Theoretical Benefits of Fluctuations in Physiological Signals

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FMIPW 2010 - Problem 5

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Problem Statement:

1. Do fractal structures in time and space offer the most efficient means to dissipate energy gradients?

and

2. Are these structures self-organizing?

Problem Interpretation:

Objective 1: Find justifications for fluctuations in vital signs e.g. Interbeat Intervals, Peak Pressure

Objective 2: Explore methods for generating a fractal or branching structure upon a lattice.

Mathematicians are like Frenchmen: whatever you say to them they translate into their own language and forthwith it is something entirely different

Goethe

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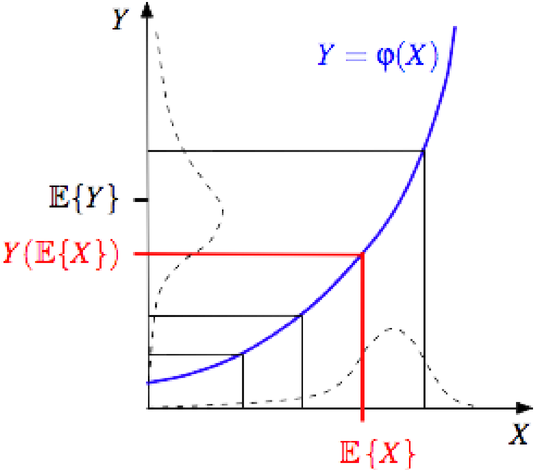
$\Rightarrow$ use Jensen's inequality.

Inequality (Jensen's)

Given a random variable X and a convex function $\varphi(x)$ then $\varphi(E(X)) \leq E(\varphi(X))$



But does such a relation exist for the cardiovascular system?

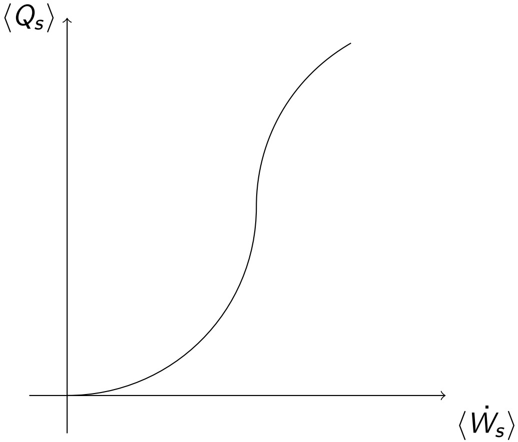


Figure: A hypothetical (mythical?) convex relation between Average Power-per-cycle $\langle \dot{W}_s \rangle$ vs. Average Flow Rate, $\langle Q_s \rangle$

Model[1]: Fluid flow through a valve:

$$\dot{Q} + aQ|Q| + bQ = P(t)$$

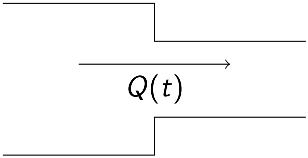


Figure: Flow through a valve

[1] Liang et al (2009). Multi-scale modeling of the human cardiovascular system with applications to aortic valvular and arterial stenoses. Med. Biol. Eng. Comput. 47: 743-755

Set up an optimal control problem for maximizing flow

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Consider:

$$\max_{P(t)} \int_0^{T_s} Qdt \;;$$

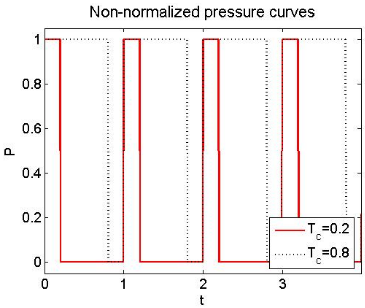
subject to

$$\int_0^{T_s} PQdt = W_s \sim \text{fixed} \tag{3}$$

and recall

$$\dot{Q} + aQ|Q| + bQ = P(t)$$

Reduce the $P(t)$ functional form to one-parameter square waves.



