Benchmarking Non-First-Come-First-Served Component Allocation in an Assemble-To-Order System

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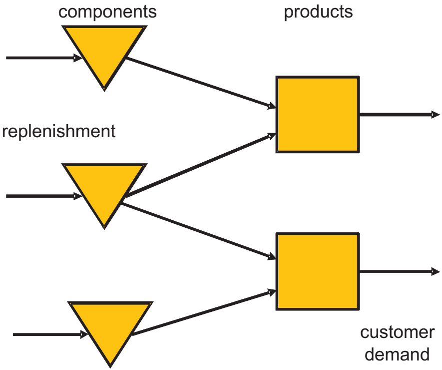
Table of Contents

Assemble-To-Order System (ATOS)

- Two levels: Products and components.



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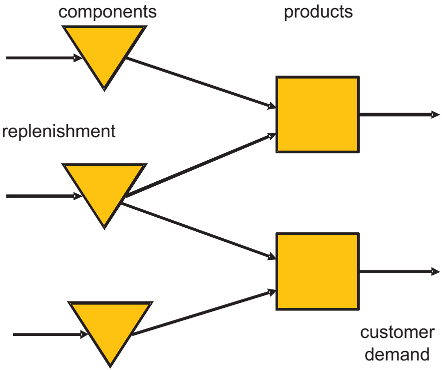

- In the middle of single-echelon and two-echelon.

Assemble-To-Order System (ATOS)

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Assemble-To-Order System (ATOS)

- Assumptions:
  - Periodic review.
  - Independent base stock policy for each component.
  - Consignment policy: once a unit of component is assigned to an order, it is not available to other orders anymore even if it still stays in the inventory.
- Optimization problems:
  - Base stock level optimization.
  - Component allocation optimization.

Last-Come-First-Served-Within-One-Period (LCFP)

- In a period, the unfulfilled orders come from $t_1, t_1 + 1, \cdots, t - 1, t$:
  - FCFS: Fulfill the orders in the sequence $t_1, t_1 + 1, \cdots, t - 1, t$.
  - LCFP: Fulfill the orders in the sequence $t, t_1, t_1 + 1, \cdots, t - 1$.

Product-Based-Priority-Within-Time-Windows (PTW)

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- The fulfillment follows the priority list.
- Example: Let $w_1 = 0, w_2 = 1, w_3 = 2$. Then the sequence of satisfying the demands $P_{1,t}, P_{2,t}, P_{3,t}$ will be

$$P_{1,t}, P_{2,t-1}, P_{3,t-2}, P_{1,t+1}, P_{2,t}, P_{3,t-1}, P_{1,t+2}, P_{2,t+1}, P_{3,t}.$$

Demand Fulfillment Rates of the LCFP Rule

- The amount of inventory committed to the demand $D_{i,t}$ should be

$$E_{i,t} = \mathrm{Min}\{(S_i - D_i[t - L_i - 1, t - 1])^+ + D_{i,t-L_i-1}, D_{i,t}\},$$

while in FCFS, this amount is

$$\mathrm{Min}\{(S_i - D_i[t - L_i, t - 1])^+, D_{i,t}\}.$$

Demand Fulfillment Rates of the LCFP Rule (Zero Time Window)

Lemma

The available on-hand inventory at the end of period t is $(S_i - D_i[t - L_i, t])^+$ under the LCFP rule, which is the same as that under the FCFS rule.

Theorem

The demand $D_{i,t}$ will be satisfied exactly in period t if and only if $(S_i - D_i[t - L_i - 1, t - 1])^+ + D_{i,t-L_i-1} \geq D_{i,t}$ under the LCFP rule.

