Writing CFT correlation functions as AdS scattering amplitudes

João Penedones
Perimeter Institute for Theoretical Physics

JP, arXiv:1011.1485
Okuda, JP, arXiv:1002.2641
Heemskerk, JP, Polchinski, Sully, arXiv:0907.0151
Gary, Giddings, JP, arXiv:0903.4437

Connections in Geometry and Physics
Toronto, May 15th, 2011

Outline

- Introduction
- Mellin amplitudes
- Flat space limit of AdS
- Open questions

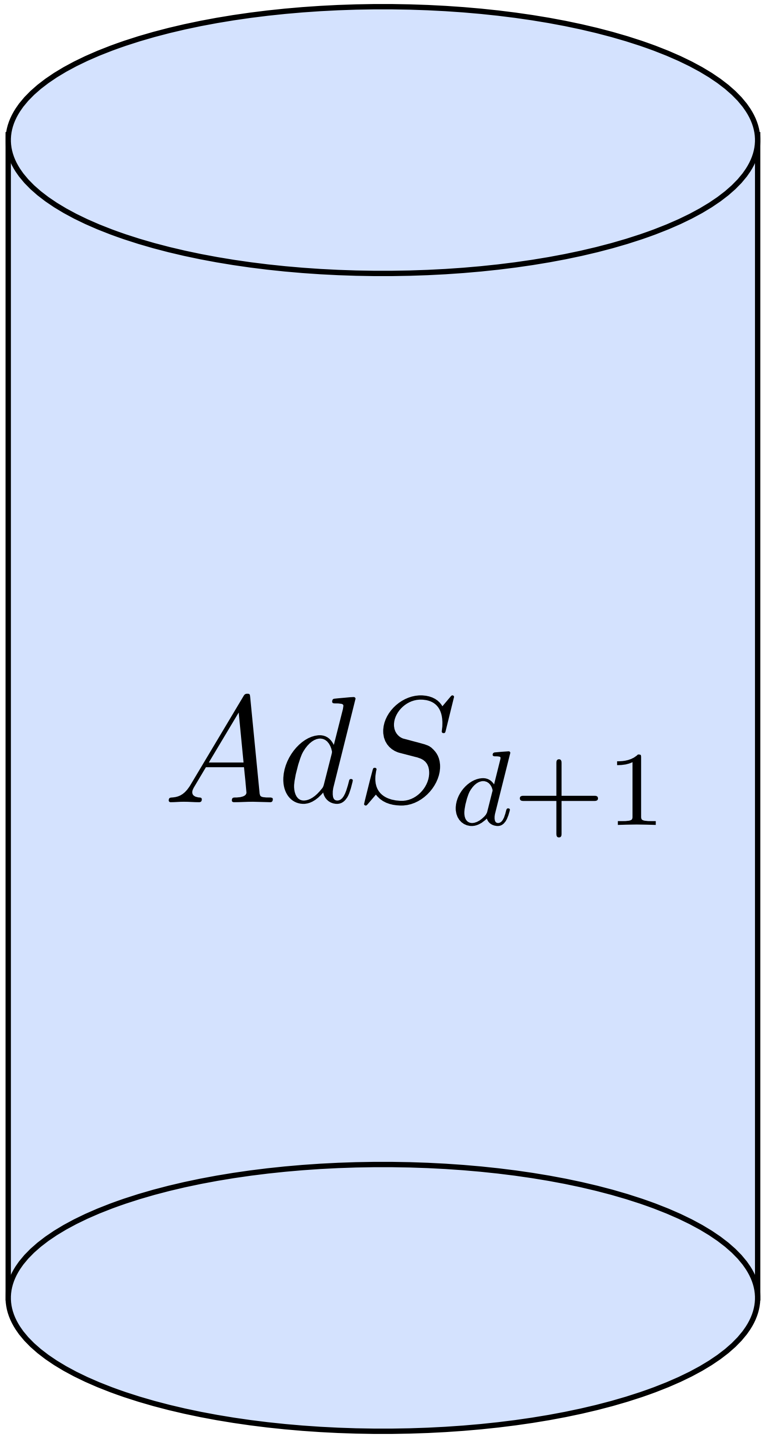
Introduction

Quantum Gravity with AdS boundary conditions

$\updownarrow$

Conformal Field Theory on the conformal boundary of AdS

$\mathbb{R} \times S^{d-1}$ or $\mathbb{R}^d$



local fields ϕ_i in AdS $\longleftrightarrow$ CFT primary operators $\mathcal{O}_i$

