Escape orbits shaping narrow planetary rings: A billiard example

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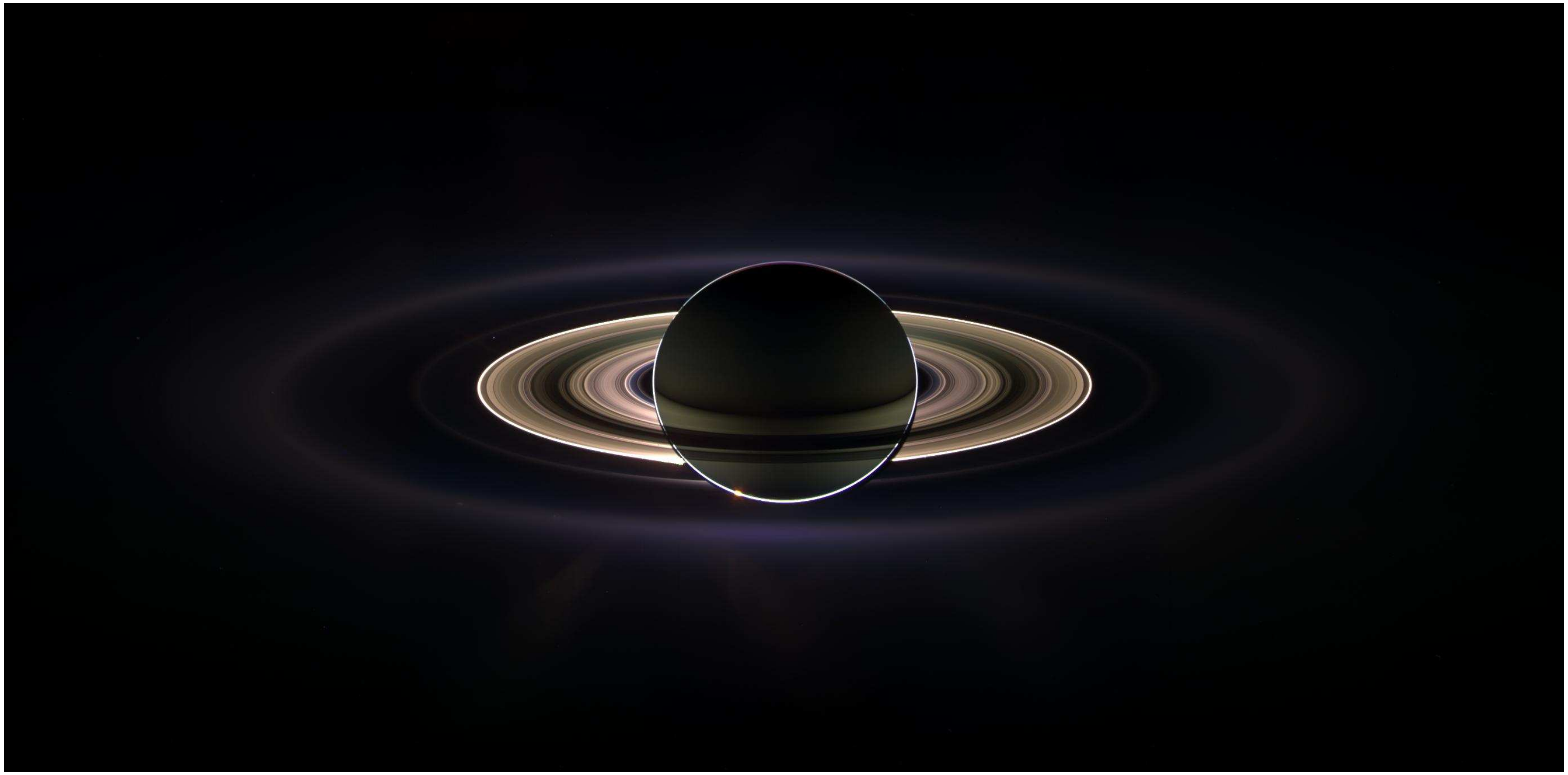
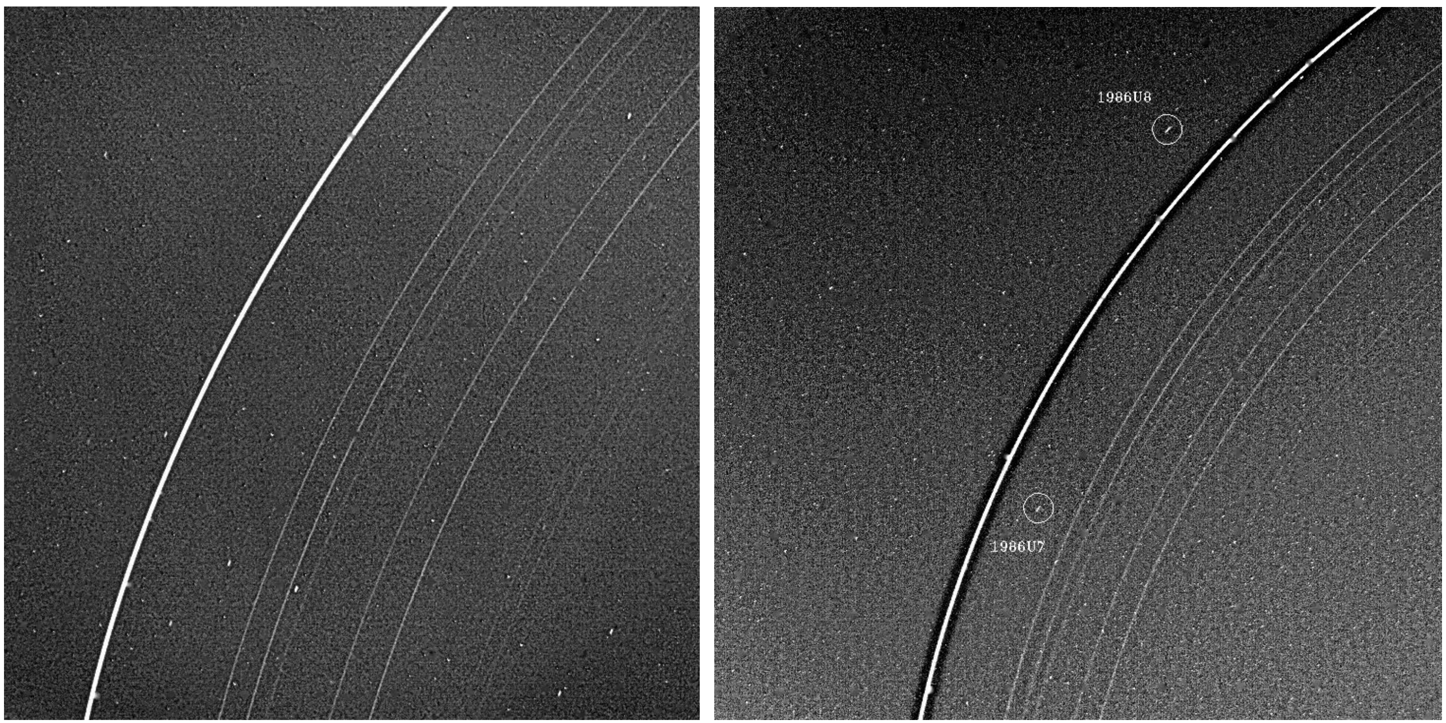
Observations: narrow planetary rings



Saturn rings
PIA08329 (NASA/JPL/Space Science Institute)