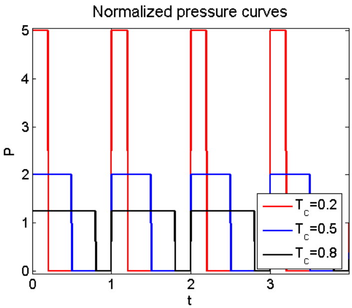


Figure: P - waveforms: Amplitude-preserving (non-normalized) on the left, Area-preserving (normalized) on the right.

However results were inconclusive and did not give us the relations between W_s and Q that we were looking for.

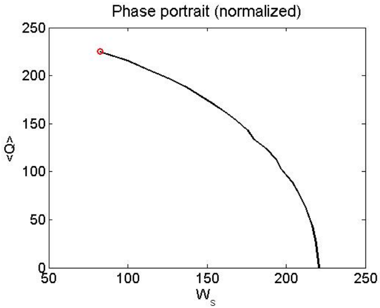


Figure: Flow as a function of $W_s(T_s)$ for area-preserving P-waveforms

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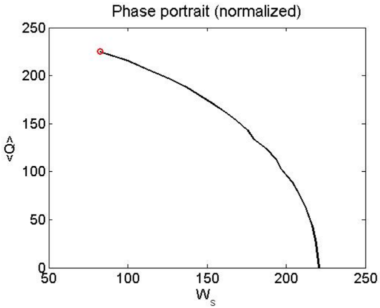


Figure: Flow as a function of $W_s(T_s)$ for area-preserving P-waveforms

$\Rightarrow$ Change the objective function!

Maximize smoothness:

Consider:

$$\min_{P(t)} \left\{ J(P) = \left[\int_0^{T_s} (Q - \bar{Q})^2 dt \right]^{1/2} \right\};$$

where

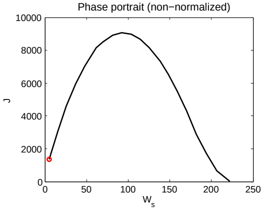
$$\bar{Q} = \frac{1}{T_s} \int_0^{T_s} Q dt \tag{6}$$

subject to

$$\int_0^{T_s} PQ dt = W_s \sim \text{fixed} \tag{7}$$

and

$$\dot{Q} + aQ|Q| + bQ = P(t)$$



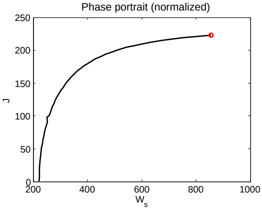


Figure: Q-variation as a function of $W_s(T_s)$: Amplitude-preserving P-waveforms on the left, Area-preserving P-waveforms on the right.

Model elaborations - include elastic wall of the aorta.

$$\dot{Q} + aQ|Q| + bQ = P(t) - P_a$$

$$\theta(P_a)\dot{P}_a + \frac{P_a}{R_a} = Q$$

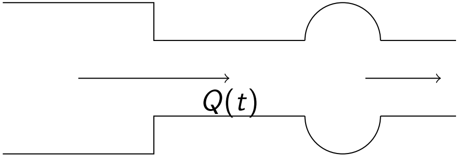


Figure: Flow through a valve + aorta

However results remain essentially the same.

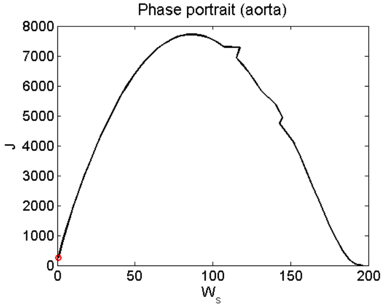


Figure: Q-variation as a function of $W_s(T_s)$ for the valve+aorta model: Amplitude-preserving P-waveforms on the left, Area-preserving P-waveforms on the right.

Objective 1 Restatement: Find justifications for fluctuations in vital signs e.g. Interbeat Intervals, Peak Pressure based on Jensen's inequality.