Introduction

The AdS description of a CFT is useful if it is

- **Weakly coupled** $\longleftrightarrow$ "large N" factorization

$$\langle \mathcal{O}_1 \ldots \mathcal{O}_n \rangle_{\text{connected}} \sim \kappa^{n-2} \,, \qquad \kappa^2 \sim G R^{1-d} \ll 1$$

- **Local** - effective field theory in AdS with small number of fields valid up to some UV cutoff ℓ much smaller than the AdS radius R

$$\ell \sim \frac{1}{\text{mass}} \sim \frac{R}{\Delta}$$

$\rightarrow$ Large gap in spectrum of dimensions $\Delta \gg 1$

local fields ϕ_i in AdS $\longleftrightarrow$ CFT primary operators $\mathcal{O}_i$

Spectrum of Effective CFT

$\Delta \gg 1$ Other single-particle primary operators

AdS coupling constant

Multi-particle primaries

$$\dim\left(\mathcal{O}_1 \partial^n \mathcal{O}_2\right) = \Delta_1 + \Delta_2 + n + \mathcal{O}\left(1/N^2\right)$$

$\mathcal{O}(1)$ Few single-particle primaries spin ≤ 2

Unitary bound

0

[Heemskerk, JP, Polchinski, Sully '09]
[Fitzpatrick, Katz, Poland, Simmons-Duffin '10]
[El-Showk, Papadodimas '11]

A Conjecture

[Heemskerk, JP, Polchinski, Sully '09]

Any CFT that has a large-N expansion, and in which all single-trace operators of spin greater than two have parametrically large dimensions, has a local bulk dual.

Large-N expansion $\longleftrightarrow$ Weakly coupled bulk dual $\frac{l_P}{R} \ll 1$

Single-trace operator $\longleftrightarrow$ Single-particle state

Large-N vector models have weakly coupled non-local bulk duals (AdS higher spin theories).

[Sezgin, Sundell 02]
[Klebanov, Polyakov 02]
[Fradkin, Vasiliev 87, ...]

Is there a CFT dual of GR?

$\Delta \gg 1$ Other "single-trace"

Gravitational coupling

"Multi-trace" primaries

$$\dim\left(T\partial^n T\right) = 2d + n + \mathcal{O}\left(GR^{1-d}\right)$$
$$\dim\left(T\partial^n T\partial^m T\right) = 3d + n + m + \mathcal{O}\left(GR^{1-d}\right)$$

