $G(\mathcal{X}_n^{(0)})$ in $\mathbb{R}^2$: n nodes $\mathbf{X}_1^{(0)}, \ldots, \mathbf{X}_n^{(0)}$ uniformly distributed at random in $\mathcal{B} = \left[-\frac{\sqrt{n/\lambda}}{2}, \frac{\sqrt{n/\lambda}}{2}\right]^2$.

- Undirected link between u and v iff $\|\mathbf{X}_u^{(0)} - \mathbf{X}_v^{(0)}\| \leq 1$.

- As n and $|\mathcal{B}| \to \infty$ but density $\lambda = \frac{n}{|\mathcal{B}|}$ is kept constant,

$$G(\mathcal{X}_n^{(0)}) \xrightarrow{D} G(\mathcal{X}^{(0)}),$$

a homogeneous Poisson point process.

Continuum Percolation and Critical Density

- For $G(\mathcal{X}^{(0)} \cup \{\mathbf{0}\})$,
  - Percolation probability $p_\infty(\lambda) = \Pr$(component containing the origin has an infinite number of nodes)
  - Critical density $\lambda_c = \inf\{\lambda > 0 : p_\infty(\lambda) > 0\}$
- If $\lambda < \lambda_c$, largest component of $G(\mathcal{X}_n, 1)$ contains $O(\ln n)$ nodes a.a.s. (asymptotic almost surely) — subcritical phase
- If $\lambda > \lambda_c$, there exists unique connected component containing $\Theta(n)$ nodes of $G(\mathcal{X}_n^{(0)})$ a.a.s. — supercritical phase
  - This largest component called giant component — $\mathcal{C}(G(\mathcal{X}_n^{(0)}))$.

RGG with Node Mobility

- Random geometric graph with mobile nodes $G(\mathcal{X}^{(t)})$:
  - Each node u moves according to mobility model $M(t), t = 0, 1, 2, ...$
  - u and v can communicate with each other (share a link) at time t iff $d_t(u, v) \triangleq ||\mathbf{X}_u^{(t)} - \mathbf{X}_v^{(t)}|| \leq 1$
- When $M(t)$ results in Poisson spatial distribution of $\{\mathcal{X}^{(t)}\}$ for all time, critical density of $G(\mathcal{X}^{(t)})$ is same as one for static model
  - If network at time 0 is (not) percolated, then network is (not) percolated at any time.
  - If $\lambda > \lambda_c$, then for each $t \geq 0$, $G(\mathcal{X}^{(t)})$ is percolated, i.e. exists a giant component in $G(\mathcal{X}^{(t)})$ with probability 1.

Modelling Mobile Wireless Networks

- Map mobile wireless networks to stationary wireless networks with dynamic links
  - Nodes positions in stationary network are same as initial positions of mobile network
  - Link is on (active) in stationary network whenever end nodes of link are within communication range in mobile network
- Dynamic behavior viewed as "mobility-induced-fading process"

Wireless Networks with Unreliable Links

- Model for wireless networks with unreliable links: $G(\mathcal{H}_\lambda, 1, p_e(\cdot))$:
  - Given $G(\mathcal{H}_\lambda, 1)$, each link (i, j) is active with probability $p_e(d_{ij})$.
  - $G(\mathcal{H}_\lambda, 1, p_e(\cdot))$ consists of active links and their end nodes.
- Link quality varies with time—shadowing and fading
- Dynamic unreliable links: Markov On-off process $\{W_{ij}(d_{ij}, t)\}$:
  - $W_{ij}(d_{ij}, t) = 0/1$ if link (i, j) is inactive/active at time t.
  - $\{W_{ij}(d_{ij}, t)\}$ probabilistically identical for same d_{ij} – $\{W(d, t)\}$.
- Percolation-based connectivity and latency results (INFO-COM'08)

Constrained I.I.D. Mobility Model

- Given initial positions $\{\mathcal{X}^{(0)}\}$
- At each time $t = 0, 1, 2, ...,$ $\mathbf{X}_u^{(t+1)}$ is uniformly distributed at random in $\mathcal{A}(\mathbf{X}_u^{(0)}, a)$—circular region centered at $\mathbf{X}_u^{(0)}$ with radius $a > 0$
- $\mathbf{X}_u^{(t)}$ are mutually independent among all nodes and independent of all previous locations $\mathbf{X}_u^{(t')}, t' = 1, ..., t-1$.
- As $a \to \infty$, constrained i.i.d. mobility model becomes an unconstrained i.i.d. model

First Meeting and Exiting Times

- Given $\{\mathcal{X}^{(0)}\}$ and mobility model $M(t)$
- First meeting time of nodes i and j:

$$T_m(i,j) \triangleq \inf\{t \geq 0 : d_t(i,j) \leq 1\}$$

- First exiting time of nodes i and j:

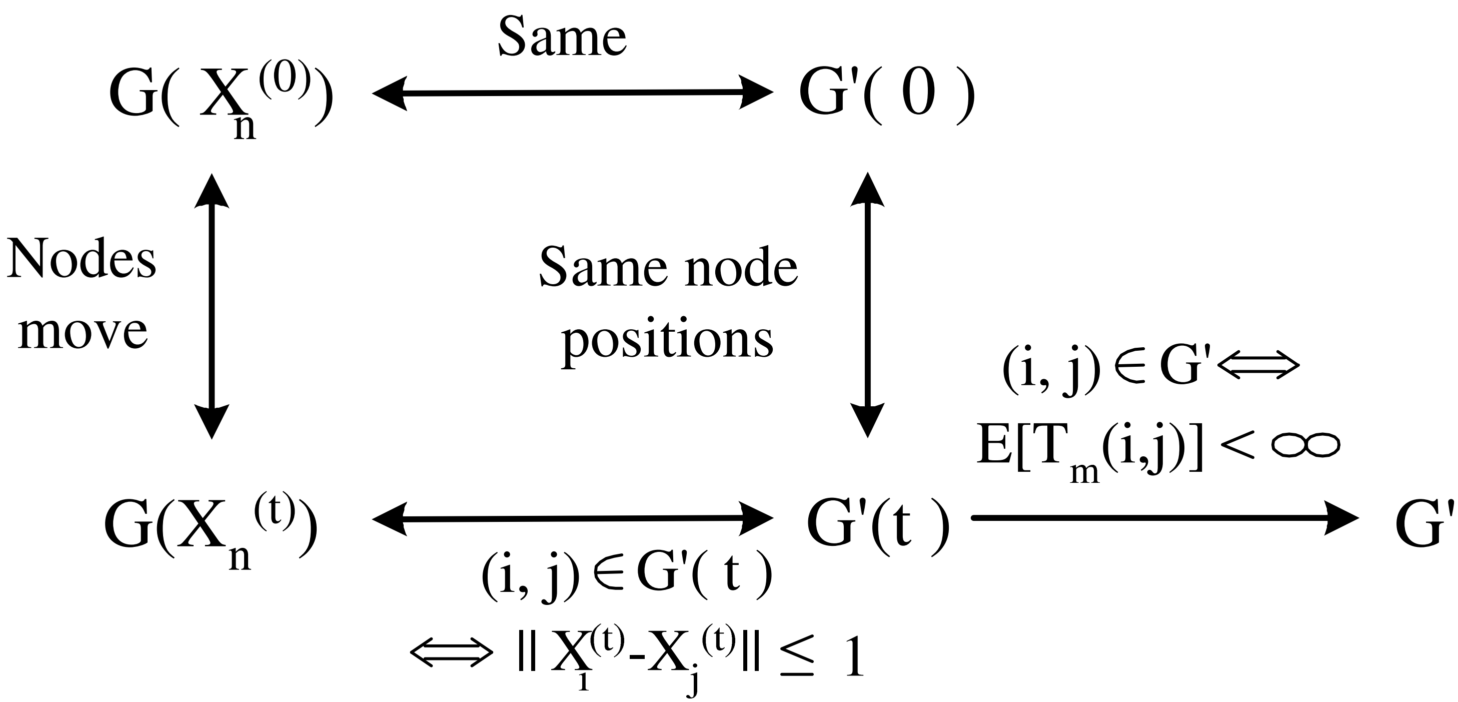
$$T_e(i,j) \triangleq \inf\{t \geq 0 : d_t(i,j) > 1\}$$