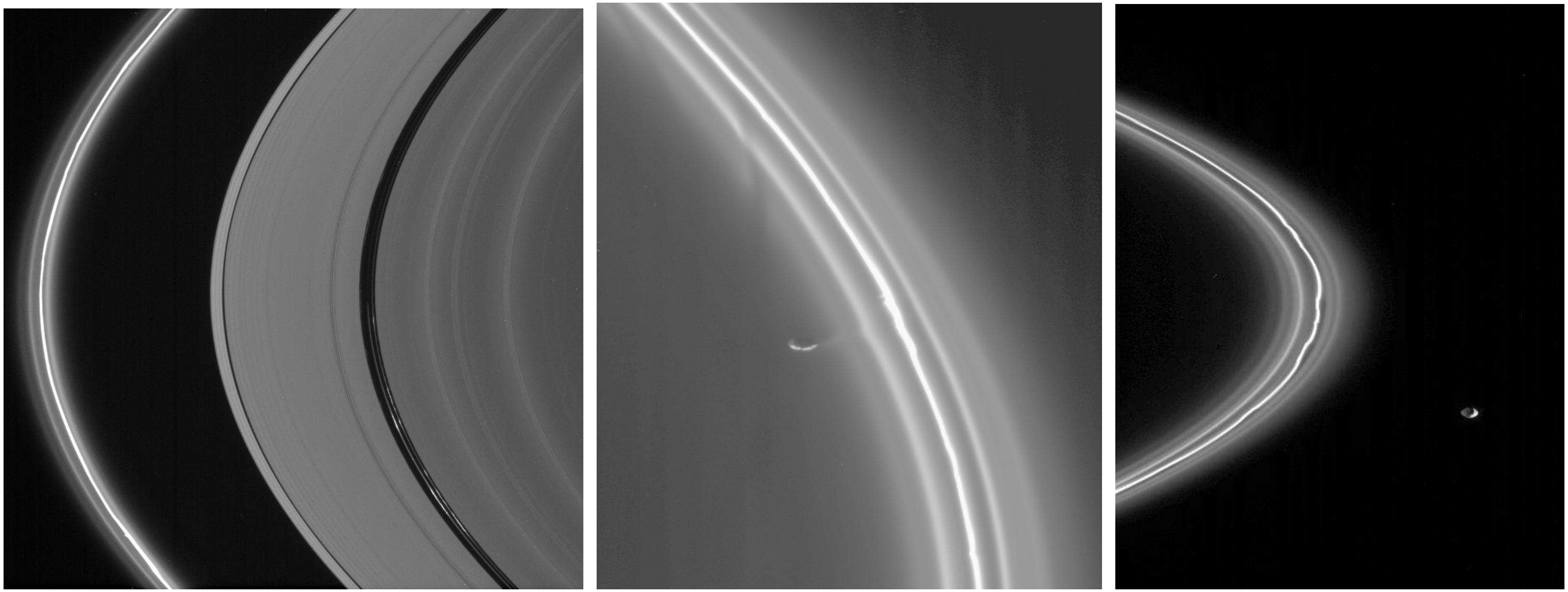
Observations: narrow planetary rings



Uranus rings

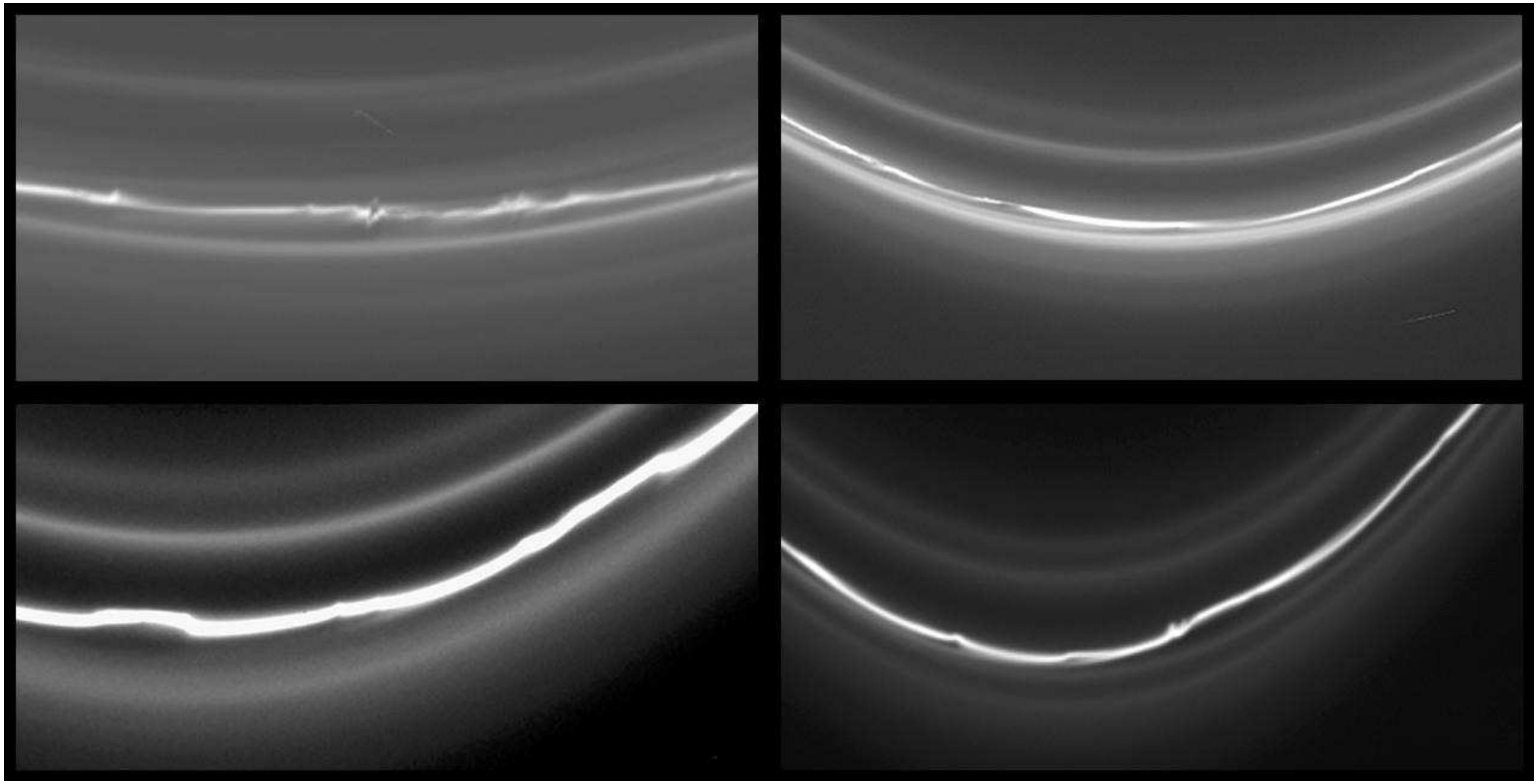
PIA01977, PIA01976 (NASA/JPL/Space Science Institute)

Observations: narrow planetary rings



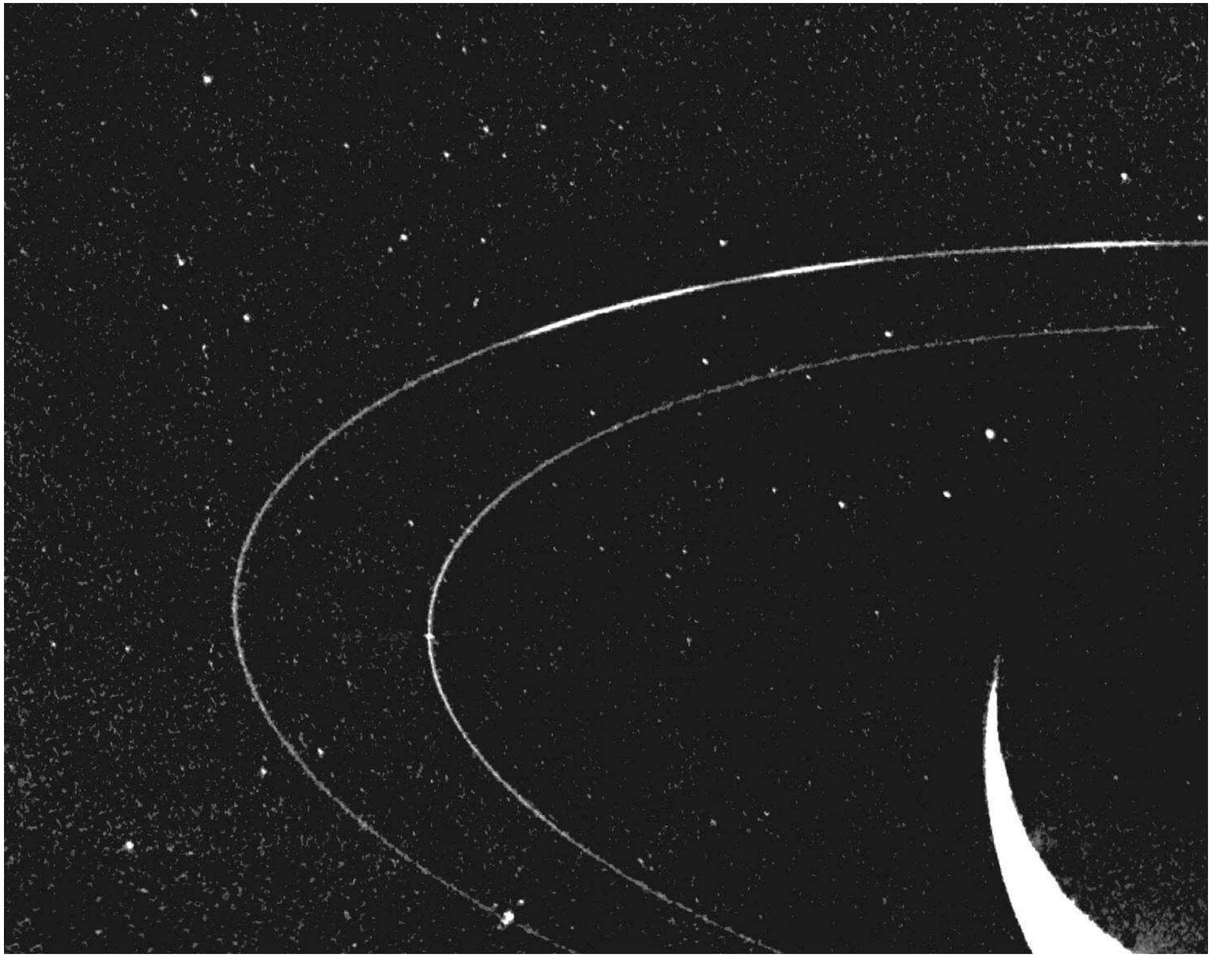
Saturn’s F ring and Encke gap; Prometheus and Pandora
PIA08305, PIA06143, PIA07523 (NASA/JPL/Space Science Institute)

Observations: narrow planetary rings



Structure in Saturn’s F ring
PIA07522 (NASA/JPL/Space Science Institute)

Observations: narrow planetary rings



Neptune rings and arcs
PIA01493 (NASA/JPL/Space Science Institute)

Open questions

We have a first-order understanding of the dynamics and key processes in rings, (...)*Unfortunately, the models are often idealized (for example, treating all particles as hard spheres of the same size) and cannot yet predict many phenomena in the detail observed by spacecraft (for example, sharp edges). Non-intuitive collective effects give rise to unusual structures*.

(...) One such example is the case of shepherding satellites. The F ring is not exactly placed where the shepherding torques would balance. Of the Uranian rings, shepherds were found only for the largest ε (epsilon) ring; even so, they are too small to hold it in place for the age of the solar system. Another issue is that the sharp edges of rings are too sharp!

Larry Esposito, Planetary rings (Cambridge University Press, 2006).

Open questions

Some open issues are:

- Rings with sharp–edges, narrow and eccentricity
- Multiple ring components: Strands
- Clumps and arcs
- Kinks and bendings
- Stability, life times, origin, ...

Scattering approach to narrow rings

Consider the full gravitational $(N+1)$-body problem in an inertial frame

$$\begin{aligned}\mathcal{H} &= \sum_{i=0}^{N}\left[\frac{1}{2M_i}|\vec{P}_i|^2 - \frac{GM_0M_i}{|\vec{R}_i - \vec{R}_0|}\right] - \sum_{i<j\neq 0}^{N}\frac{GM_iM_j}{|\vec{R}_i - \vec{R}_j|} \\ &= \mathcal{H}_{\mathrm{K_m}} + \mathcal{V}_{\mathrm{m-m}} + \mathcal{H}_{\mathrm{K_{rp}}} + \mathcal{V}_{m-rp} + \mathcal{V}_{rp-rp}\end{aligned}$$

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1st approx.: No interaction among ring particles.