$2d$

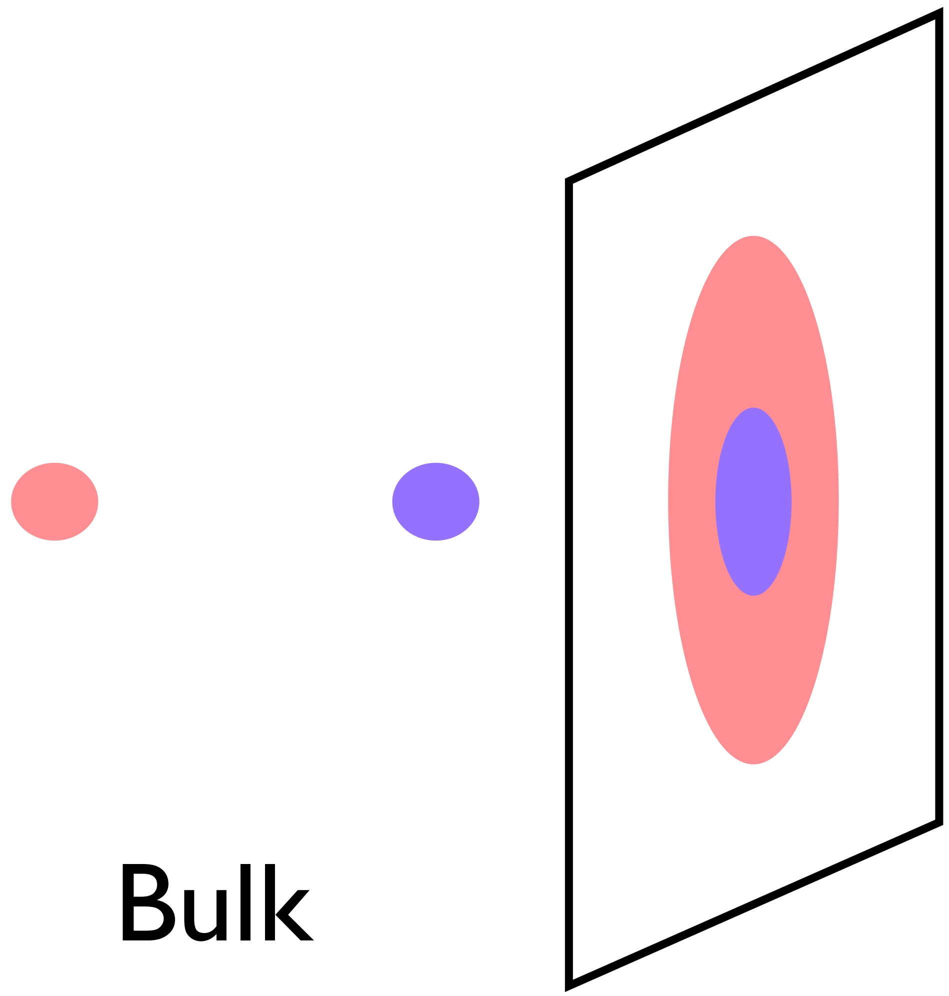
d Stress-energy tensor

0

Weak coupling $\Leftrightarrow$ $\boxed{|\Lambda| \ll M_{\mathrm{P}}^2}$

Bulk Locality

How can a field theory generate an extra dimension?