- By definition
  - $T_m(i,j) = 0$ and $T_e(i,j) > 0$ when $d_0(i,j) \leq 1$
  - $T_m(i,j) > 0$ and $T_e(i,j) = 0$ when $d_0(i,j) > 1$

Dynamic and Long-Term Connectivity Graphs

- Given $\{\mathcal{X}^{(0)}\}$ and mobility model $M(t)$
- Dynamic connectivity graph $G'(t)$
  - Nodes are located at $\{\mathcal{X}^{(0)}\}$
  - Link exists between i and j iff $d_t(i, j) \leq 1$
- Long-term connectivity graph G'
  - Nodes are located at $\{\mathcal{X}^{(0)}\}$
  - Link exists between i and j iff $E[T_m(i, j)] < \infty$

Relationships



- In both $G'(t)$ and G', nodes fixed to initial random positions rather than being mobile

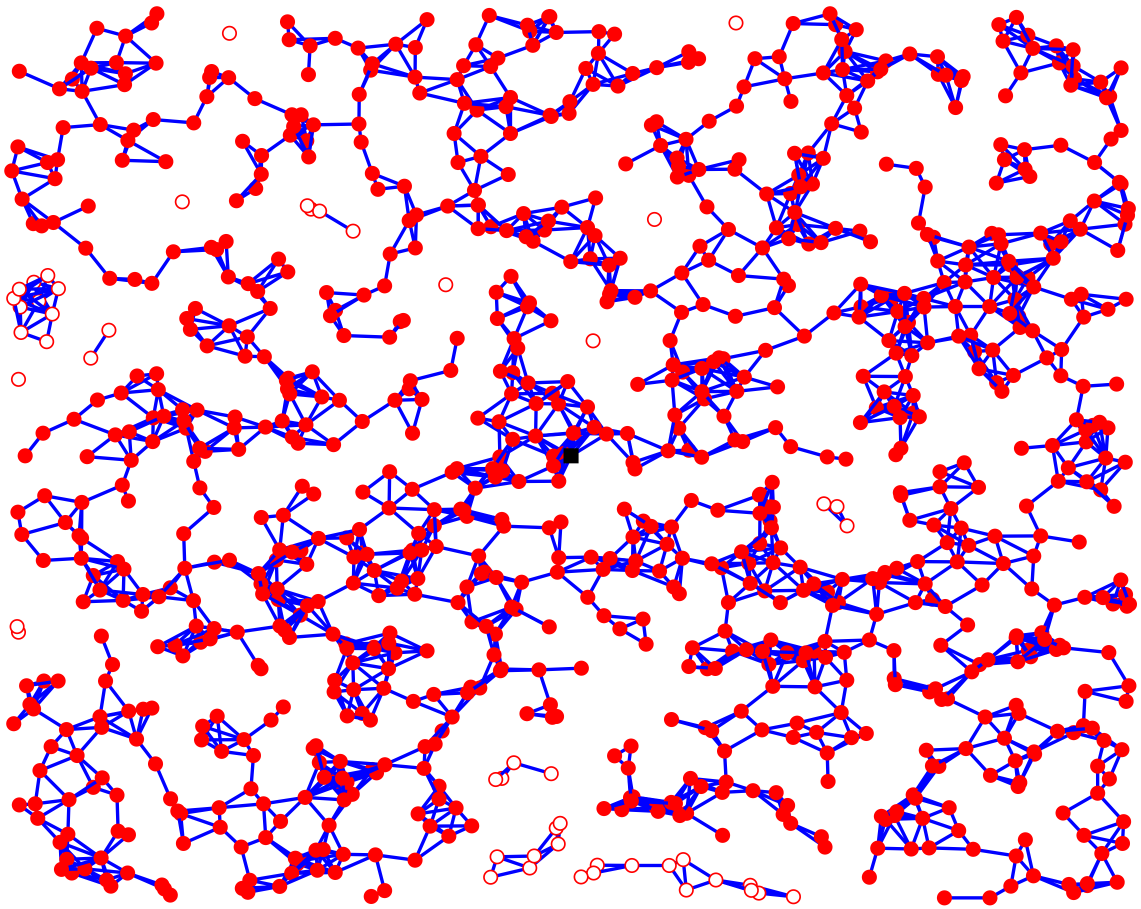
Information Dissemination in G'

- When u and v are connected in G', at least one path from u to v consisting of links in G'
- If u broadcasts a message, v can receive it within finite expected time
  - End nodes of each link along the path have finite expected first meeting time
- When u and v are not connected in G', v cannot receive message (in finite expected time)
- Assume G' is percolated