Demand Fulfillment Rates of the LCFP Rule (Positive Time Window)

Theorem

The demand $D_{i,t}$ will be satisfied within a time window $w \geq 1$ if and only if $(S_i - D_i[t - L_i - 1, t - 1])^+ + D_{i,t-L_i-1} \geq D_{i,t}$ (i.e. $E_{i,t} = D_{i,t}$), or, $(S_i - D_i[t - L_i - 1, t - 1])^+ + D_{i,t-L_i-1} < D_{i,t}$ (i.e. $E_{i,t} < D_{i,t}$) and $S_i - D_i[t - L_i + w, t] - \sum_{s=1}^{w} E_{i,t+s} \geq 0$, under the LCFP rule.

Demand Fulfillment Rates of the PTW Rule (Zero Time Window)

Theorem

When the PTW rule is applied, the net inventory just before satisfying the demand $a_{ij}P_{j,t}$ in period $t + w_j$ is:

$$S_i - D_i[t - L_i + w_j, t - 1]$$
$$- \sum_{k:k<j} \sum_{s:s \geq t, s+w_k \leq t+w_j} a_{ik} P_{k,s}$$
$$+ \sum_{k:k>j} \sum_{s:s<t, s+w_k \geq t+w_j} a_{ik} P_{k,s}.$$

Demand Fulfillment Rates of the PTW Rule (Positive Time Window)

Theorem

When the PTW rule is applied, the net inventory just before satisfying the demand $a_{ij}P_{j,t}$ in period $t + w_j + \delta_j$ is:

$$S_i - D_i[t - L_i + w_j + \delta_j, t - 1]$$
$$- \sum_{k:k<j} \sum_{s:s\geq t, s+w_k \leq t+w_j} a_{ik}P_{k,s}$$
$$+ \sum_{k:k>j} \sum_{s:s<t, s+w_k \geq t+w_j} a_{ik}P_{k,s}.$$

Base Stock Level Optimization of the LCFP Rule

$$\text{Min} \sum_{i \in \mathcal{M}} c_i S_i$$

$$\text{s.t.} \quad P\{(S_i - D_i^{L_i+1})^+ + D_{i,t-L_i-1} \geq D_{i,t}, \forall i : a_{ij} > 0\} \geq \alpha_j \quad \forall j.$$

Base Stock Level Optimization of the LCFP Rule

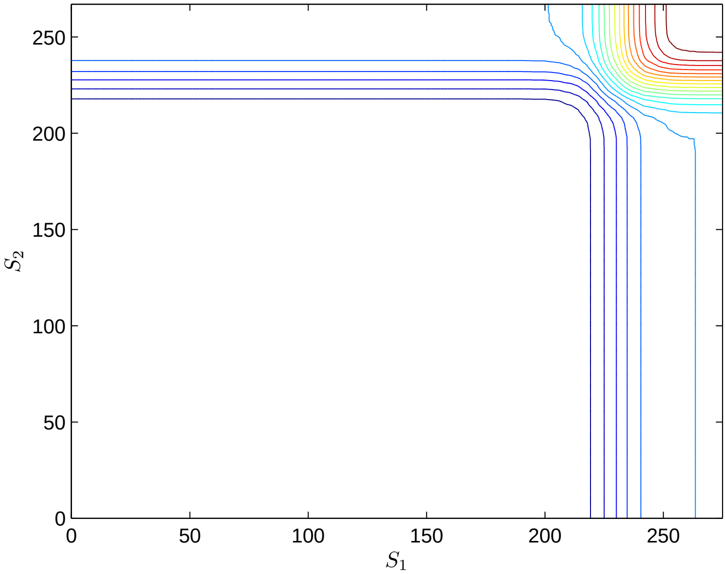
Observation

Assume the LCFP rule is applied, and the demands in the same period follow a multi-variate normal distribution, and the demands from different periods are i.i.d. Let $\mathcal{X}$ be defined as:

$$\{S : P\{(S_i - D_i^{L_i+1})^+ + D_{i,t-L_i-1} \geq D_{i,t}, \forall i : a_{ij} > 0\} \geq \alpha_j \quad \forall j\},$$

where $S = (S_i)_{i \in \mathcal{M}} \in \mathbb{R}_+^{|\mathcal{M}|}$ is the vector of nonnegative base stock levels. The set $\mathcal{X}$ is not necessarily convex.