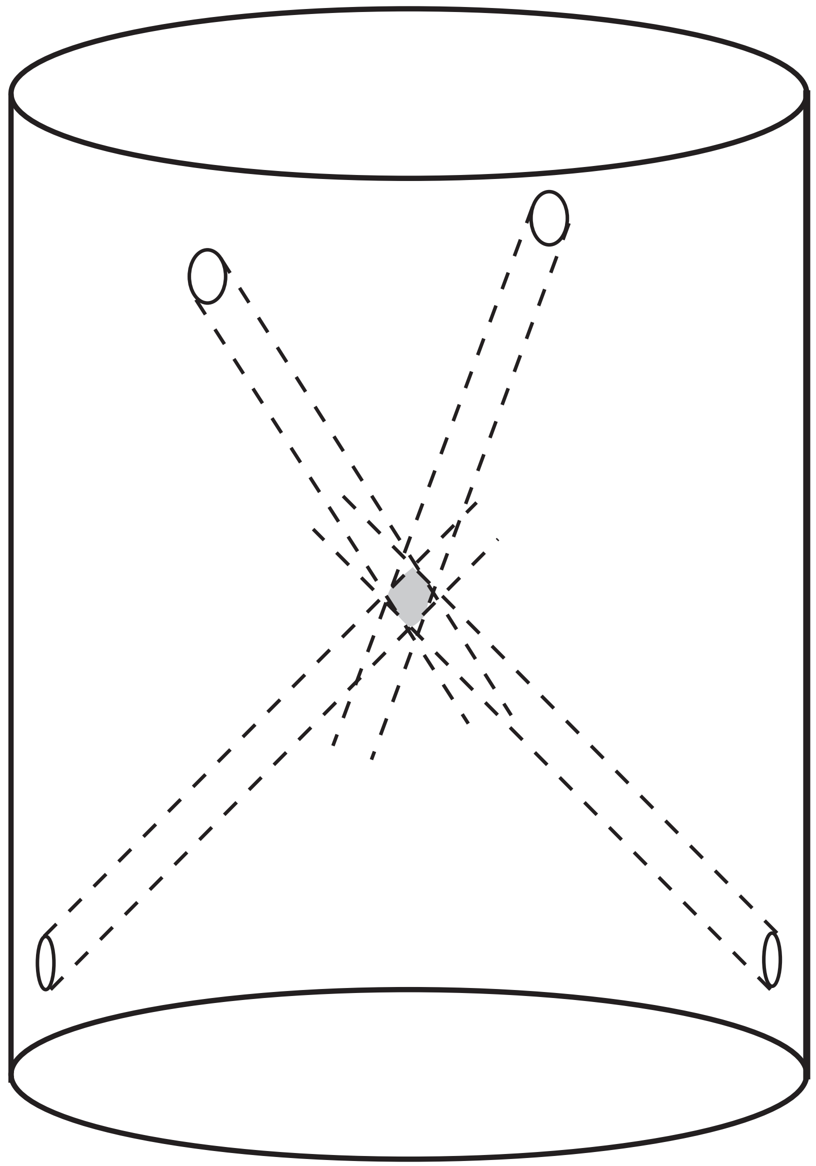


How can we probe local bulk physics in the CFT?

Bulk Locality

Main difficulty: local bulk physics is encoded in CFT correlation functions in a non-trivial way.

How to extract the bulk S-matrix?



Idea: prepare wave-packets that scatter in small region of AdS

[Polchinski 99]
[Susskind 99]
[Gary, Giddings, JP '09]
[Okuda, JP '10]

Best language: Mellin amplitudes

Mellin Amplitudes

Mellin Amplitudes

[Mack '09]

Correlation function of scalar primary operators

$$A(x_i) = \langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle$$

$$A(x_i) = \mathcal{N} \int_{-i\infty}^{i\infty} [d\delta] \, M(\delta_{ij}) \prod_{i<j}^{n} \Gamma(\delta_{ij}) \left(x_{ij}^2\right)^{-\delta_{ij}}$$

Constraint $\displaystyle \sum_{j \neq i}^{n} \delta_{ij} = \Delta_i = \dim[\mathcal{O}_i]$

integration variables = # ind. cross-ratios = $\dfrac{n(n-3)}{2}$

Mellin Amplitudes

[Mack '09]

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Analogy with Scattering Amplitudes

Introduce k_i such that $-k_i^2 = \Delta_i$ and $\sum_{i=1}^n k_i = 0$

then $\delta_{ij} = k_i \cdot k_j$ automatically solves the constraints.

Define $s_{ij} = -(k_i + k_j)^2 = \Delta_i + \Delta_j - 2\delta_{ij}$ [Mack '09]

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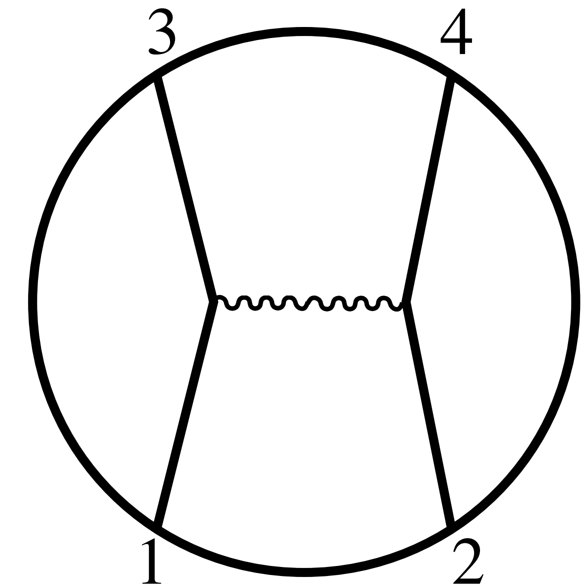
The Mellin amplitude is crossing symmetric and meromorphic with simple poles at ($n = 4$)

$$M(s_{ij}) \approx \frac{C_{13k} C_{24k} \, P_{l_k}^{m}(\gamma_{13})}{s_{13} - (\Delta_k - l_k + 2m)} \qquad m = 0, 1, 2, \ldots$$

$$\mathcal{O}_1 \mathcal{O}_3 \sim C_{13k} \mathcal{O}_k$$
$$\mathcal{O}_2 \mathcal{O}_4 \sim C_{24k} \mathcal{O}_k$$