Information Dissemination in $G'(t)$

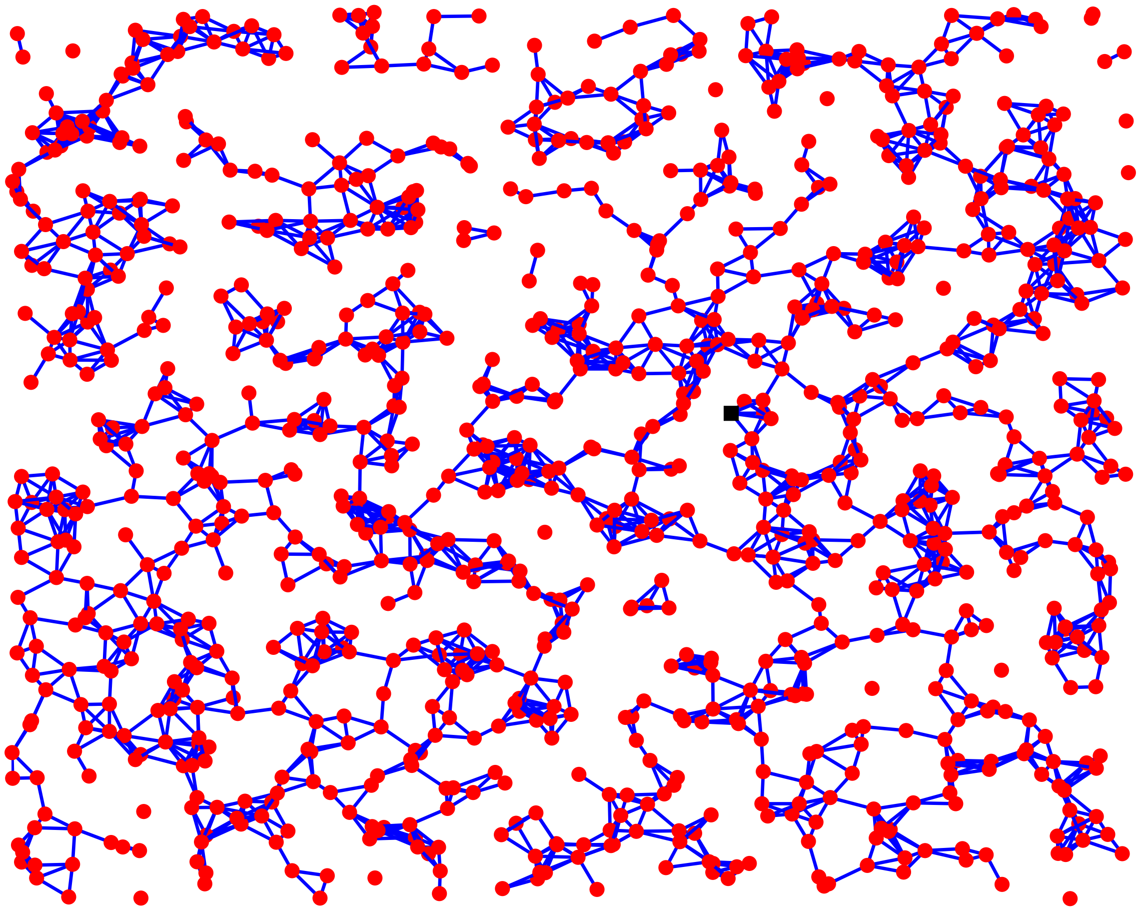
- Supercritical phase: $G(\mathcal{X}^{(t)})$ is percolated for all t
  - One node inside $\mathcal{C}(G(\mathcal{X}^{(0)}))$ broadcasts message at time 0
  - Ignoring propagation delay, all nodes in $\mathcal{C}(G(\mathcal{X}^{(0)}))$ receive message instantaneously.
  - Nodes in $\mathcal{C}(G')$ but not in $\mathcal{C}(G(\mathcal{X}^{(0)}))$ can receive message later

Information Dissemination in $G'(t)$



- 1000 nodes on $[-12, 12]^2$, constraint radius $=$ 5
- Source (black nodes) broadcasts message M at $t = 0$.
- Blue links—exist, red nodes—received, white nodes—not received

Information Dissemination in $G'(t)$

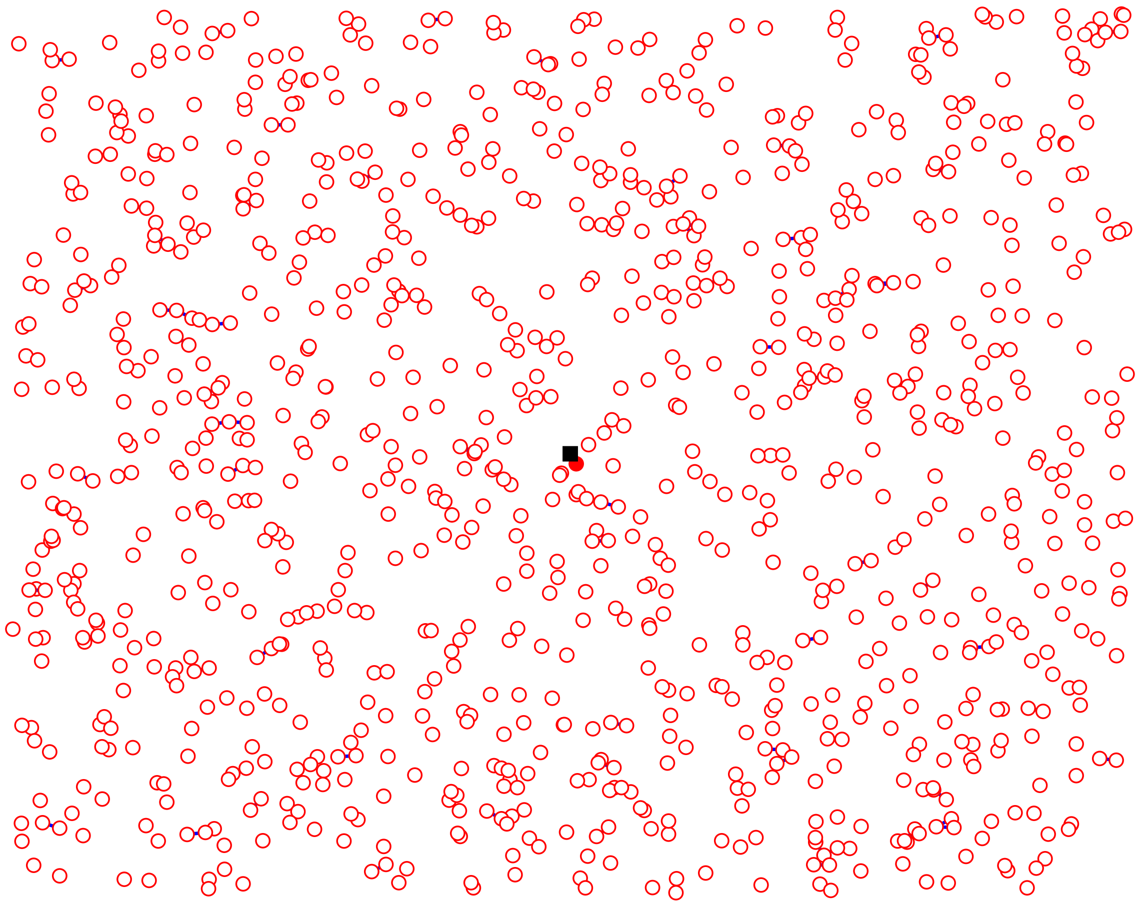


- 1000 nodes on $[-12, 12]^2$, constraint radius $= 5$
- By $t = 2$, all nodes in $\mathcal{C}(G')$ have received M.
- Links: blue—on, green—off; Nodes: red—received, white—not received