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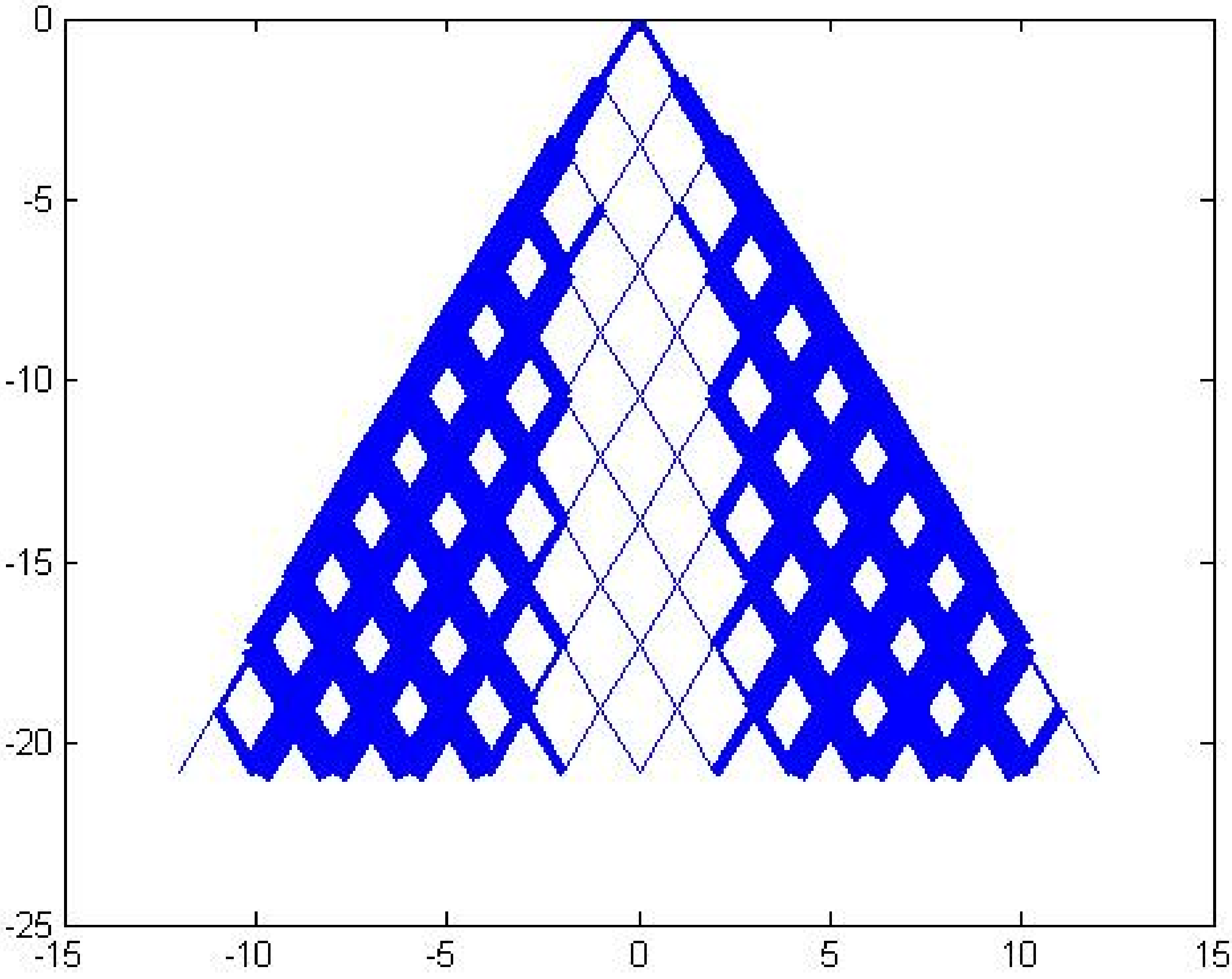
Conclusions: Using a simple heart model, we have shown