$$\gamma_{13} = \frac{1}{2}(s_{12} - s_{14})$$

Example: Graviton exchange in AdS5



Minimally coupled massless scalars

$$\Delta_i = d = 4$$

[D'Hoker, Freedman, Mathur, Mathusis, Rastelli '99]

$$
\begin{aligned}
A(x_i) \propto\ & 9D_{4444}(x_i) - \frac{4}{3x_{13}^6} D_{1414}(x_i) - \frac{20}{9x_{13}^4} D_{2424}(x_i) - \frac{23}{9x_{13}^2} D_{3434}(x_i) \\
& + \frac{16(x_{14}^2 x_{23}^2 + x_{12}^2 x_{34}^2)}{3x_{13}^6} D_{2525}(x_i) + \frac{64(x_{14}^2 x_{23}^2 + x_{12}^2 x_{34}^2)}{9x_{13}^4} D_{3535}(x_i) \\
& + \frac{8(x_{14}^2 x_{23}^2 + x_{12}^2 x_{34}^2 - x_{24}^2 x_{13}^2)}{x_{13}^2} D_{4545}(x_i) \ .
\end{aligned}
$$

D-function

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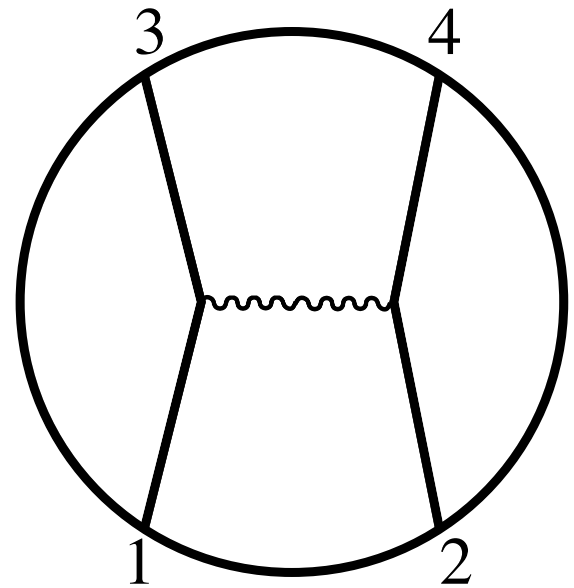

$$A(x_i) \propto 9D_{4444}(x_i) - \frac{4}{3x_{13}^6}D_{1414}(x_i) - \frac{20}{9x_{13}^4}D_{2424}(x_i) - \frac{23}{9x_{13}^2}D_{3434}(x_i)$$
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$$+\frac{8(x_{14}^2x_{23}^2 + x_{12}^2x_{34}^2 - x_{24}^2x_{13}^2)}{x_{13}^2}D_{4545}(x_i)\ .$$