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1st approx.: No interaction among ring particles.

2nd approx.: In the planetary case $M_{\mathrm{rp}} \ll M_{\mathrm{m}} \ll M_0$.

Thus, we replace the full many-body problem by a collection of independent one–particle time-dependent effective interactions:

$$H = \frac{1}{2}|\vec{P}|^2 + V_0(\vec{X}, t) + V_{\mathrm{eff}}(\vec{X}, t)$$

$\Rightarrow$ Restricted $(N_m + 1 + 1)$–body problem

Scattering approach to narrow rings

Basic ideas:

1. Ensemble ($N_r \gg 1$) non-interacting particles whose dynamics is given by

$$H = \frac{1}{2}|\vec{P}|^2 + V_0(\vec{X}, t) + V_{\text{eff}}(\vec{X}, t)$$

2. Phase–space regions where escape to infinity dominates the dynamics (scattering)

3. Organizing centers in phase space (periodic orbits or tori) are stable.

Then, rings shall be obtained by projecting onto the $X - Y$ space, at fixed time, the phase-space position of *all particles that are dynamically trapped.*

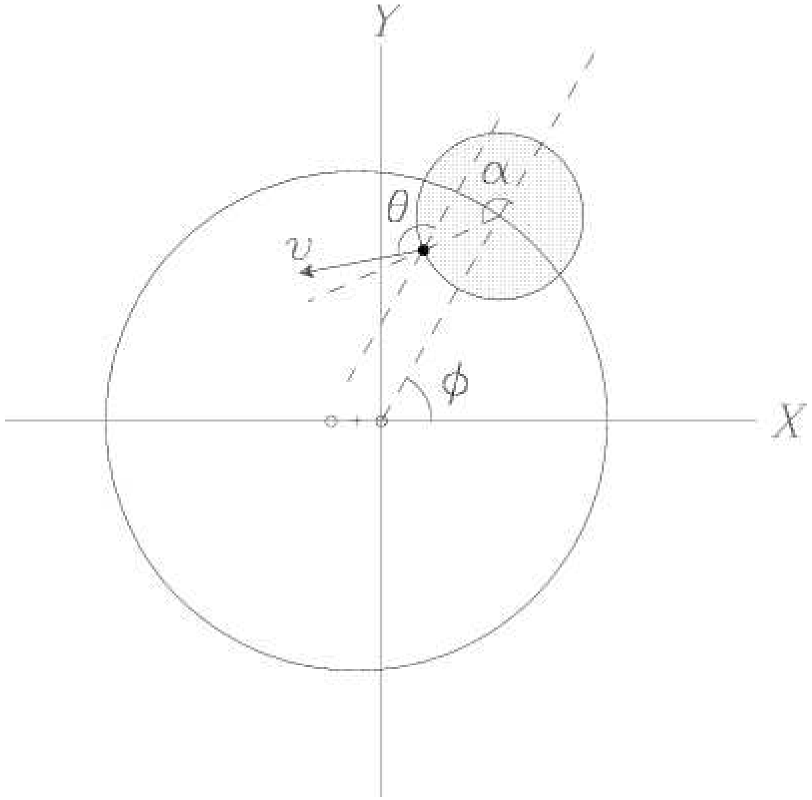
Ring particles evolution is given by

$$H(\vec{X},\vec{P},t) = \frac{1}{2}|\vec{P}|^2 + V_0(\vec{X},t) + V_{\rm eff}(\vec{X},t)$$

The simplest case: planar billiard on a Kepler orbit

$$\begin{aligned} V_0(|\vec{X}|) &= 0, \\ V_{\rm eff}(|\vec{X}-\vec{R}_d(\phi(t))| > d) &= 0, \\ V_{\rm eff}(|\vec{X}-\vec{R}_d(\phi(t))| \le d) &= \infty, \end{aligned}$$

$$R_d(\phi) = \frac{a(1-\varepsilon^2)}{1+\varepsilon\cos\phi}, \quad R_d^2\dot{\phi} = [a(1-\varepsilon^2)]^{1/2}$$

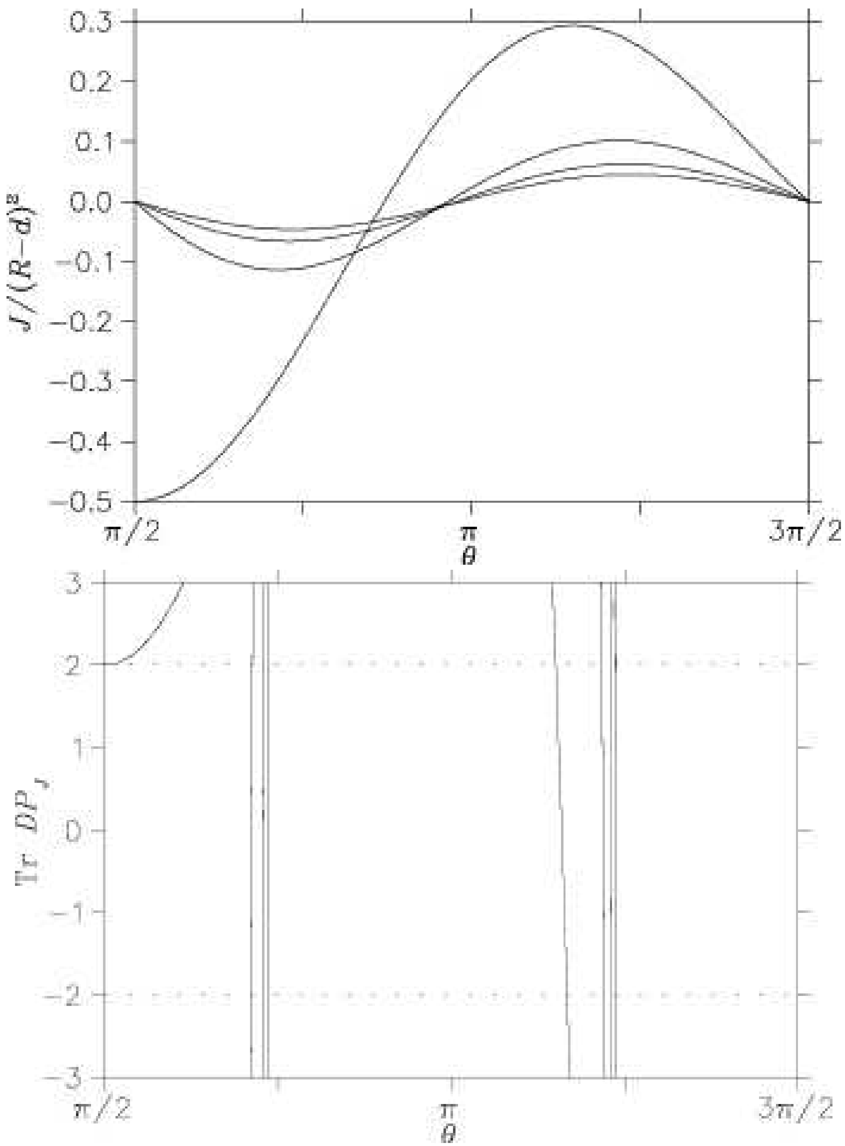


The Hamiltonian is:

$$J = \frac{1}{2}\frac{a^2}{R_d(\phi)^2}|\vec{p}|^2 + V_{\rm eff}\left(\frac{R_d(\phi)}{a}|\vec{x}-\vec{r}_d|\right)$$

$$-\dot{\phi}(xp_y - yp_x) - \dot{\phi}\frac{1}{R_d(\phi)}\frac{{\rm d}R_d(\phi)}{{\rm d}\phi}(xp_x + yp_y)$$

Circular case: Periodic orbits and stability



Radial periodic orbits:

$$\frac{J_n}{(R-d)^2} = \frac{2\cos^2\theta + \Delta\phi\sin(2\theta)}{(\Delta\phi)^2}$$

with $\Delta\phi = (2n-1)\pi + 2\theta$

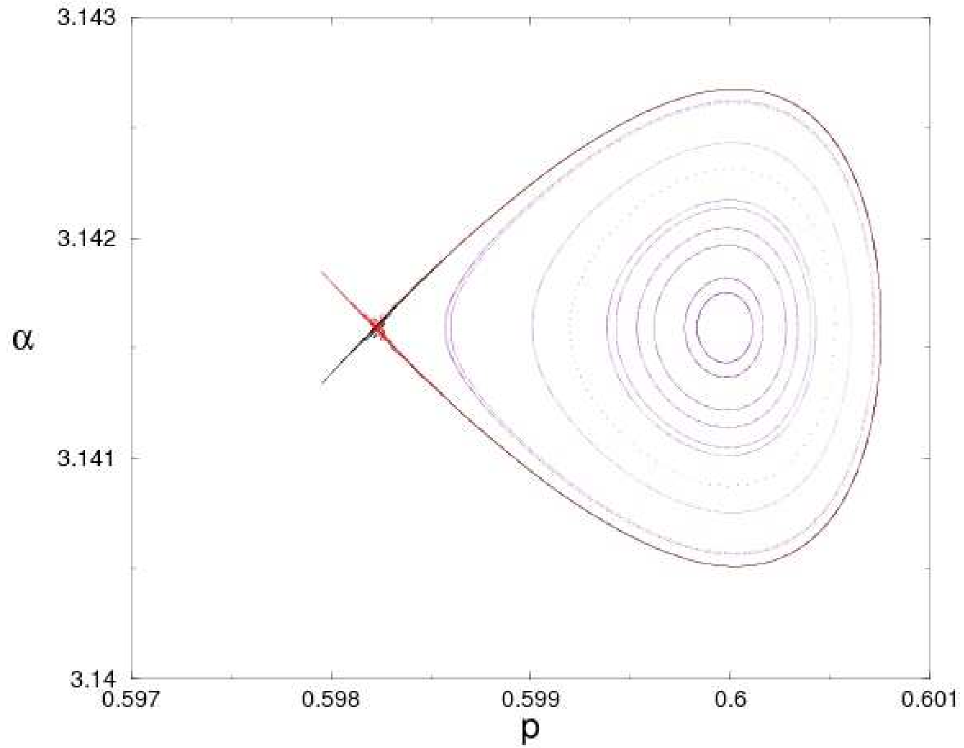
Stability:

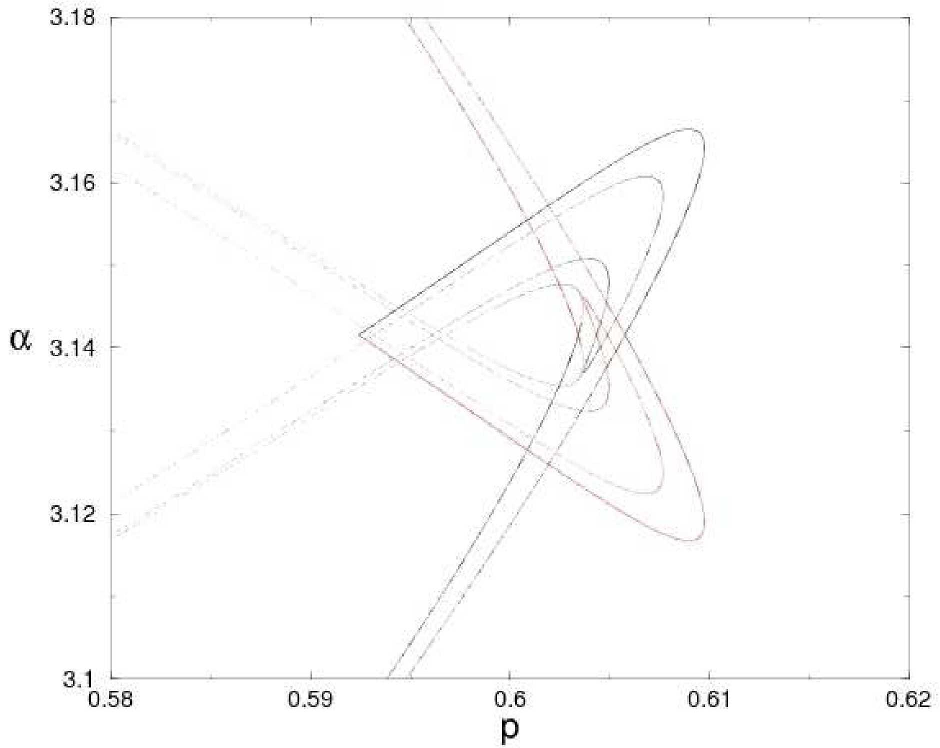
$$\mathrm{Tr}\, D\mathcal{P}_J = 2 + \frac{(\Delta\phi)^2(1-\tan^2\theta)}{d/R} - \frac{4(1+\Delta\phi\tan\theta)}{d/R}$$

Changes of stability at $\mathrm{Tr}\, D\mathcal{P}_J = \pm 2$

Circular case: Periodic orbits and stability

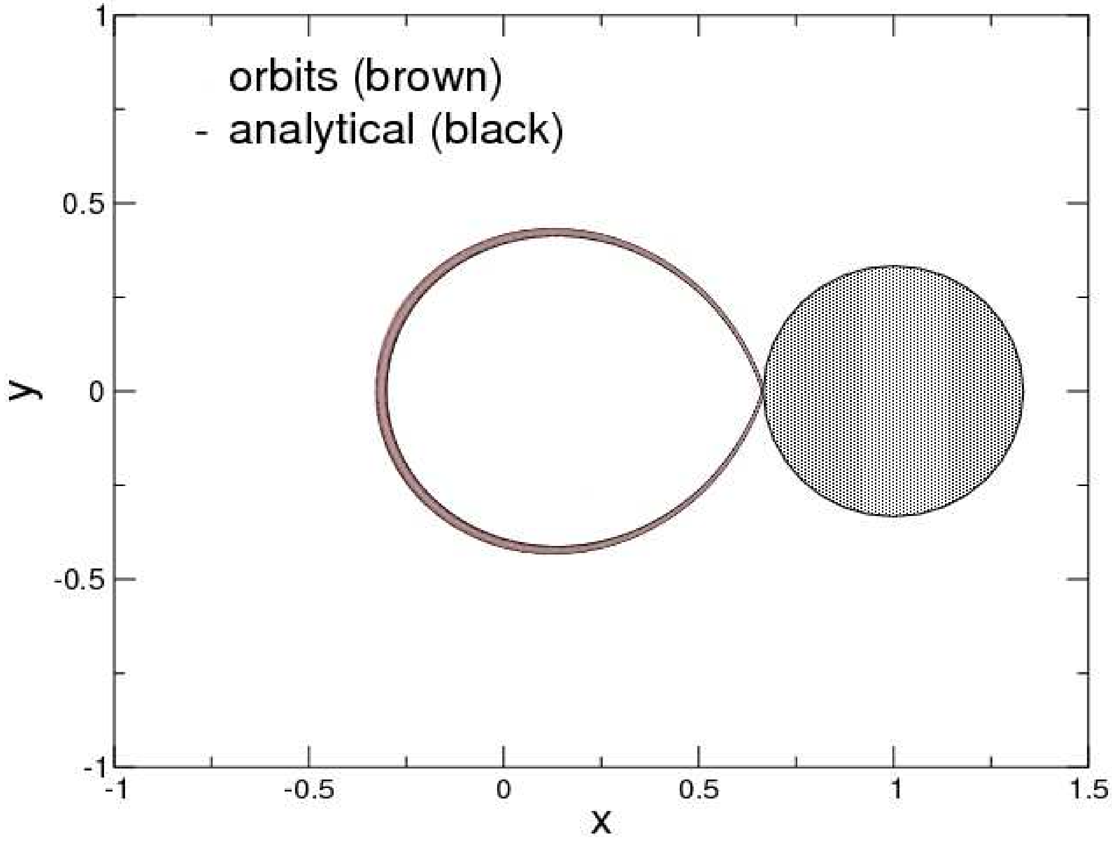
$J/(R-d)^2 = 0.29325$ $\qquad$ $J/(R-d)^2 = 0.29218$





$$p = -d - R\cos\alpha - v\sin(\alpha - \theta)$$

Circular case: Occurrence of rings



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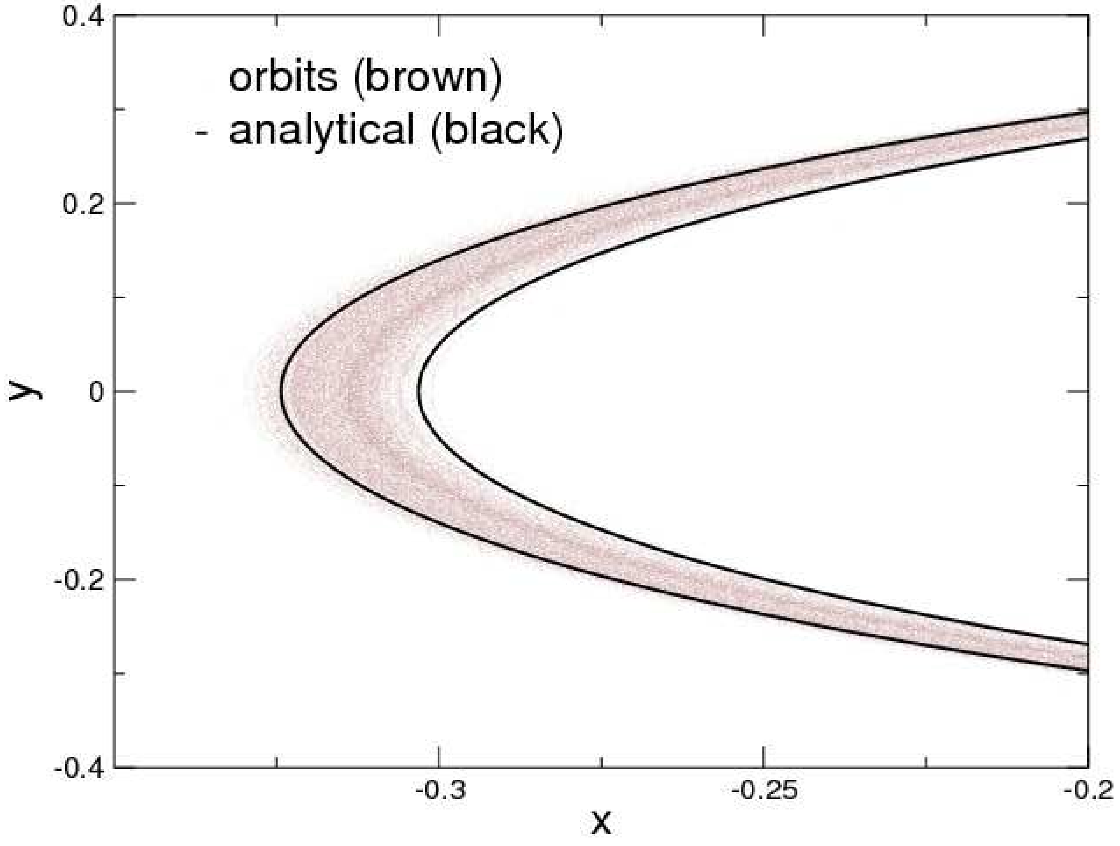
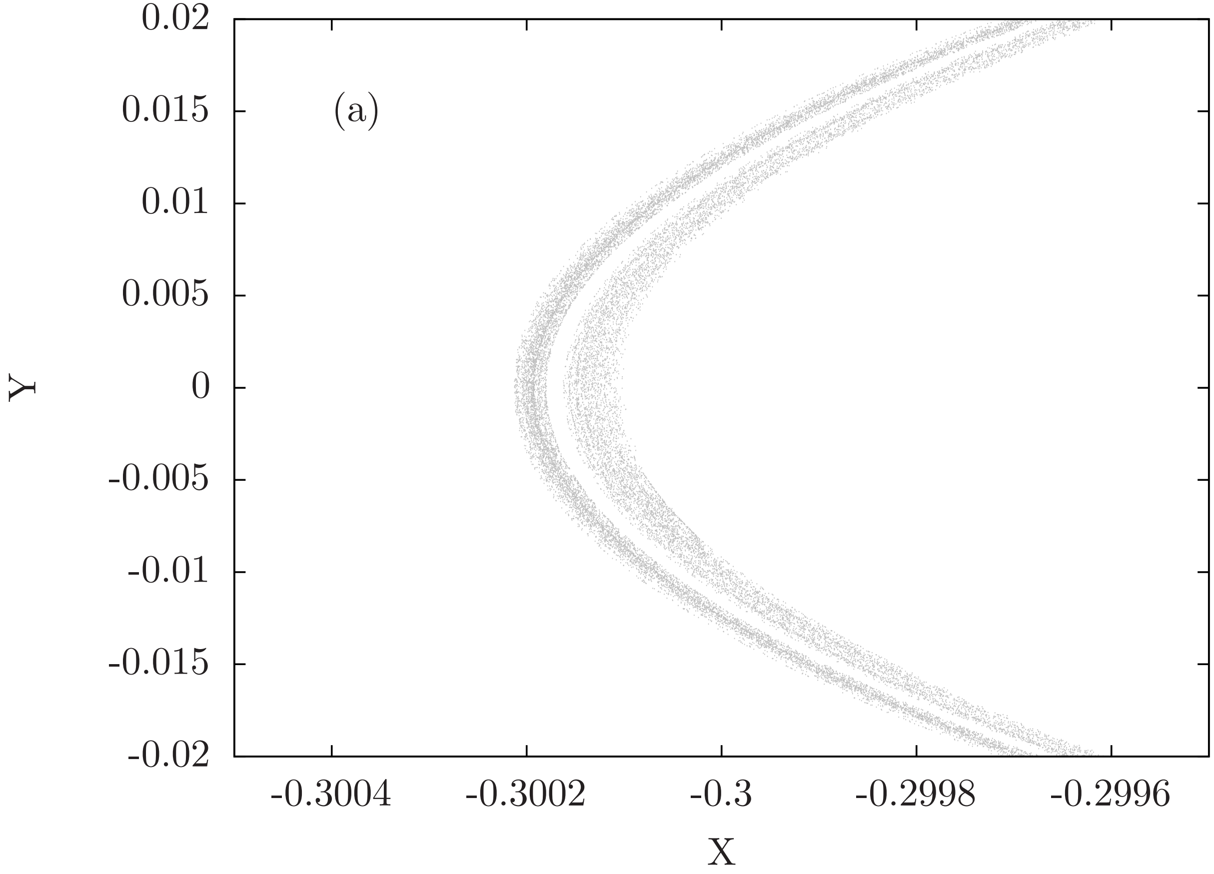