Information Dissemination in $G'(t)$

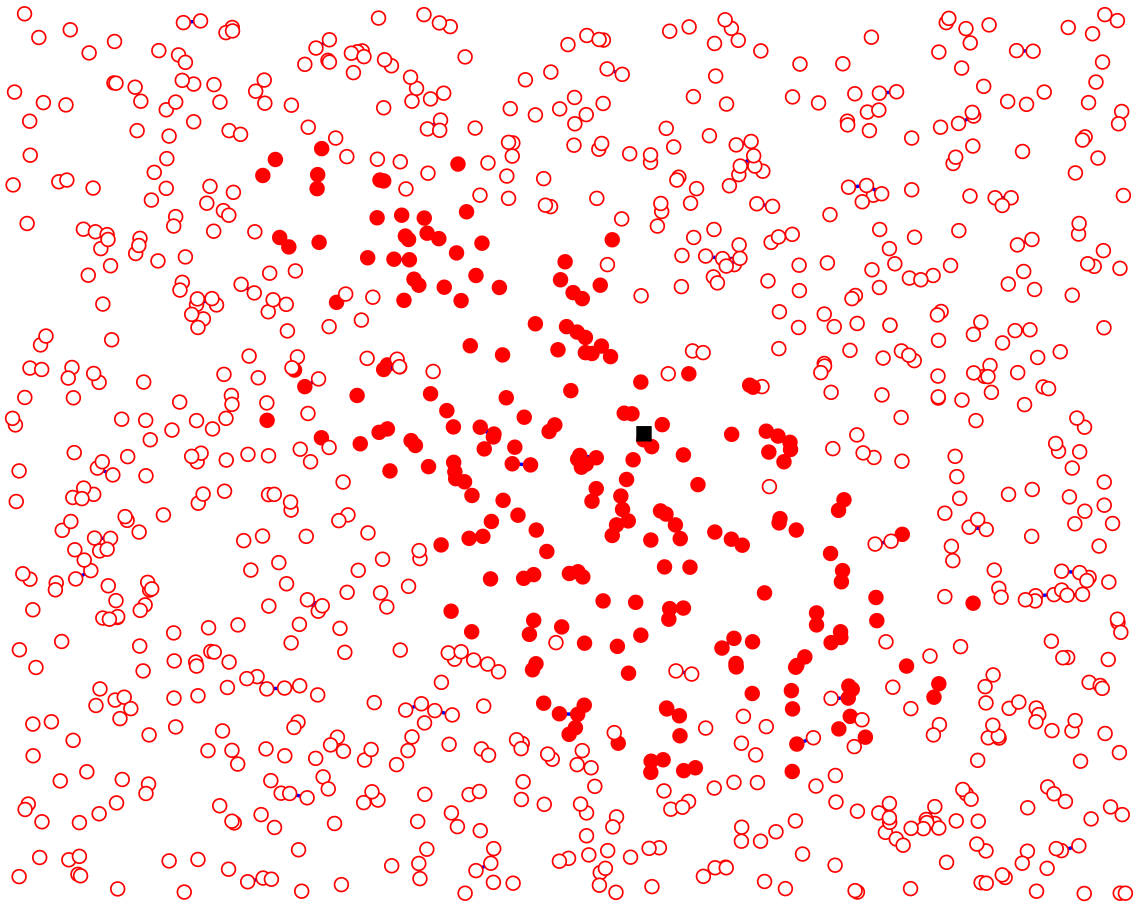
- Supercritical phase: $G(\mathcal{X}^{(t)})$ is percolated for all t
  - One node inside $\mathcal{C}(G(\mathcal{X}^{(0)}))$ broadcasts message at time 0
  - Ignoring propagation delay, all nodes in $\mathcal{C}(G(\mathcal{X}^{(0)}))$ receive message instantaneously.
  - Nodes in $\mathcal{C}(G')$ but not in $\mathcal{C}(G(\mathcal{X}^{(0)}))$ can receive message later
- Subcritical phase: $G(\mathcal{X}^{(t)})$ is not percolated at any t.
  - If two nodes u and v are in $\mathcal{C}(G')$, information can be eventually transmitted from u to v
  - Large delay

Information Dissemination in $G'(t)$



- 1000 nodes on $[-30, 30]^2$, constraint radius $= 5$
- Source (black nodes) broadcasts message M at $t = 0$.
- Blue links—exist, red nodes—received, white nodes—not received

Information Dissemination in $G'(t)$



- 1000 nodes on $[-30, 30]^2$, constraint radius $=$ 5.
- By $t = 25$, only 200 nodes have received message M.
- Blue links—exist, red nodes—received, white nodes—not received

Latency of Information Dissemination

- $\mathcal{I}_u^{(t_0)}(t)$: set of nodes that have received source message broadcast at time t_0 by u up to time t

- $T^{(t_0)}(u,v)$: first time node v receives message, i.e.,

$$T^{(t_0)}(u,v) \triangleq \inf\{t : v \in \mathcal{I}_u^{(t_0)}(t)\}$$

- $T^{(t_0)}(u,v)$ is latency of information dissemination from u to v

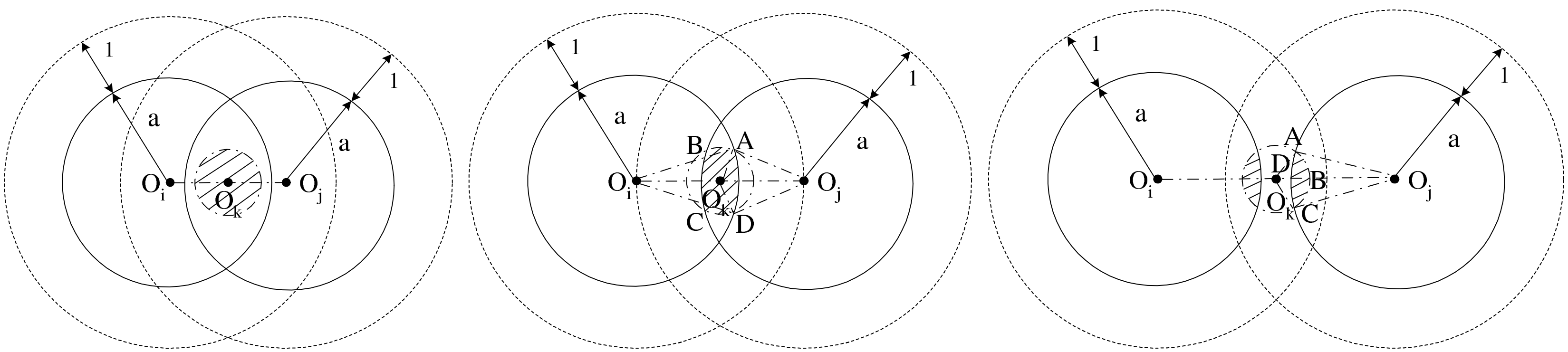
- Assume source node u initiates broadcast at time 0, write $T^{(t_0)}(u,v)$ as $T(u,v)$

Condition for Finite First Meeting Time

Lemma 1: Given $\{\mathcal{X}^{(0)}\}$ and constrained i.i.d. mobility model with constraint radius $a > 0$, for any two nodes i and j,

$$0 < E[T_m(i,j)] < \infty \text{ iff } 1 < d_0(i,j) < 2a+1$$

- Proof:



(a) $||O_i - O_j|| < 2a - 1$ (b) $2a - 1 \le ||O_i - O_j|| < 2a$ (c) $2a \le ||O_i - O_j|| < 2a + 1$

Condition for Finite First Exit Time

Lemma 2: Given $\{\mathcal{X}^{(0)}\}$ and constrained i.i.d. mobility model with constraint radius $a > \frac{1}{2}$, for any two nodes i and j

$$0 < E[T_e(i,j)] < \infty \text{ if } d_0(i,j) \leq 1$$

- Proof is similar to previous one

Percolation in Constrained I.I.D. Model

Proposition 1: Given $G(\mathcal{X}^{(0)})$ and constrained i.i.d. mobility model with constraint radius $a > 0$, critical density for G' is

$$\lambda_c(a) = \frac{\lambda_c}{(2a+1)^2}$$

where λ_c is critical density for $G(\mathcal{X}^{(0)})$.