how fluctuations can result in a smoother flow rate.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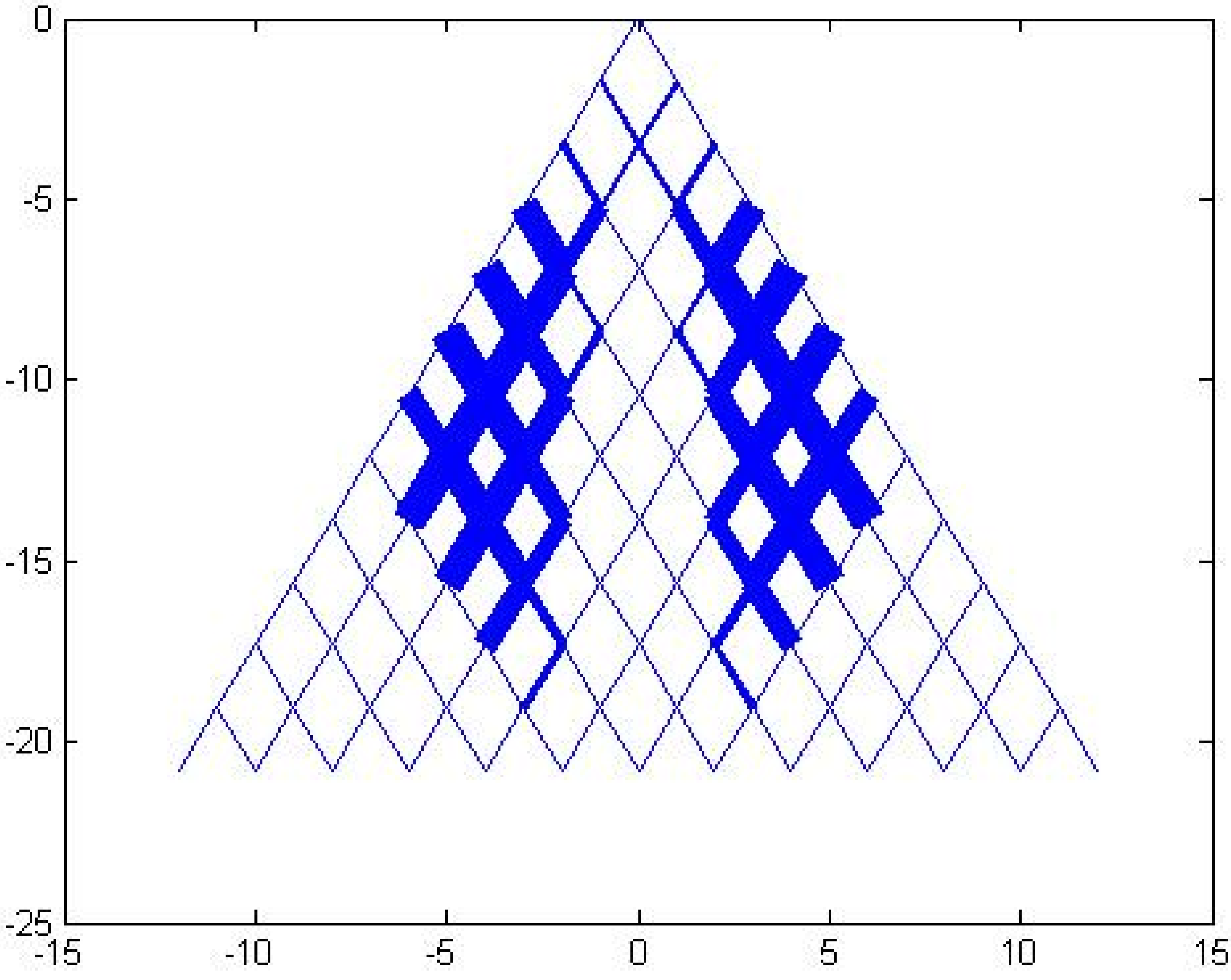