Eliptic case: Strands and Arcs

We consider now elliptic Kepler motion ($2 + 1/2$ degrees of freedom)



$$\varepsilon = 0.00165$$

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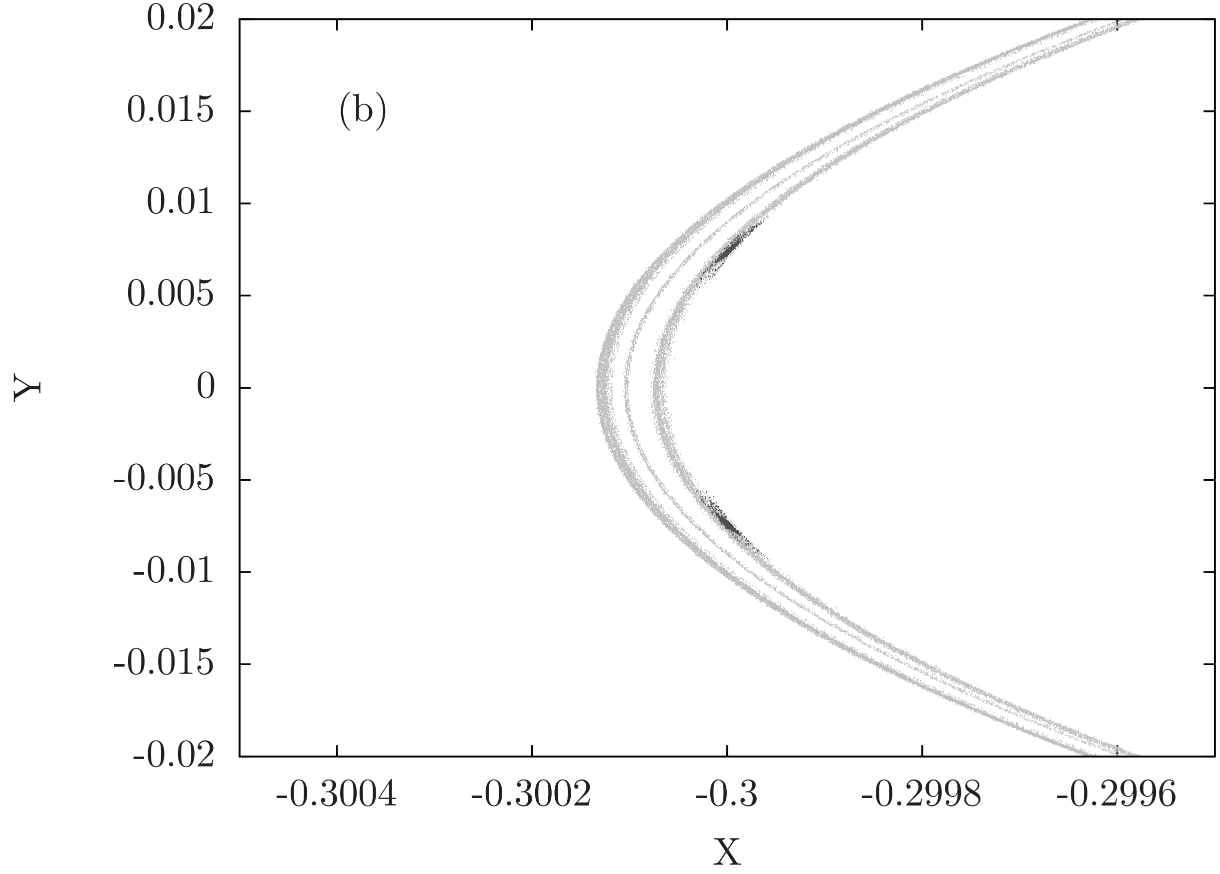

$$\varepsilon = 0.00167$$

Eliptic case: Strands and Arcs

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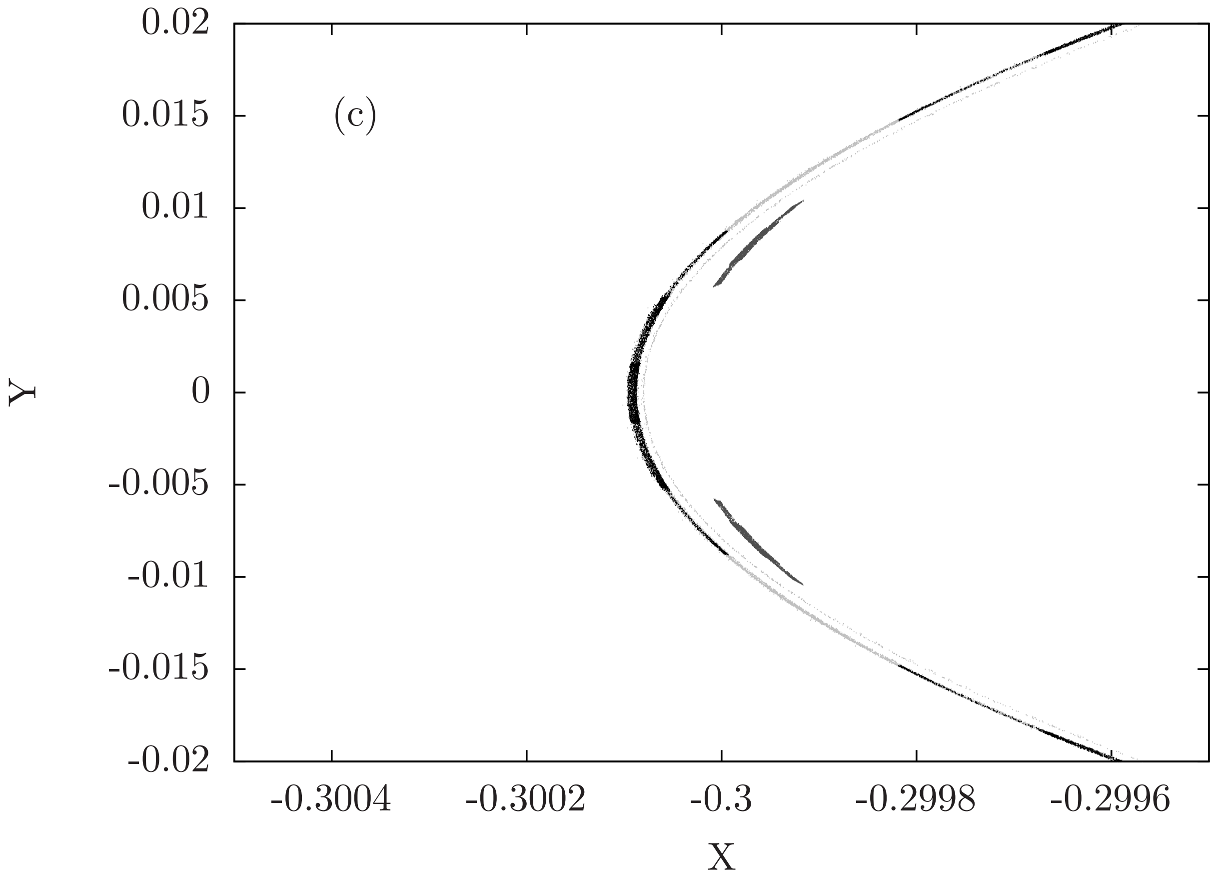
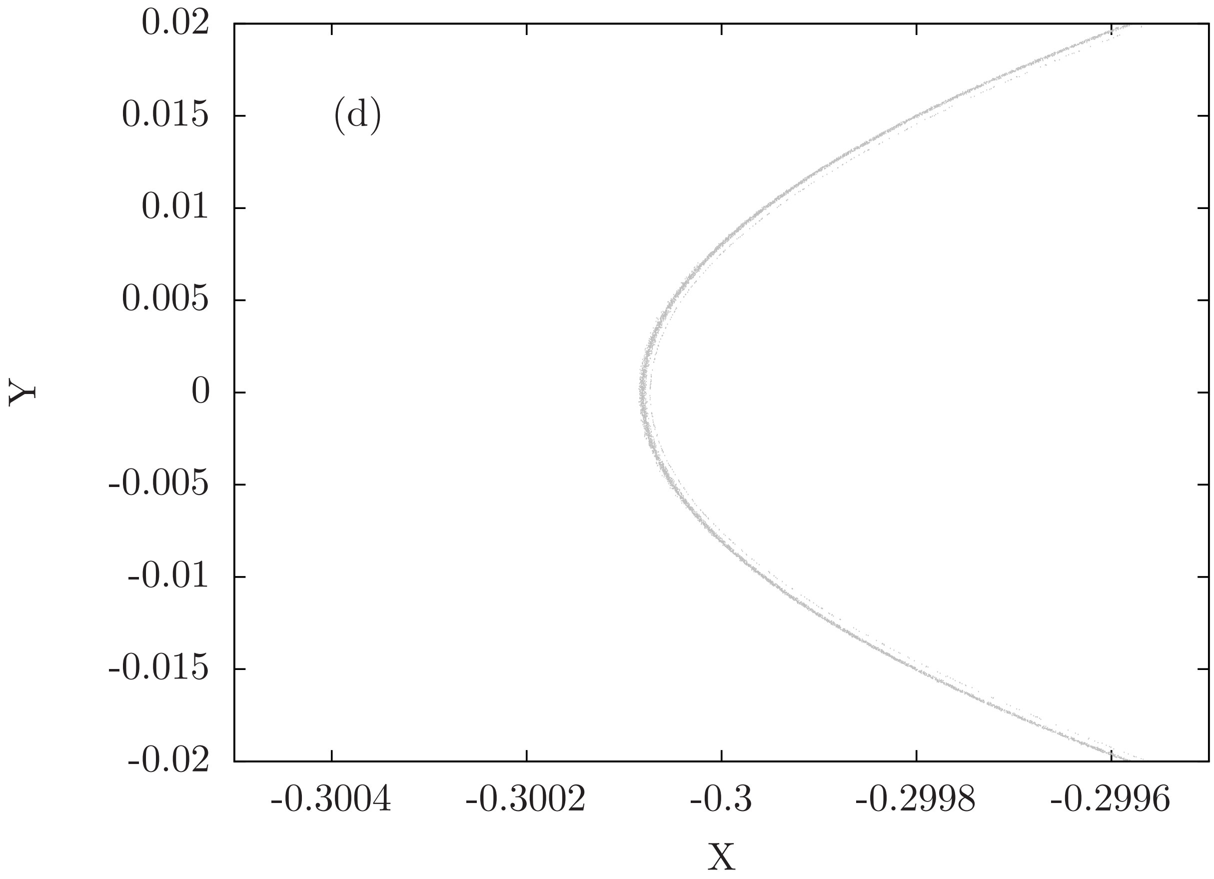