Illustration



Base stock Level Optimization of the PTW Rule

$$\text{Min} \sum_{i \in \mathcal{M}} c_i S_i$$

$$\text{s.t.} \quad P\{X_{it}^j \leq S_i, \forall i : a_{ij} > 0\} \geq \alpha_j \quad \forall j.$$

where

$$\begin{aligned} X_{it}^j \quad = \quad & D_i[t - L_i + w_j, t - 1] \\ & + \textstyle\sum_{k:k \leq j} \sum_{0 \leq q \leq w_j - w_k} a_{ik} P_{k,t+q} \\ & - \textstyle\sum_{k:k > j} \sum_{0 < q \leq w_k - wj} a_{ik} P_{k,t-q}. \end{aligned}$$

Base stock Level Optimization of the PTW Rule

Theorem

Assume the PTW rule is applied, and the demands in the same period follow a multi-variate normal distribution, and the demands from different periods are i.i.d. Let $\mathcal{X}$ be defined as:

$$\{S : P\{X_{it}^{j} \leq S_i, \forall i : a_{ij} > 0\} \geq \alpha_j \quad \forall j\},$$

where $S = (S_i)_{i \in \mathcal{M}} \in \mathbb{R}_{+}^{|\mathcal{M}|}$ is the vector of nonnegative base stock levels. The set $\mathcal{X}$ is convex.

Solution Strategies

- Use the Sample Average Approximation algorithm to solve the base stock level optimization of the LCFP rule.

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- Use a line search algorithm to solve the base stock level optimization of the PTW rule.

Observation of Component Allocation Optimizaiton under FCFS

Theorem

For a periodic review ATO system with component base stock policy and FCFS allocation, let x_{jk} be the number of product j assembled in period $t+k$ for the demand $P_{j,t}$. Then the set of feasible component allocation decisions $x = (x_{jk})_{j,k}$ is characterized by:

$$X = \left\{ (x_{jk})_{j,k} : \begin{array}{ll} \sum_{k=0}^{L+1} x_{jk} = P_{j,t} & \forall j \in \mathcal{N} \\ \sum_{\mu=0}^{k} \sum_{j=1}^{n} a_{ij} x_{j\mu} \le O_i^k & \forall i \in \mathcal{M}, k < k^*, k \in \mathcal{L} \\ \sum_{\mu=0}^{k} \sum_{j=1}^{n} a_{ij} x_{j\mu} = D_{i,t} & \forall i \in \mathcal{M}, k \ge k^*, k \in \mathcal{L} \\ x_{jk} \in \mathbb{Z}_+ & \forall j \in \mathcal{N}, k \in \mathcal{L} \end{array} \right\},$$

where $O_i^k = \mathrm{Min}\{(S_i - D_i[t - L_i + k, t-1])^+, D_{i,t}\}$ and $k^* = \mathrm{Min}\{k \in \mathcal{L} : O_i^k = D_{i,t}\}$ and $\mathbb{Z}_+$ is the set of nonnegative integers.

Benchmark for the Demand Fulfillment Rates under FCFS

$$
\begin{array}{rcll}
C_1(S, \xi(\omega)) & = & \text{Min} & f_1(S, \xi(\omega), x, z) \\
 & & \text{s.t.} & P_{j,t} - \sum_{k=0}^{w_j} x_{jk} \leq P_{j,t} z_j \quad \forall j \in \mathcal{N} \\
 & & & z_j \in \{0, 1\} \quad \forall j \in \mathcal{N} \\
 & & & x \in X,
\end{array}
$$

where $z = (z_j)_{j \in \mathcal{N}}$ and $f_1(S, \xi(\omega), x, z) = \sum_{j=1}^{n} \frac{1}{n} z_j$.