- Proof:
  - By Lemma 1, there exists a link between i and j in G' iff $d_0(i,j) < 2a+1$
  - Use scaling property of random geometric graphs

Latency in Constrained I.I.D. Model

Theorem 3: Given $G(\mathcal{X}^{(0)})$ under constrained i.i.d. mobility model with $a > 1/2$ and $\lambda > \lambda_c(a)$, for $u, v \in \mathcal{C}(G')$, ignoring propagation delay, $\exists$ constant $0 < \gamma < \infty$

(i) if $\lambda < \lambda_c$, i.e., $G(\mathcal{X}^{(t)})$ is not percolated at any time,

$$\Pr\left(\lim_{d_0(u,v)\to\infty} \frac{T(u,v)}{d_0(u,v)} = \gamma\right) = 1$$

(ii) if $\lambda > \lambda_c$, i.e., $G(\mathcal{X}^{(t)})$ is percolated at any time,

$$\Pr\left(\lim_{d_0(u,v)\to\infty} \frac{T(u,v)}{d_0(u,v)} = 0\right) = 1$$

- Latency of information dissemination scales
  - Linearly with initial Euclidean distance between sender and receiver if $G(\mathcal{X}^{(t)})$ is in subcritical phase
  - Sub-linearly with distance if $G(\mathcal{X}^{(t)})$ is in supercritical phase

Mobility-Induced Fading Process

- Associate "mobility-induced fading process" $W_{i,j}(t)$ with each link $(i,j) \in G'$
  - $W_{i,j}(t) = 0$ if $d_t(i,j) > 1$ (i.e., $(i,j) \notin G'(t)$)
  - $W_{i,j}(t) = 1$ if $d_t(i,j) \leq 1$ (i.e., $(i,j) \in G'(t)$)
- $W_{i,j}(t)$ has i.i.d. inactive periods $Y_k(i,j)$ and i.i.d. active periods $Z_k(i,j)$.
  - $E[Y_k(i,j)] = E[T_m(i,j)]$
  - $E[Z_k(i,j)] = E[T_e(i,j)]$

First Passage Percolation

- Similar to first passage percolation problems
- $T_{(i,j)}$: delay with link (i,j)
  - Random variable depends on $W_{i,j}(t)$.
- Define

$$T(u,v) = \inf_{l(u,v)\in\mathcal{L}(u,v)} \left\{ \sum_{(i,j)\in l(u,v)} T_{(i,j)} \right\}$$

  - $l(u,v)$: path from u to v in G'
  - $\mathcal{L}(u,v)$: set of all such paths
  - $T(u,v)$: message delay on path having smallest delay

Lemma on Convergence

- Let

$$\tilde{\mathbf{X}}_i \triangleq \operatorname*{argmin}_{\mathbf{X}_j^{(0)} \in \mathcal{C}(G')} \{||(i,0) - \mathbf{X}_j^{(0)}||\},$$

$$T_{l,m} \triangleq T(\tilde{\mathbf{X}}_l, \tilde{\mathbf{X}}_m), 0 \leq l \leq m$$

Lemma 3: Let

$$\gamma \triangleq \lim_{m \to \infty} \frac{E[T_{0,m}]}{m}$$

Then,

$$\gamma = \inf_{m \geq 1} \frac{E[T_{0,m}]}{m}, \text{ and } \lim_{m \to \infty} \frac{T_{0,m}}{m} = \gamma \text{ with probability 1}$$

- Proof based on Subadditive Ergodic Theorem

Subadditive Ergodic Theorem

Theorem 2 (Liggett'85): Let $\{S_{l,m}\}$ be a collection of random variables indexed by integers $0 \le l < m$. Suppose $\{S_{l,m}\}$ has the following properties:

(i) $S_{0,m} \le S_{0,l} + S_{l,m}, 0 \le l \le m$;

(ii) $\{S_{(m-1)j,mj}, m \ge 1\}$ is a stationary process for each j;

(iii) $\{S_{l,l+j}, j \ge 0\} = \{S_{l+1,l+1+j}, j \ge 0\}$ in distribution for each l;

(iv) $E[|S_{0,m}|] < \infty$ for each m.

Then

(a) $\alpha \triangleq \lim_{m\to\infty} \frac{E[S_{0,m}]}{m} = \inf_{m\ge1} \frac{E[S_{0,m}]}{m}$, $S \triangleq \lim_{m\to\infty} \frac{S_{0,m}}{m}$ exists with probability 1, and $E[S] = \alpha$.

Furthermore, if

(v) the stationary process in (ii) is ergodic,

then

(b) $S = \alpha$ with probability 1.

Lemma on Positiveness and Finiteness

Lemma 4: Let γ be defined as in Lemma 1, if $\lambda < \lambda_c$, then

$$0 < \gamma < \infty$$

- Proof based on following Exponential Decay Proposition

Proposition 2: Given $G(\mathcal{X}^{(0)})$ with $\lambda_c(a) < \lambda < \lambda_c$, let $B(h) = [-h, h]^2$, $h \in \mathbb{R}^+$. Then there exist $c_1, c_2 > 0$ such that for any $t > 0$, $\Pr(\tilde{\mathbf{X}}_0^{(0)} \leftrightsquigarrow B(h)^c) \leq c_1 e^{-c_2 h}$, where $\{\tilde{\mathbf{X}}_0^{(0)} \leftrightsquigarrow B(h)^c\}$ denotes event that the node closest to the origin at time 0 and some nodes in $B(h)^c$ are connected.

Proof of Theorem 1-(i)

- Consider any two nodes $u, v \in \mathcal{C}(G')$.
- Suppose $G(\mathcal{X}^{(0)})$ is subcritical. Then, as $d_0(u,v) \to \infty$, u and v cannot lie within the same component of $G'(0)$, so that $T(u,v) > 0$.
- Assume $\mathbf{X}_u^{(0)} = \mathbf{0}$, and take line $\mathbf{X}_u^{(0)}\mathbf{X}_v^{(0)}$ as x-axis.
- Let m be closest integer to $x(v)$—x-axis coordinate of node $\mathbf{X}_v^{(0)}$.
- Now $T_{0,m} = T(\mathbf{X}_u^{(0)}, \tilde{\mathbf{X}}_m)$.
- If $\mathbf{X}_v^{(0)} = \tilde{\mathbf{X}}_m$, then $T(u,v) = T_{0,m}$, and since $m-1 \le d_0(u,v) \le m+1$,
$$\frac{T_{0,m}}{m+1} \le \frac{T(u,v)}{d_0(u,v)} \le \frac{T_{0,m}}{m-1}.$$
- If $\mathbf{X}_v^{(0)} \ne \tilde{\mathbf{X}}_m$, then $\tilde{\mathbf{X}}_m$ must be adjacent to $\mathbf{X}_v^{(0)}$.
- Because $||(m,0) - \mathbf{X}_v^{(0)}|| \le \frac{1}{2}$ (m is closest integer to $x(v)$), $||(m,0) - \tilde{\mathbf{X}}_m|| \le \frac{1}{2}$ ($\tilde{\mathbf{X}}_m$ is closest node to $(m,0)$).