$$\varepsilon = 0.00168$$

Eliptic case: Strands and Arcs

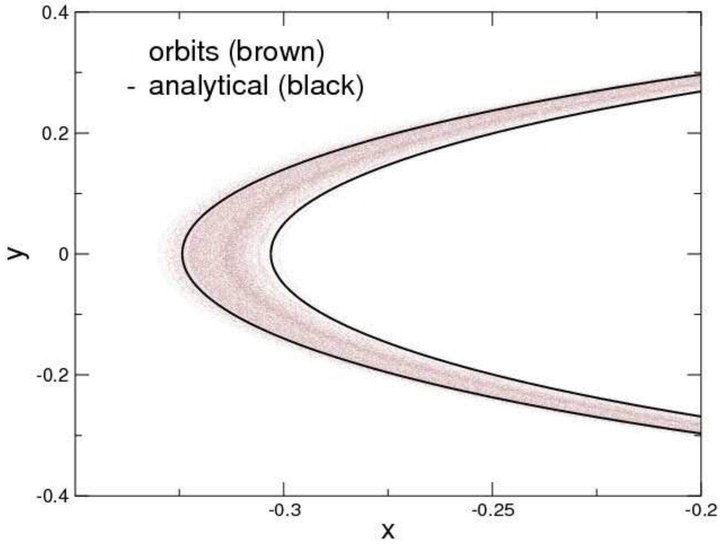
We consider now elliptic Kepler motion ($2 + 1/2$ degrees of freedom)

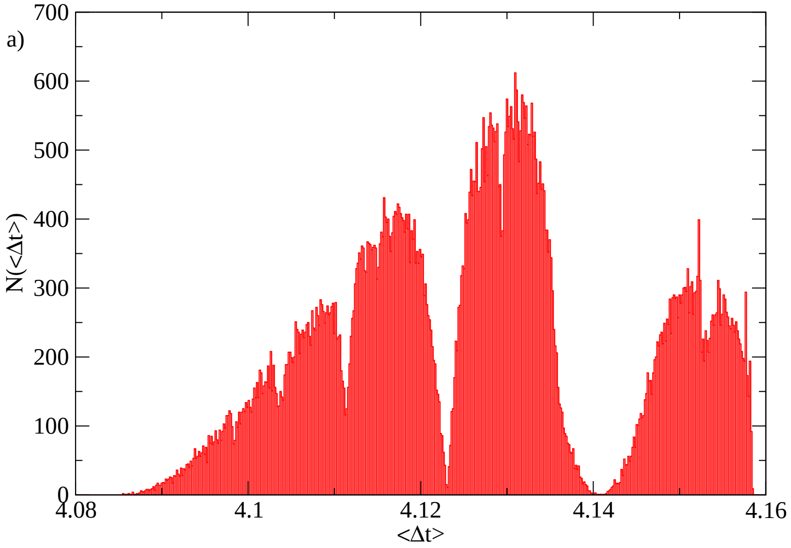


$$\varepsilon = 0.001683$$

Phase-space volume of trapped regions

$$\varepsilon = 0$$

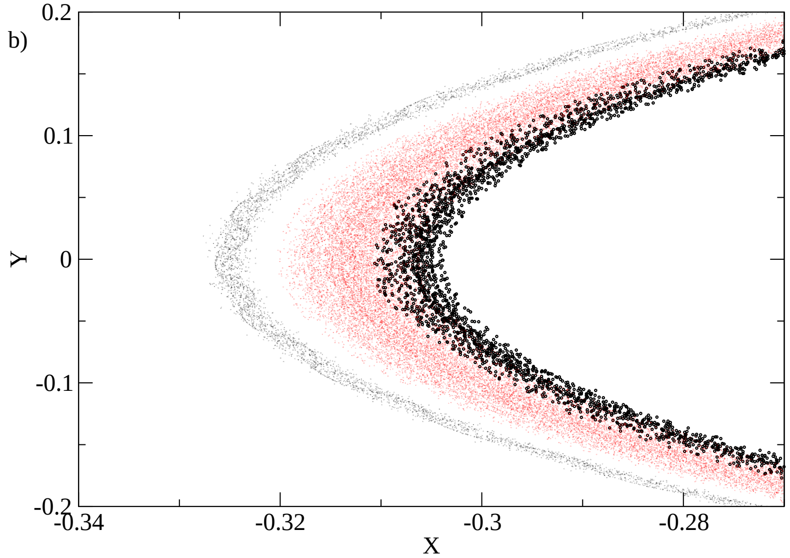


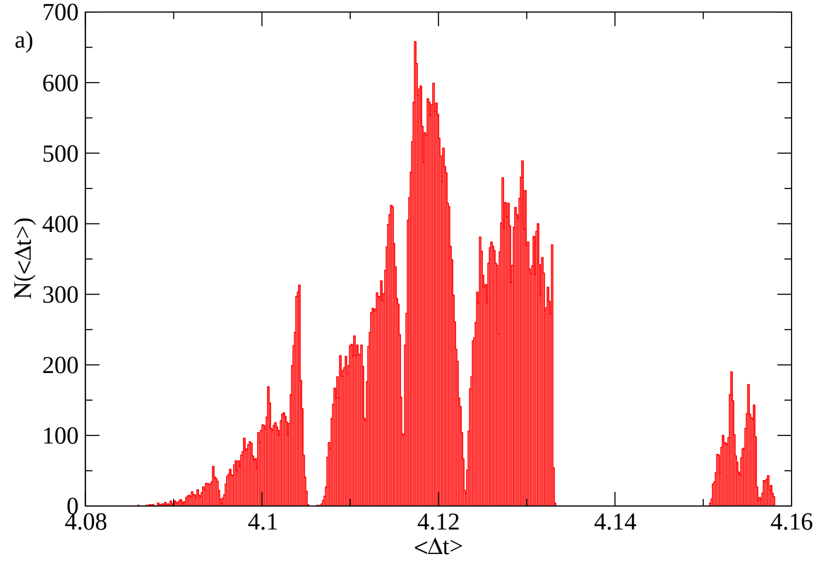


$$\exp[i\alpha],\, \cos(\alpha) = 2\mathrm{Tr}\, D\mathcal{P}_J,\, \alpha_{p:q}/(2\pi) = p/q$$