Benchmark for the Operational Costs under FCFS

$$
\begin{array}{rcll}
C_3(S,\xi(\omega)) & = & \text{Min} & f_3(S,\xi(\omega),x) \\
 & & \text{s.t.} & x \in X,
\end{array}
$$

where

$$
\begin{array}{rcl}
f_3(S,\xi(\omega),x) & = & \sum_{i=1}^{m} h_i[(S_i - D_i^{L_i})^+ - \sum_{j=1}^{n} a_{ij} P_{j,t}]^+ \\
 & & + \sum_{i=1}^{m} \sum_{k=0}^{L+1} h_i(O_i^k - \sum_{\mu=0}^{k} \sum_{j=1}^{n} a_{ij} x_{j\mu}) \\
 & & + \sum_{j=1}^{n} \sum_{k=0}^{L+1} b_j(P_{j,t} - \sum_{\mu=0}^{k} x_{j\mu})
\end{array}
$$

Instances

- Agrawal and Cohen (2001)

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Performance Measure of the LCFP Rule

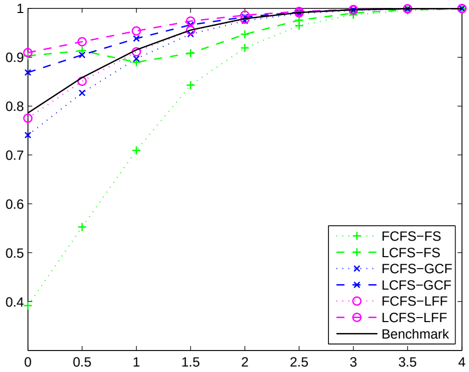
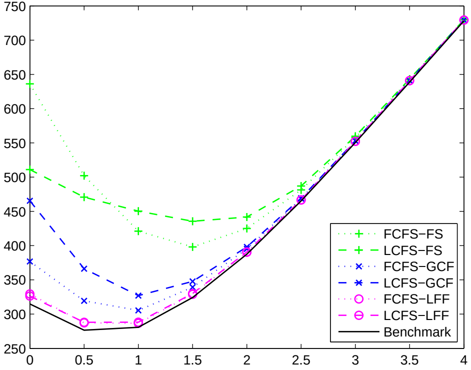


Figure : Comparison of demand fulfillment rates

Performance Measure of the LCFP Rule



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Performance Measure of the PTW Rule

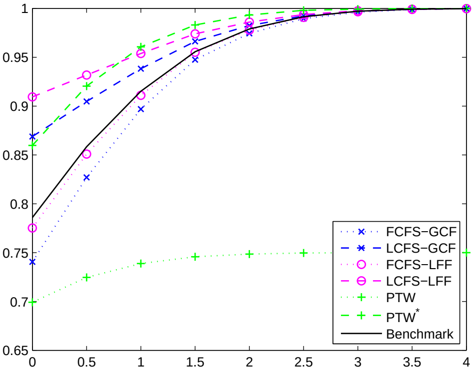

Figure : Comparison of demand fulfillment rates

Performance Measure of the PTW Rule

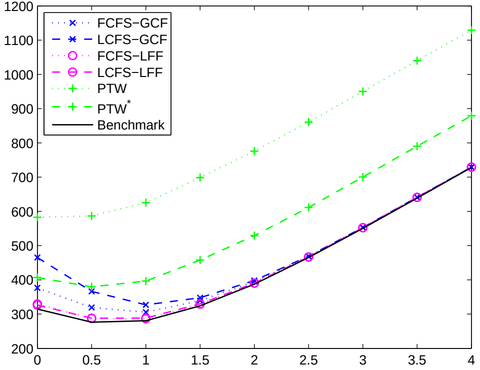

Figure : Comparison of operational costs

Conclusions

- The consignment property is the key in the analysis of the non-FCFS component allocation policies.

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- The consignment property is the key in the analysis of the non-FCFS component allocation policies.
- Chance-constrained programs naturally arise from ATO system optimization.
- The Sample Average Approximation algorithm is viable in solving small to medium instances.