Proof of Theorem 1-(i) (con'd)

- Consequently, $T_{0,m} - T(\tilde{\mathbf{X}}_m, \mathbf{X}_v^{(0)}) \le T(u,v) \le T_{0,m} + T(\tilde{\mathbf{X}}_m, \mathbf{X}_v^{(0)})$, so that

$$\frac{T_{0,m} - T(\tilde{\mathbf{X}}_m, \mathbf{X}_v^{(0)})}{m+1} \le \frac{T(u,v)}{d_0(u,v)} \le \frac{T_{0,m} + T(\tilde{\mathbf{X}}_m, \mathbf{X}_v^{(0)})}{m-1}.$$

- Since $\tilde{\mathbf{X}}_m$ is adjacent to $\mathbf{X}_v^{(0)}$, $T(\tilde{\mathbf{X}}_m, \mathbf{X}_v^{(0)}) < \infty$ by Lemma 1.

- Therefore in both cases, by Lemma 3,

$$\lim_{d_0(u,v)\to\infty} \frac{T(u,v)}{d_0(u,v)} = \lim_{m\to\infty} \frac{T_{0,m}}{m} = \gamma$$

  with probability 1.

- By Lemma 4, $0 < \gamma < \infty$.

Proof of Theorem 1-(ii)

- Now, suppose $G(\mathcal{X}^{(0)})$ is supercritical, then for $u, v \in \mathcal{C}(G')$, as $d_0(u,v) \to \infty$, it is possible that they are within $\mathcal{C}(G'(0))$.
- In this situation, $T(u,v) = 0$
- Now assume neither node u nor v is in $\mathcal{C}(G'(0))$
- Let t' be first time that some node (and therefore all nodes) in $\mathcal{C}(G'(t'))$ receives u's message
- Let $w_1 \triangleq \mathrm{argmin}_{i \in \mathcal{C}(G'(t'))} d_{t'}(i,u)$, and $w_2 \triangleq \mathrm{argmin}_{i \in \mathcal{C}(G'(t'))} d_{t'}(i,v)$.
- That is, w_1 and w_2 are nodes in $\mathcal{C}(G'(t'))$ with smallest Euclidean distances to nodes u and v, respectively.
- Since both w_1 and w_2 belong to $\mathcal{C}(G'(t'))$, $T^{(t')}(w_1, w_2) = 0$.

Proof of Theorem 1-(ii) (con'd)

- Can show $T^{(t')}(u, w_1) < \infty$ and $T^{(t')}(w_2, v) < \infty$ with probability 1.
- Moreover,

$$0 \leq \frac{T(u,v)}{d_0(u,v)} \leq \frac{T^{(t')}(u,w_1) + T^{(t')}(w_1,w_2) + T^{(t')}(w_2,v)}{d_0(u,v)} = \frac{T^{(t')}(u,w_1) + T^{(t')}(w_2,v)}{d_0(u,v)} < \infty.$$

- Therefore

$$\Pr\left(\lim_{d_0(u,v)\to\infty} \frac{T(u,v)}{d_0(u,v)} = 0\right) = 1$$

- Applying same technique for case where only one of u and v is in $\mathcal{C}(G'(0))$, obtain the same result.

Latency with Propagation Delay

Corollary 1: Given $G(\mathcal{X}^{(0)})$ under constrained i.i.d. mobility model with $a > 1/2$, $\lambda > \lambda_c(a)$ and propagation delay τ, for $u, v \in \mathcal{C}(G')$, $\exists$ constants $\tau < \gamma_2 \leq \gamma_1 < \infty$

(i) if $\lambda < \lambda_c$, i.e., $G(\mathcal{X}^{(t)})$ is subcritical for all time,

$$\Pr\left(\lim_{d_0(u,v)\to\infty} \frac{T(u,v)}{d_0(u,v)} = \gamma_1\right) = 1$$

(ii) if $\lambda > \lambda_c$, i.e., $G(\mathcal{X}^{(t)})$ is supercritical for all time,

$$\Pr\left(\lim_{d_0(u,v)\to\infty} \frac{T(u,v)}{d_0(u,v)} = \gamma_2\right) = 1$$

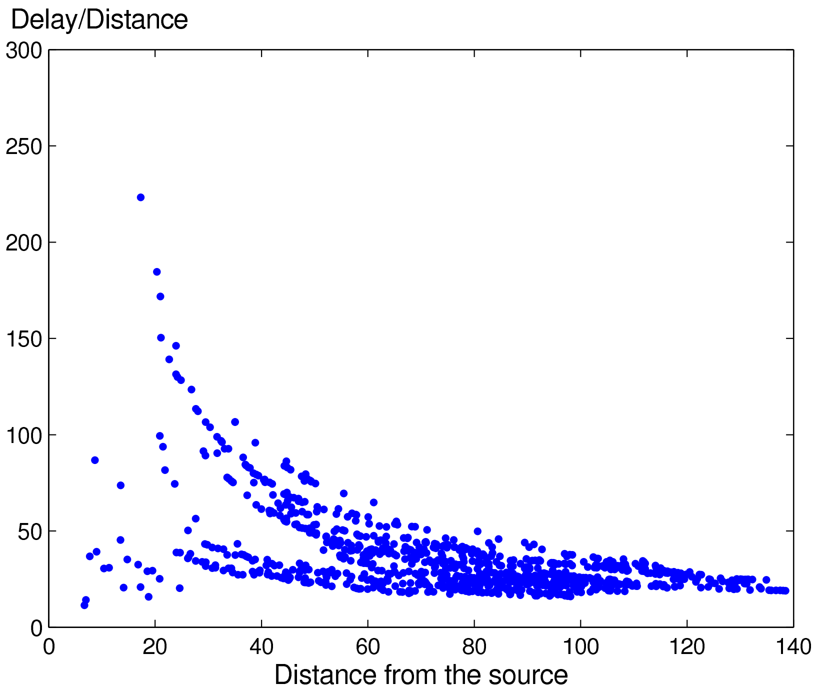
Moreover, as $\tau \to 0$, $\gamma_1 \to \gamma$ and $\gamma_2 \to 0$.

Extensions: Other Mobility Models

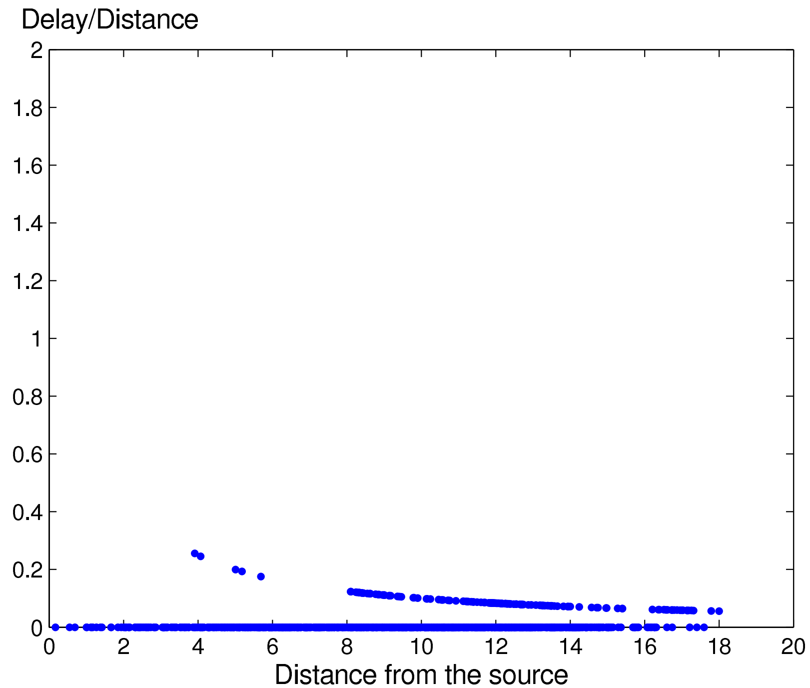
- Discrete-time Brownian motion mobility model
  - At each time t, each node u follows two-dimensional Brownian motion
  - $\mathbf{X}_u^{(t+1)} = (X_{u,1}^{(t+1)}, X_{u,2}^{(t+1)})$
  - $X_{u,1}^{(t+1)} = X_{u,1}^{(t)} + \sigma W_1$ and $X_{u,2}^{(t+1)} = X_{u,2}^{(t)} + \sigma W_2$
  - σ—variance of Brownian motion
  - W_1, W_2—two independent standard Normal random variables