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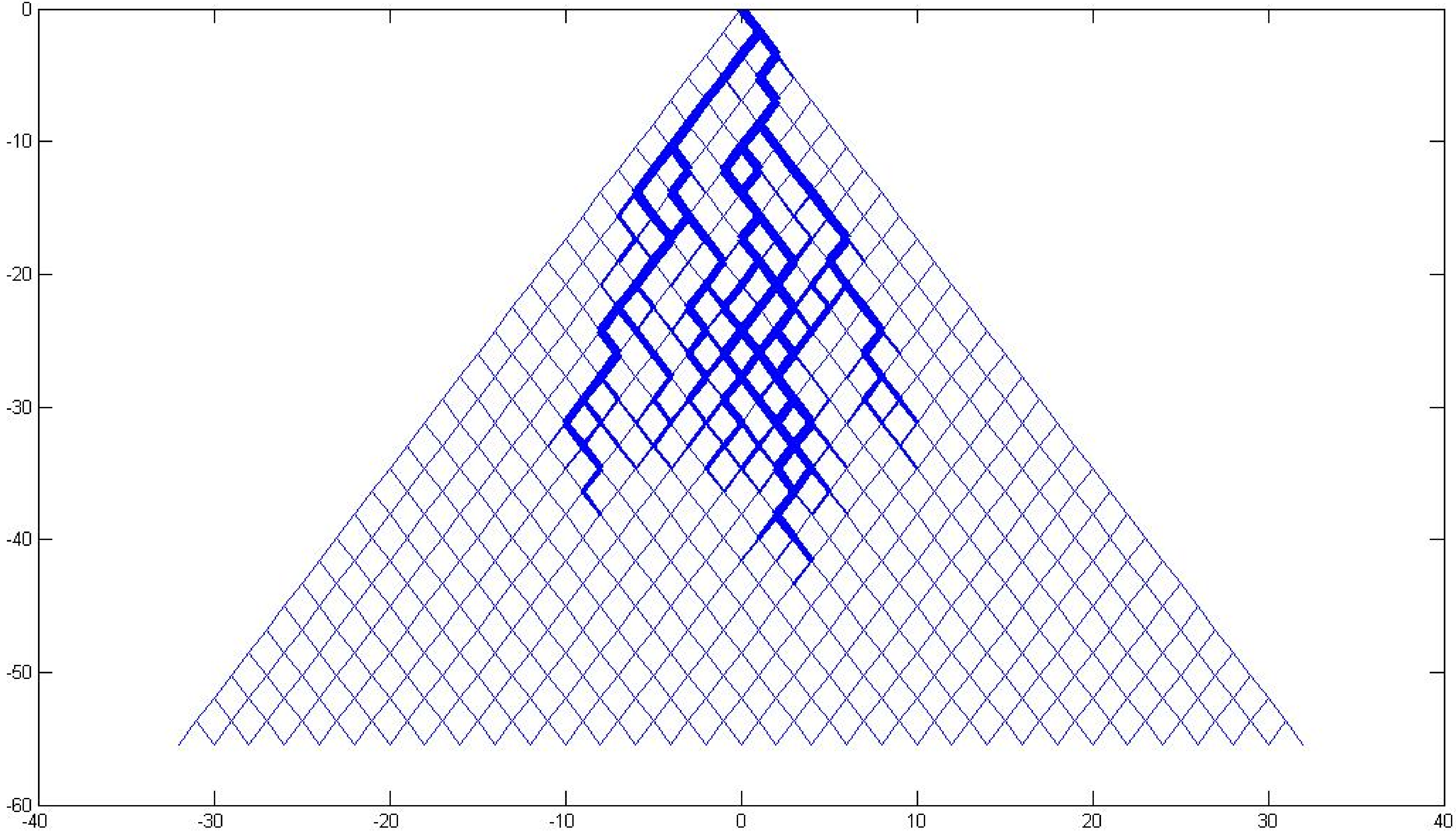

