# Hidden Illiquidity With Multiple Central Counterparties

**or**

***Why A Properly Calibrated Margin Model Underestimates Margin Requirements***

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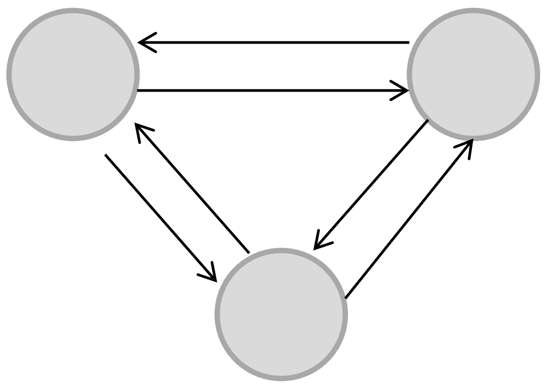
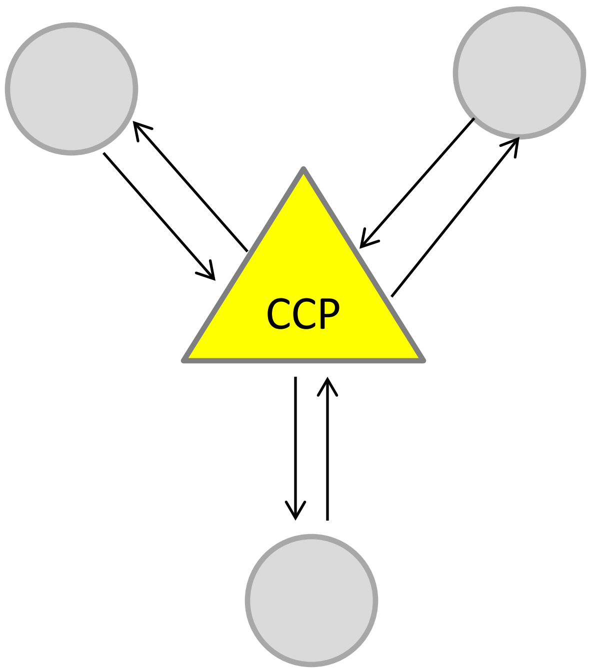
# OTC vs CCP

Over-the-counter market



Centrally cleared market

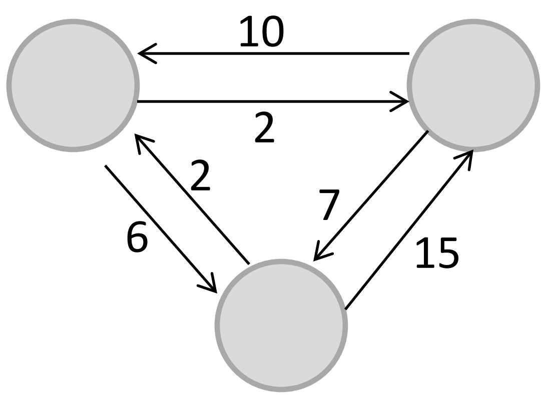


# Key Idea of the Paper

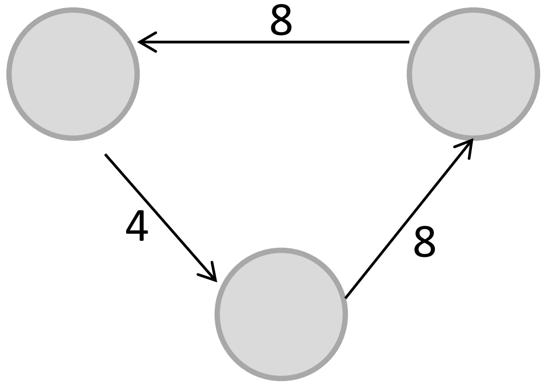
- Margin requirements need to reflect the price impact/liquidation cost/concentration risk of large illiquid positions at default
  - Need to grow superlinearly with position size
- This creates an incentive for clearing members to split their positions across CCPs
- So the CCPs need to charge more than the “right” amount of margin because of what they don’t see
- This may not work if different CCPs have different views on the “right” amount of margin, creating a race to the bottom
- Counteracting this effect requires some coordination or information sharing between CCPs and/or common members