- Random walk (Euclidean space) mobility model
  - At each time t, each node u uniformly chooses a random direction $\theta_u^{(t)} \in [0, 2\pi)$ and a random speed $v_u^{(t)} \in (0, v_{max})$
  - $\mathbf{X}_u^{(t+1)} = (X_{u,1}^{(t+1)}, X_{u,2}^{(t+1)})$
  - $X_{u,1}^{(t+1)} = X_{u,1}^{(t)} + v_u^{(t)} \cos(\theta_u^{(t)})$
  - $X_{u,2}^{(t+1)} = X_{u,2}^{(t)} + v_u^{(t)} \sin(\theta_u^{(t)})$

Latency: Constrained I.I.D. Mobility



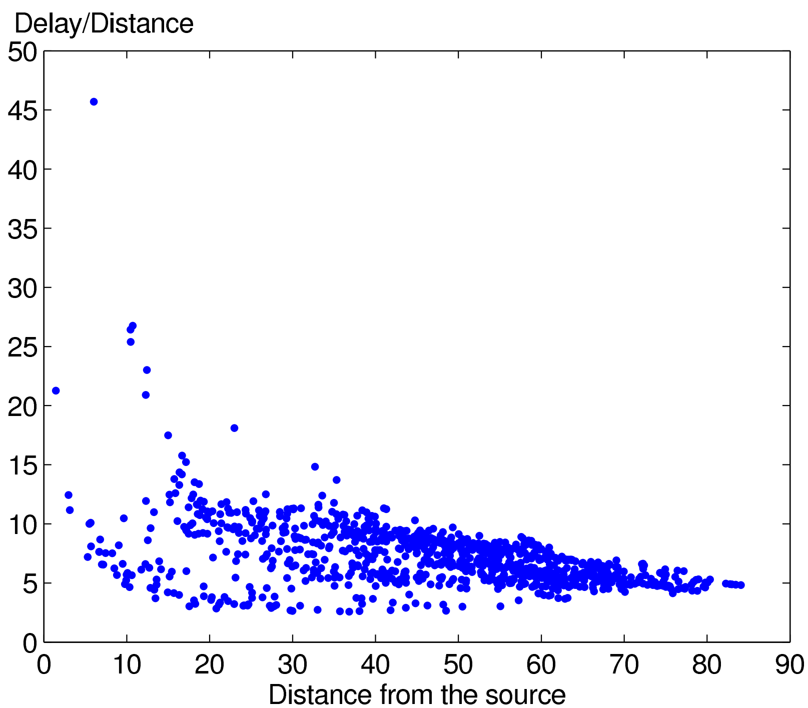
(d) Subcritical



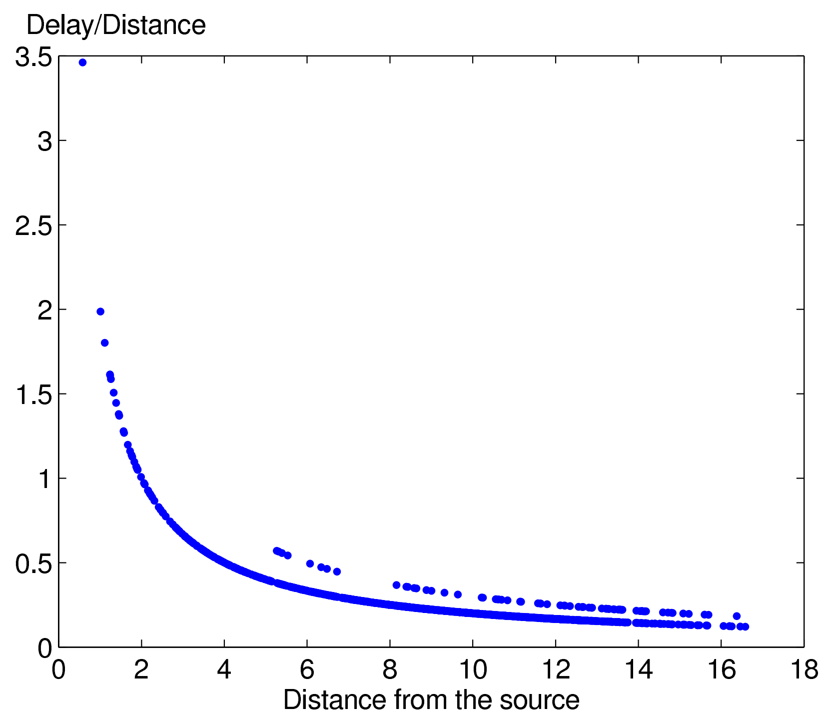
(e) Supercritical

- Constrained radius $a = 5$, propagation delay $\tau = 0$, density $\lambda = 0.1$ (subcritical) and $\lambda = 1.73$ (supercritical)