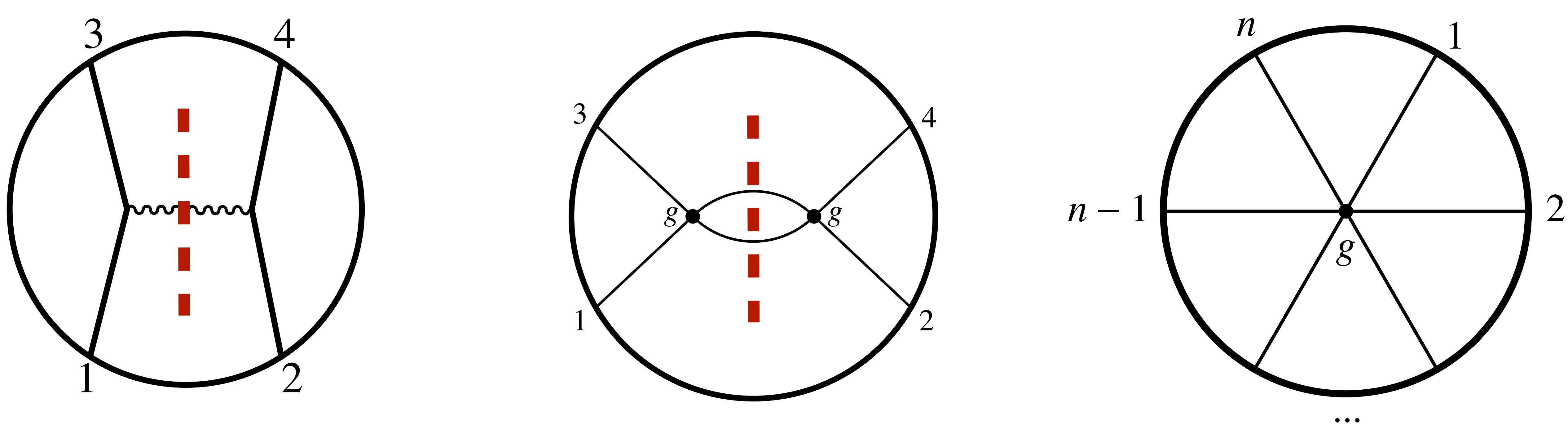
D-function

$$M(s_{ij}) \propto \frac{6\gamma_{13}^2+2}{s_{13}-2} + \frac{8\gamma_{13}^2}{s_{13}-4} + \frac{\gamma_{13}^2-1}{s_{13}-6} - \frac{15}{4}s_{13} + \frac{55}{2}$$

Double-trace operators

The double-trace operators $\mathcal{O}_i \partial^n \mathcal{O}_j$ (normal ordered product of external operators) do **not** give rise to poles in the Mellin amplitude.

All poles are associated with on-shell internal states.



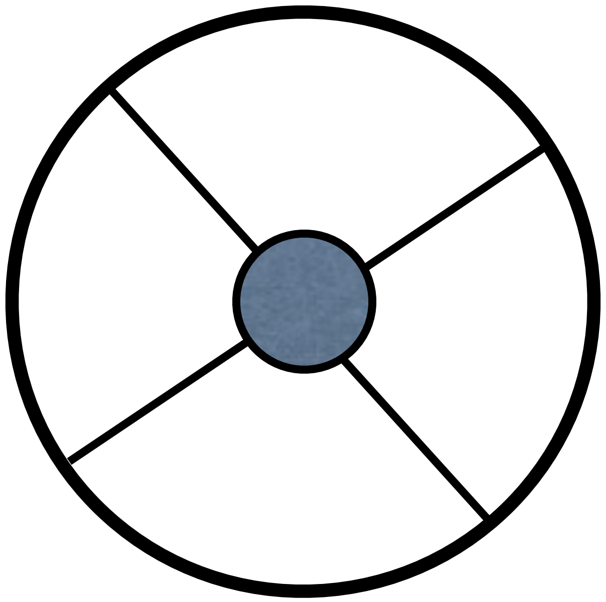
Contact diagrams in AdS give polynomial Mellin amplitudes

Mellin amplitudes are specially nice in planar CFT's (dual to tree level string theory in AdS).