# Netting Reduces Total Counterparty Risk

Over-the-counter market




Bilateral netting

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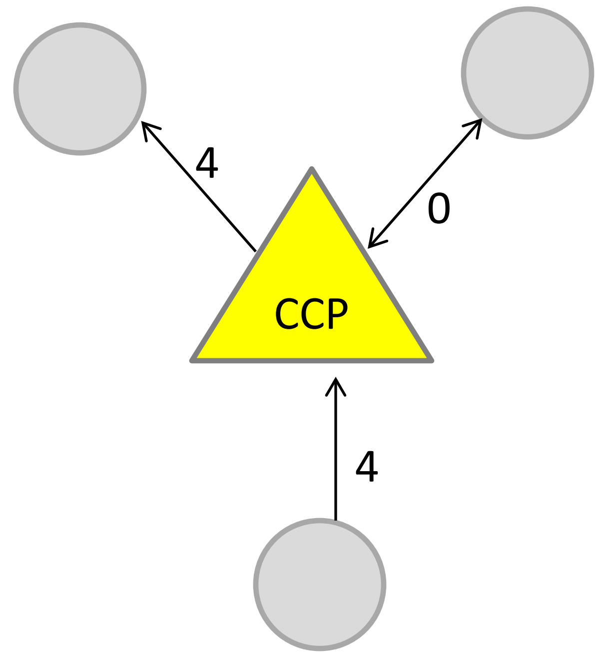


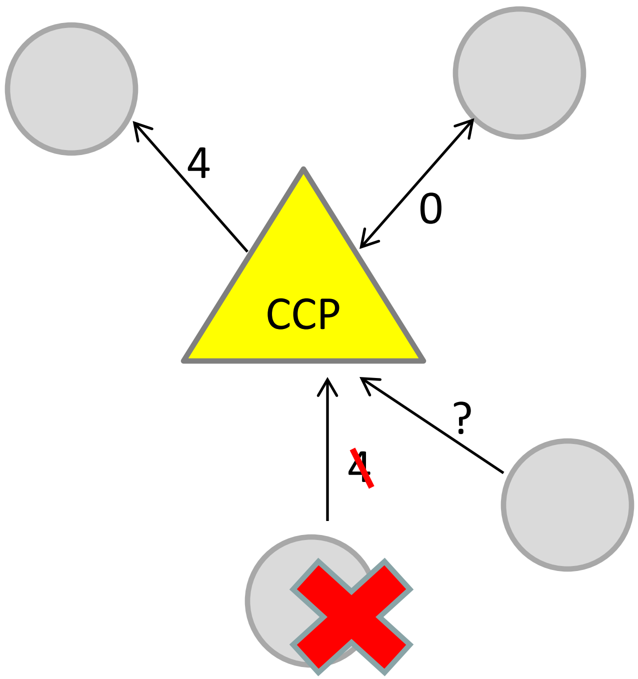

The CCP always has a matched book and zero net exposure, in theory

# But What Happens If A Clearing Member Fails?

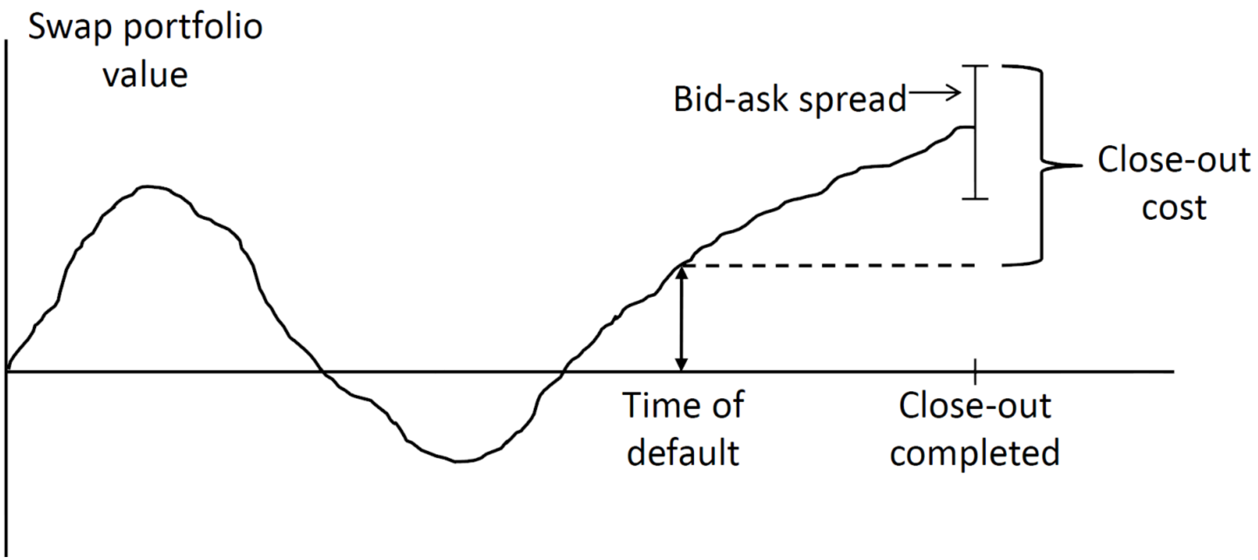
- If a clearing member fails, the CCP needs to restore a matched book but may incur a loss in doing so