Latency: with Propagation Delay



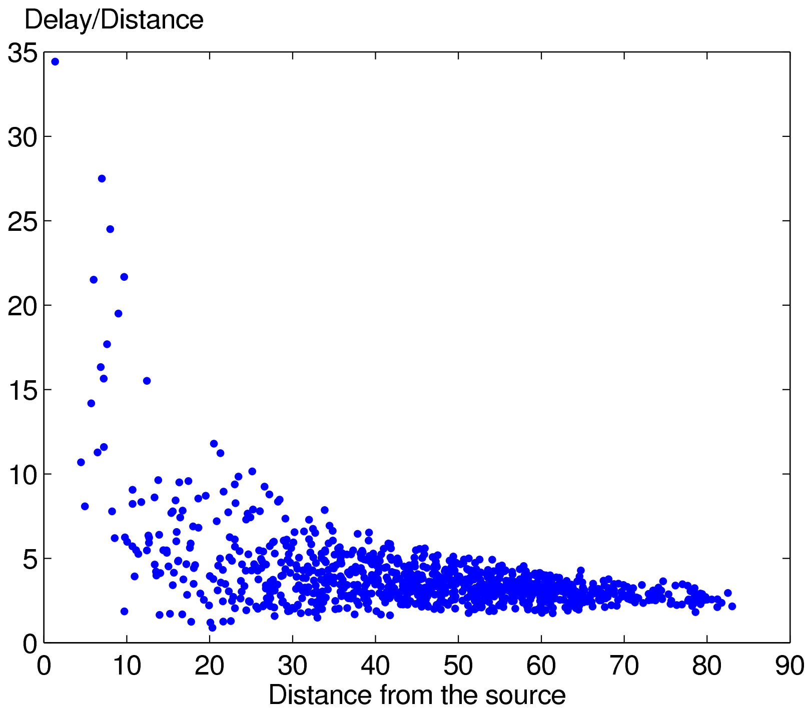
(f) Subcritical



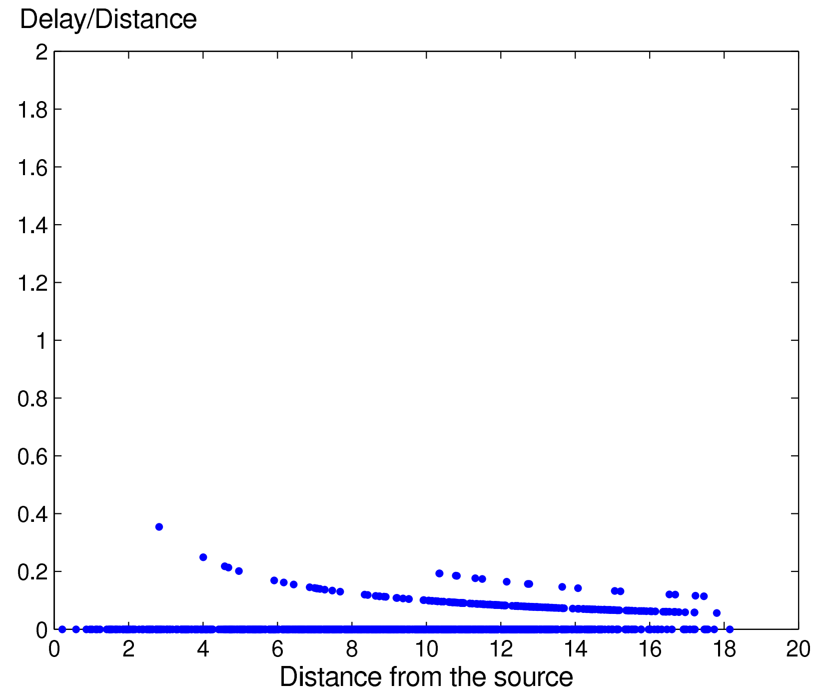
(g) Supercritical

- Constrained radius $a = 6$, propagation delay $\tau = 1$, density $\lambda = 0.1$ (subcritical) and $\lambda = 2.0$ (supercritical)

Latency: Discrete-Time Brownian Motion



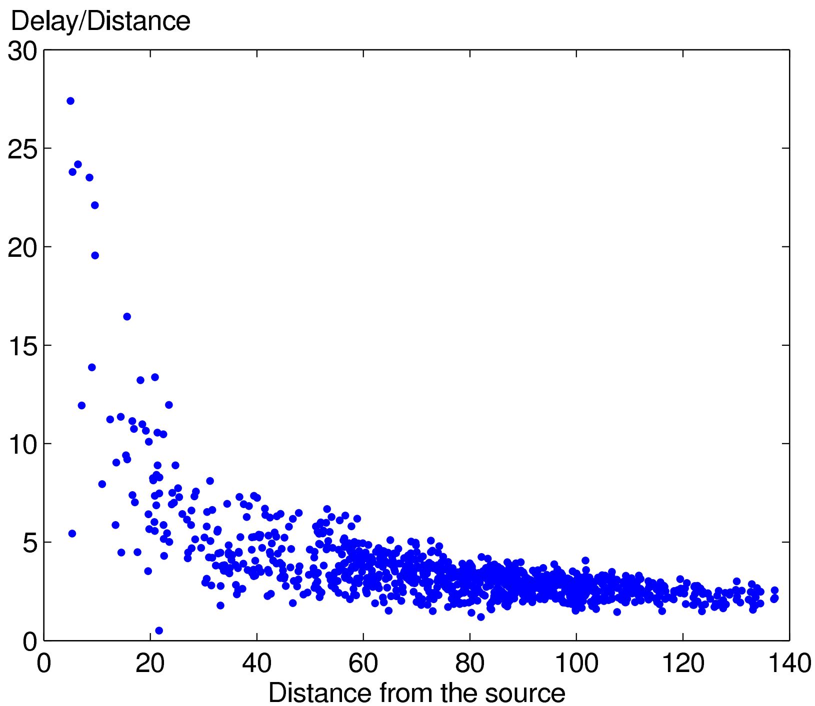
(h) Subcritical



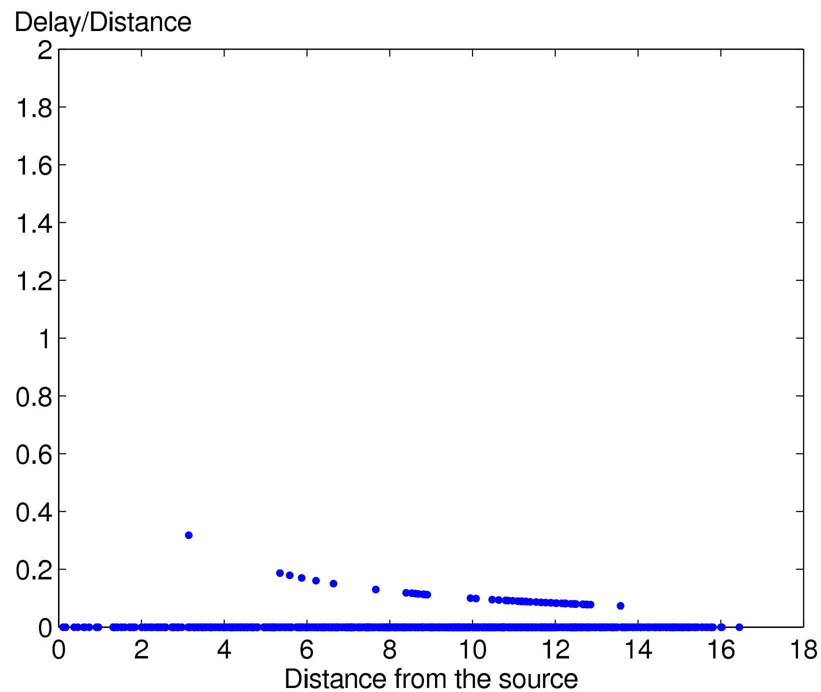
(i) Supercritical

- Brownian motion variance $\sigma = 1$, propagation delay $\tau = 0$, density $\lambda = 0.07$ (subcritical) and $\lambda = 1.73$ (supercritical)

Latency: Random Walk (Euclidean Space)



(j) Subcritical



(k) Supercritical

- $v \sim \mathcal{U}(0,2), \theta \sim \mathcal{U}(0,2\pi)$, propagation delay $\tau = 0$, density $\lambda = 0.1$ (subcritical) and $\lambda = 1.73$ (supercritical)

Conclusion

- Studied information dissemination in large-scale mobile wireless networks
- Introduced “mobility-induced fading process” to map mobile networks to stationary networks with dynamic links
- Obtained scaling behavior results on the latency
  - Linear when subcritical
  - Sublinear when supercritical

Thank you!