Flat Space Limit of AdS

Flat space limit of AdS

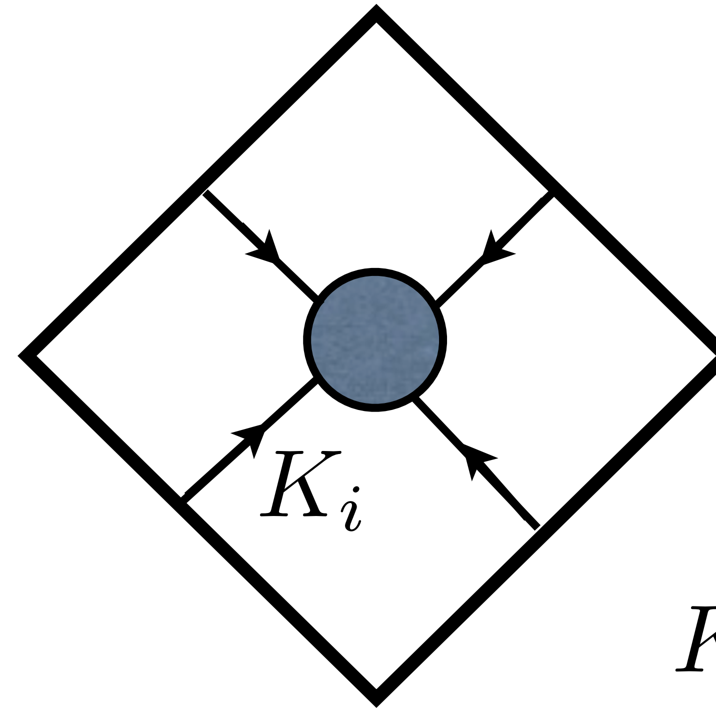
Anti-de Sitter



$$M_i^2 = \frac{\Delta_i(\Delta_i - d)}{R^2}$$

$$R \to \infty$$

Minkowski

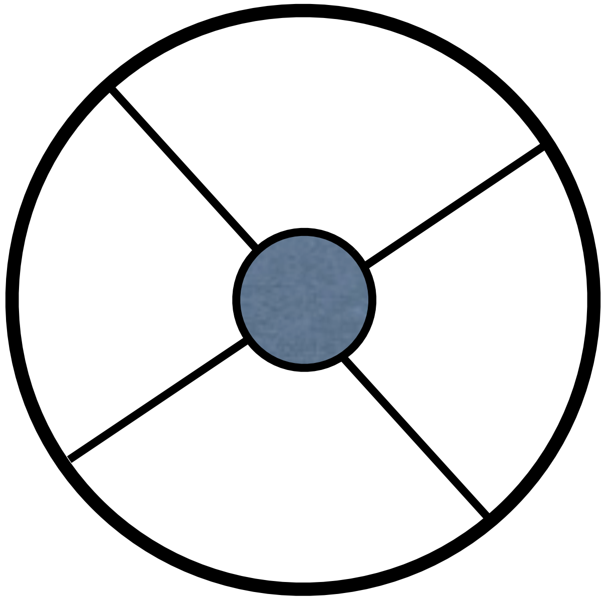


$$K_i^2 = 0$$

$$S_{ij} = -(K_i + K_j)^2$$

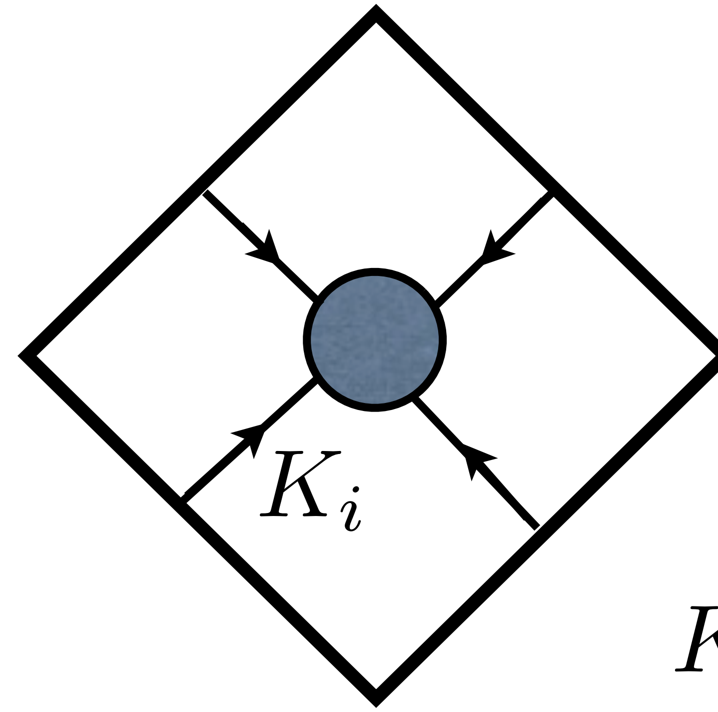
Flat space limit of AdS

Anti-de Sitter



$R \to \infty$

Minkowski



$$M_i^2 = \frac{\Delta_i(\Delta_i - d)}{R^2}$$

$$K_i^2 = 0$$

$$S_{ij} = -(K_i + K_j)^2$$

$$M(s_{ij}) \approx \frac{R^{n(1-d)/2+d+1}}{\Gamma\left(\frac{1}{2}\sum_i \Delta_i - \frac{d}{2}\right)} \int_0^\infty d\beta\, \beta^{\frac{1}{2}\sum_i \Delta_i - \frac{d}{2} - 1} e^{-\beta}\, T\left(S_{ij} = \frac{2\beta}{R^2} s_{ij}\right)$$