- The failure of a CCP could cascade to failures of other clearing members

- CCPs are a potential source of systemic risk



# Margin Protects the CCP Against Default Risk




- CCP holds margin from each clearing member to absorb potential losses over a liquidation period of 5-10 days

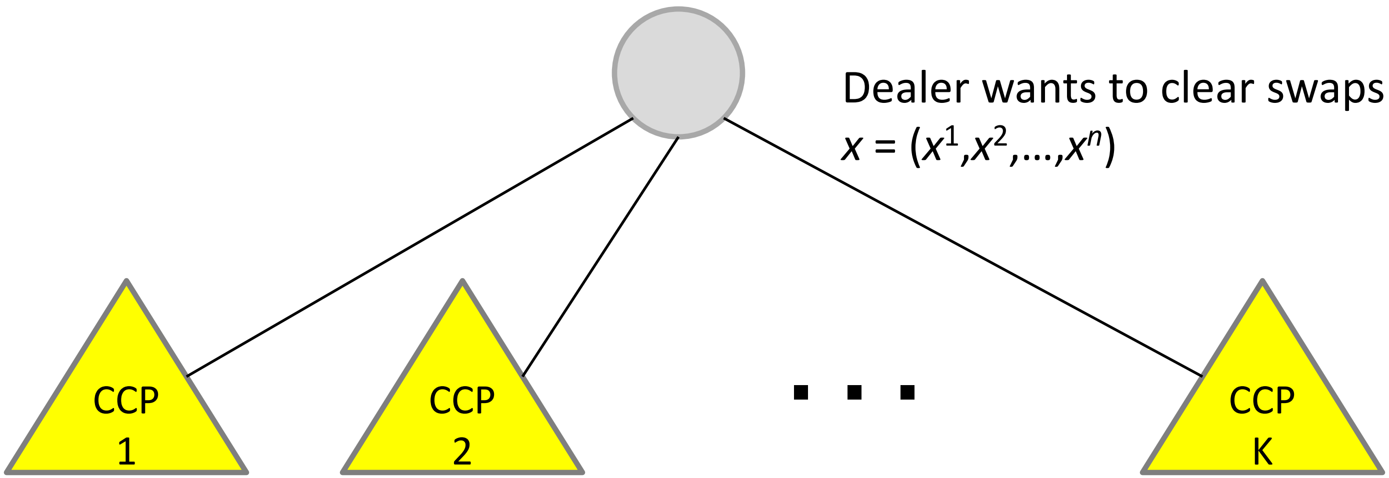
- This is “initial” margin as opposed to variation margin

- Clearing members also contribute to a default fund

# Consider Margin Proportional to Standard Deviation (Market Risk)

*n* types of swaps cleared by *K* CCPs

Dealer wants to clear swaps of size $x = (x^1, x^2, \ldots, x^n)$




| | | | |
|---|---|---|---|
| Allocation: | $x_1$ | $x_2$ | $x_K$ |
| Margin: | $a(x_1'\Sigma x_1)^{1/2}$ | $a(x_2'\Sigma x_2)^{1/2}$ | $a(x_K'\Sigma x_K)^{1/2}$ |