Phase-space volume of trapped regions

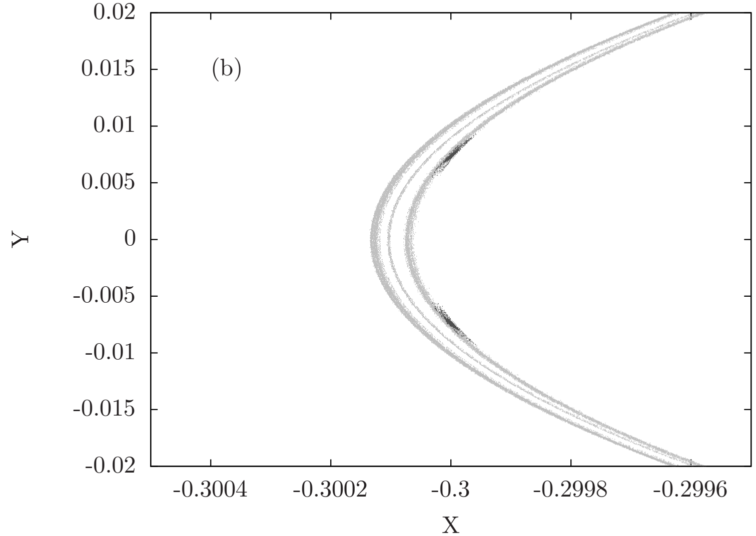
$$\varepsilon \neq 0$$



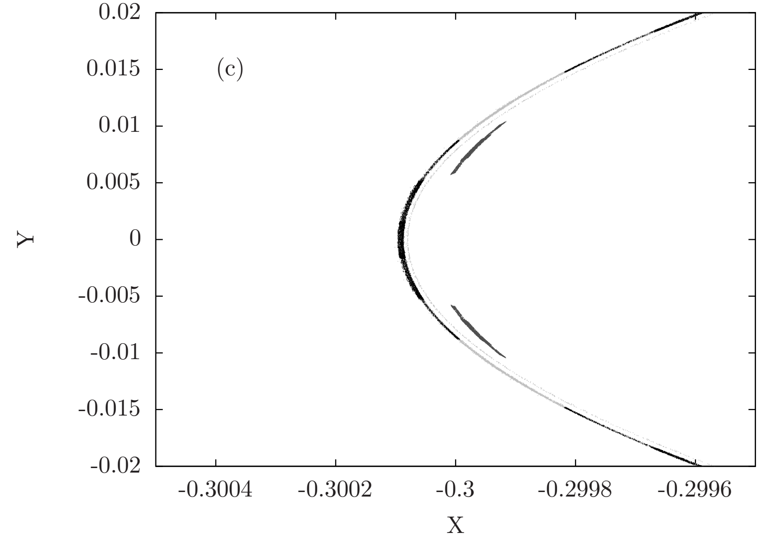


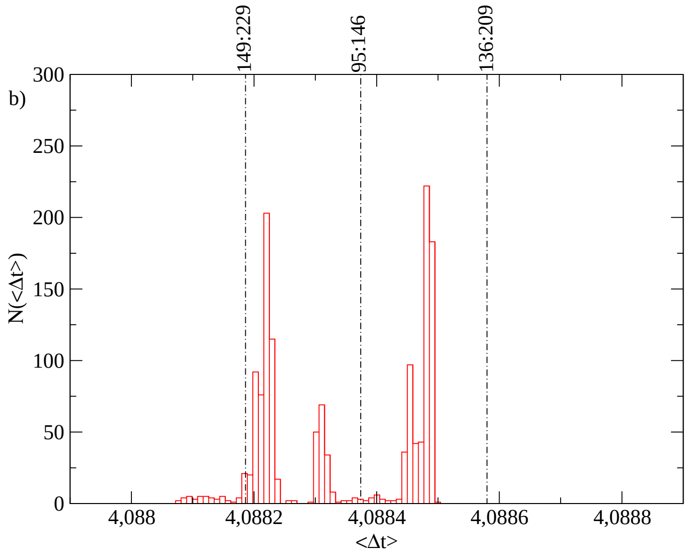
Excitation of stability resonances separates regions in phase space