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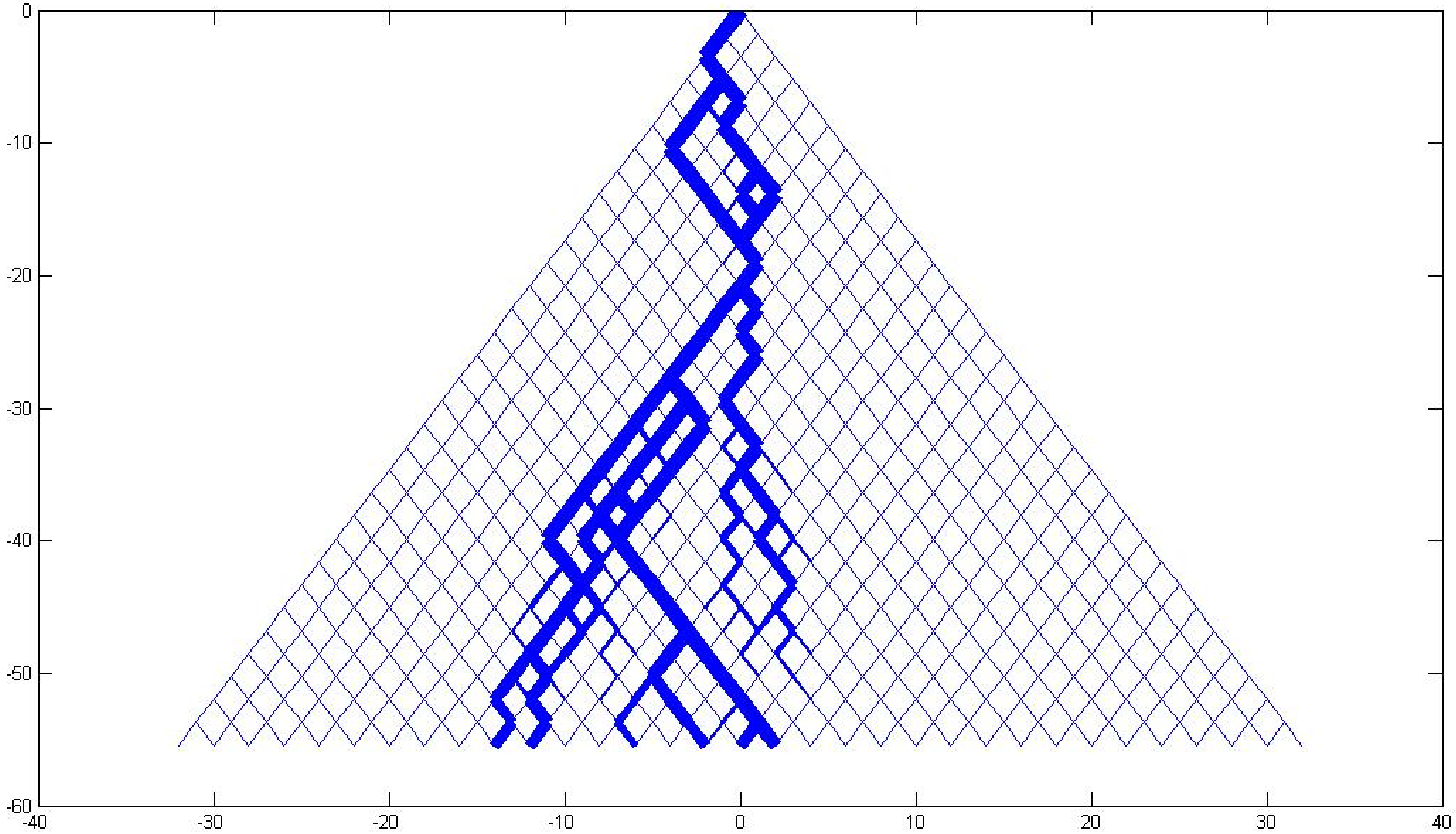

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