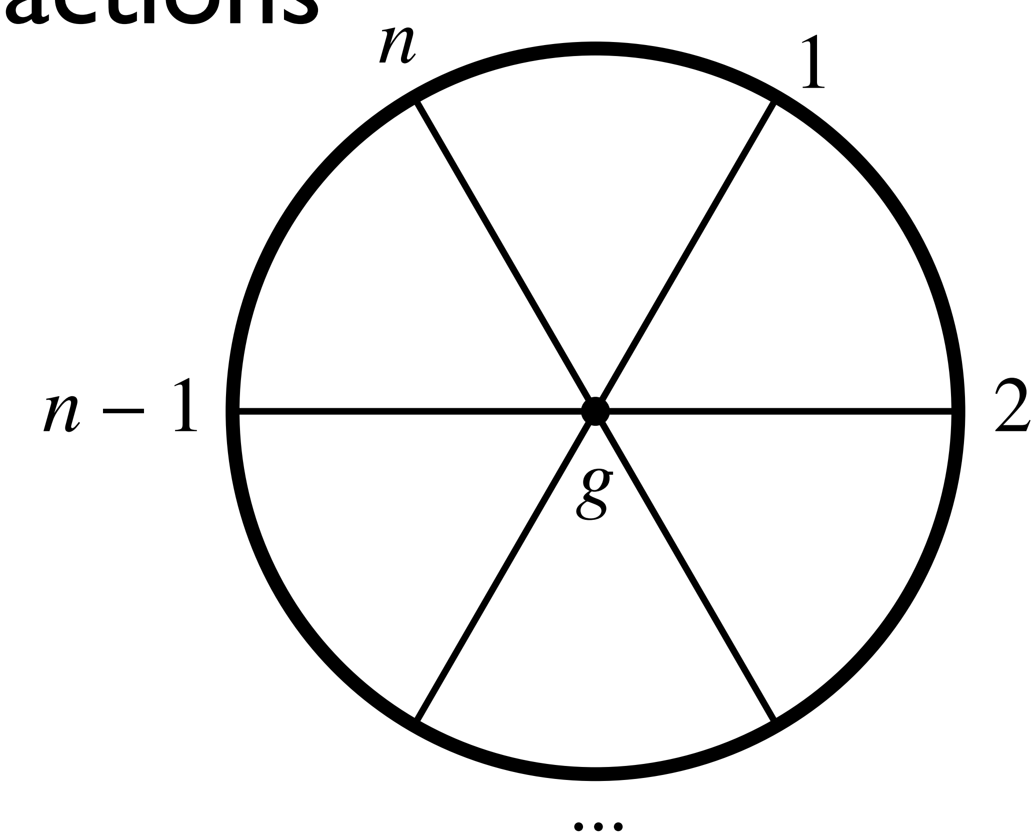
Mellin amplitude
for $s_{ij} \gg 1$

Scattering amplitude

Evidence for $M \approx \int \ldots T$

1) Works for an infinite set of interactions

$$g\nabla \ldots \nabla\phi_1 \nabla \ldots \nabla\phi_2 \ldots \nabla \ldots \nabla\phi_n$$



2) Agrees with previous results based on wave-packet constructions

[Gary, Giddings, JP '09]

[Okuda, JP '10]

3) Works in several non-trivial examples

Open Questions

- Generalize to external massive particles
  $\longrightarrow$ 3pt-functions of SYM at strong coupling

- Mellin amplitudes for external operators with spin (helicity)

- Build n-pt functions by "gluing" 3pt functions of single-trace operators (analogous to BCFW) [Raju '10]

- Feynman rules for Mellin amplitudes?

- Unitarity for Mellin amplitudes? [Fitzpatrick, Katz, Poland, Simmons-Duffin '10]
  Renormalizable vs non-renormalizable bulk interactions

- Bootstrap for CFT in higher dimensions (d>2)

- Mellin amplitudes without conformal invariance?

Thank you!