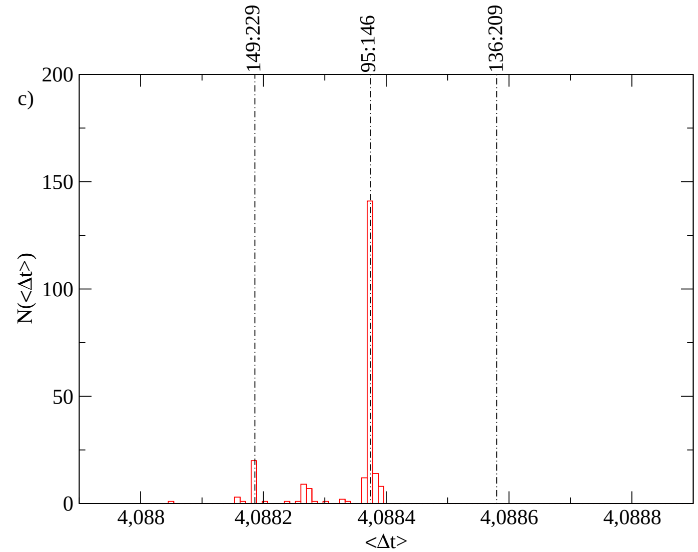
$\varepsilon = 0.00165$



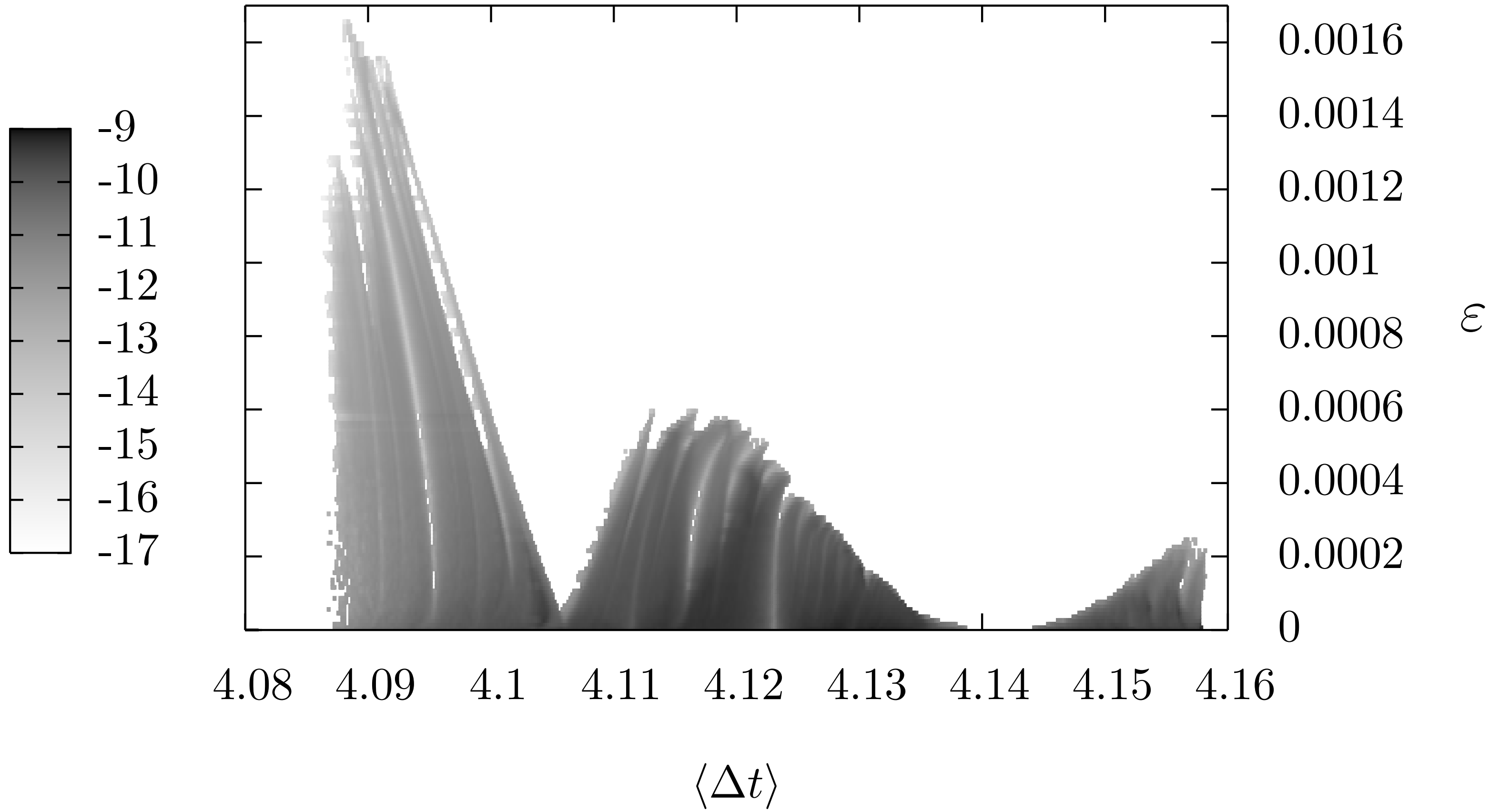
$\varepsilon = 0.00167$

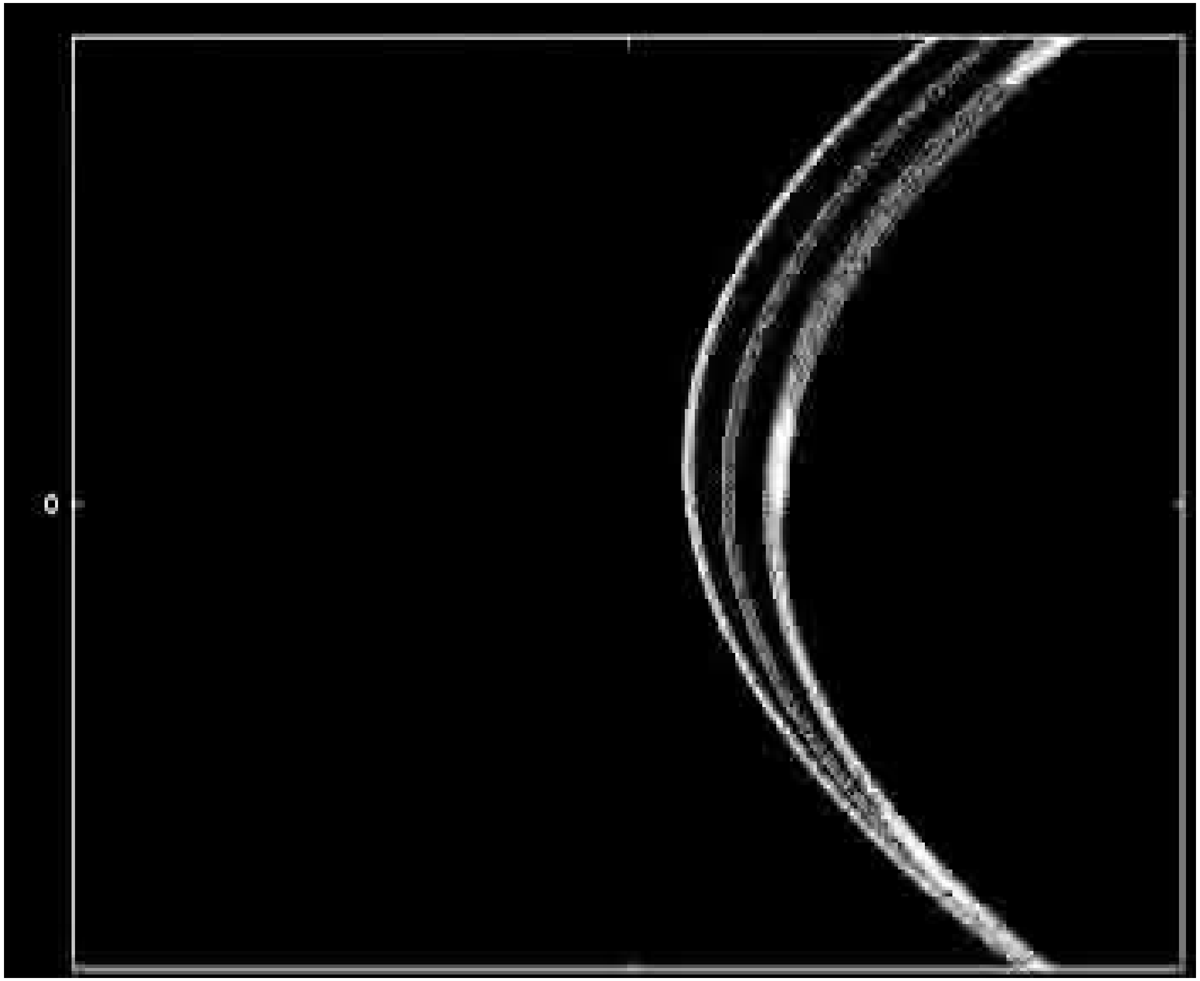






Mean-motion Resonances

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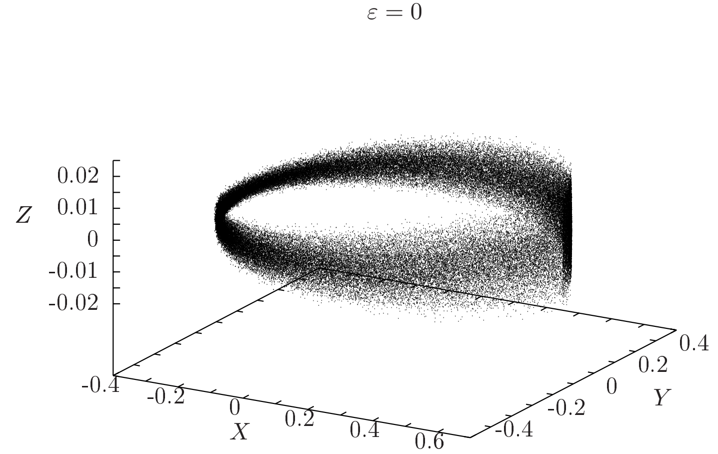
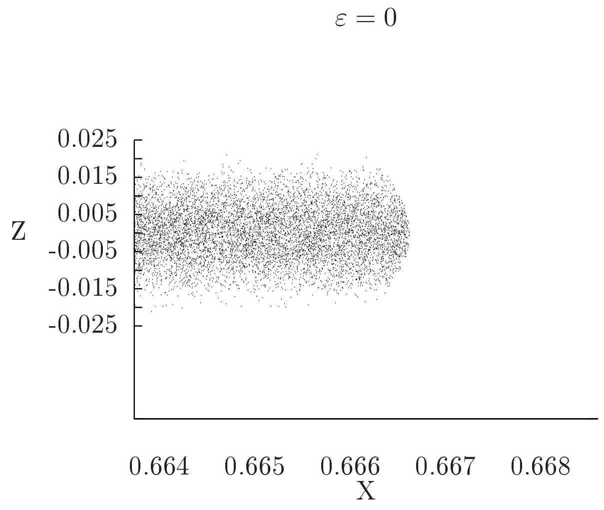
Summary

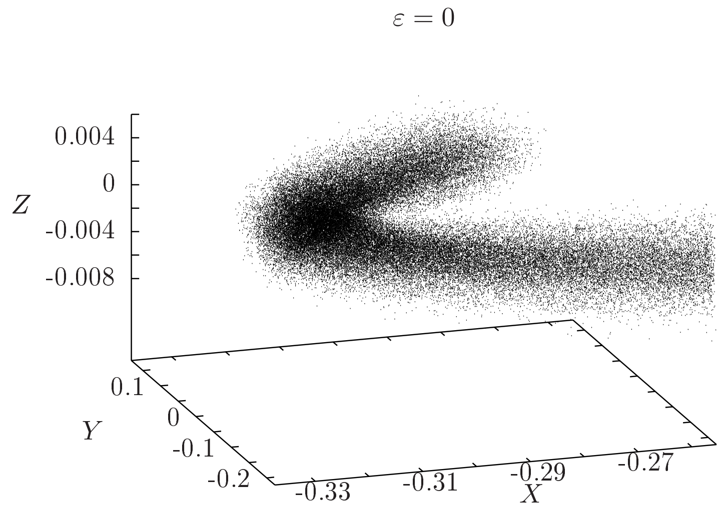
- Using a scattering approach, we obtain consistently sharp–edged narrow eccentric rings.
  Scattering dynamics $\Rightarrow$ rings have sharp edges
  Eccentric orbits as organizing centers $\Rightarrow$ eccentric rings
  Small stable regions in phase space $\Rightarrow$ narrow rings
- For more than two degrees of freedom, rings may have several components, strands. They appear by exciting low-order stability resonances which spearate the regions of trapped motion in phase space.
- Arcs (clumps) are related to the additional occurrence of mean-motion resonances within the strands.
- For two degrees of freedom our results are robust for short-range circularly rotating potentials.

Outlook (work in progress)

Q: What about the non-planar case? A 3D billiard on a planar circular orbit



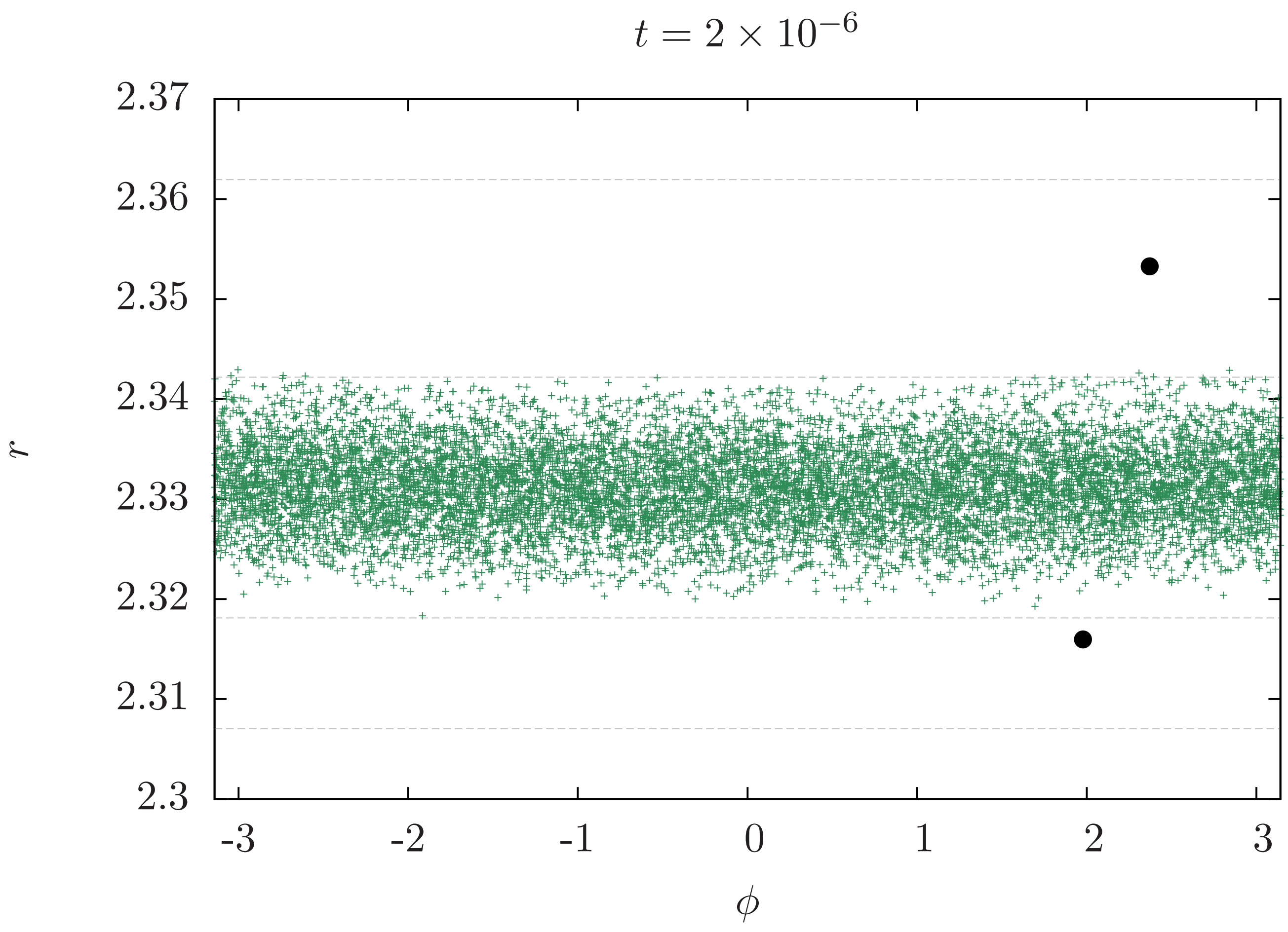




Outlook (work in progress)

Q: What about a realistic case? (in collaboration with À. Jorba)

Results for a consistent restricted (4+1)-body problem



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Thank you !