$\Sigma$= covariance matrix of 10-day price changes

$x_1+x_2+\ldots+x_K = x$

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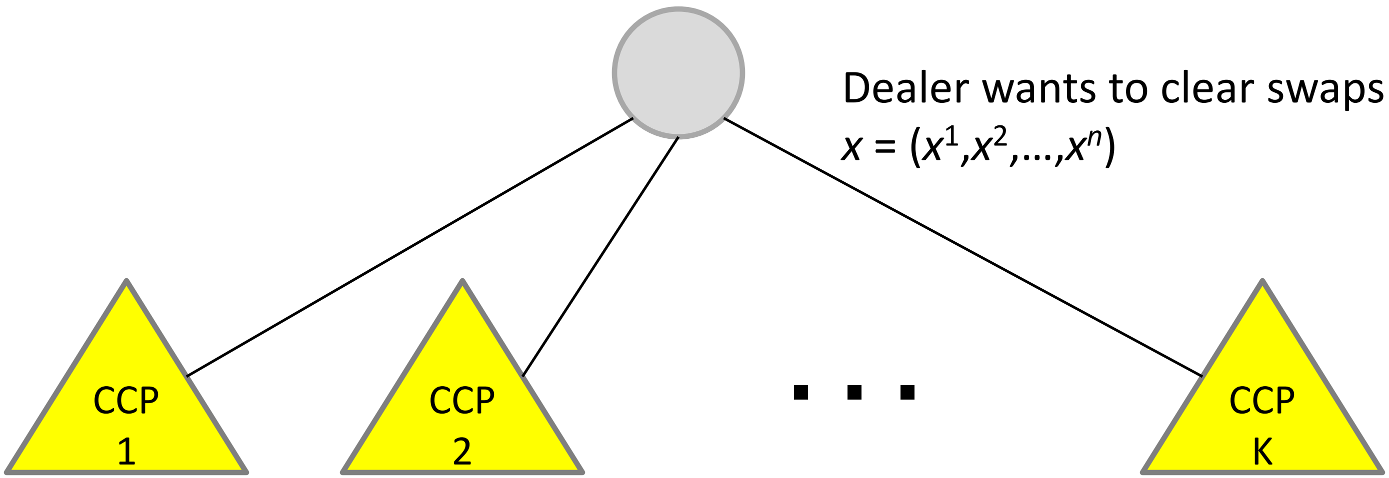


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$\Sigma$= covariance matrix of 10-day price changes

$x_1+x_2+\ldots+x_K = x$

How should the dealer allocate the position to minimize total margin?

# Incorporating Market Impact

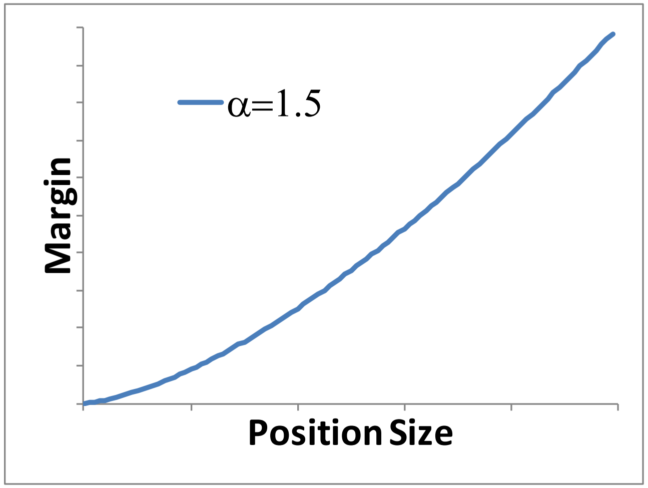
- Standard deviation is positively homogeneous: doubling the size of the swap doubles the margin requirement

$$\left(\lambda x^\top \Sigma \lambda x\right)^{1/2} = \lambda \left(x^\top \Sigma x\right)^{1/2}, \quad \lambda \geq 0$$

- But liquidating or replacing a large position will produce a more-than-proportional increase in the loss because of market impact
- Margin should be superlinear in position size; e.g.,

$$f(x) = \left(x^\top \Sigma x\right)^{\alpha/2}, \quad \alpha > 1$$

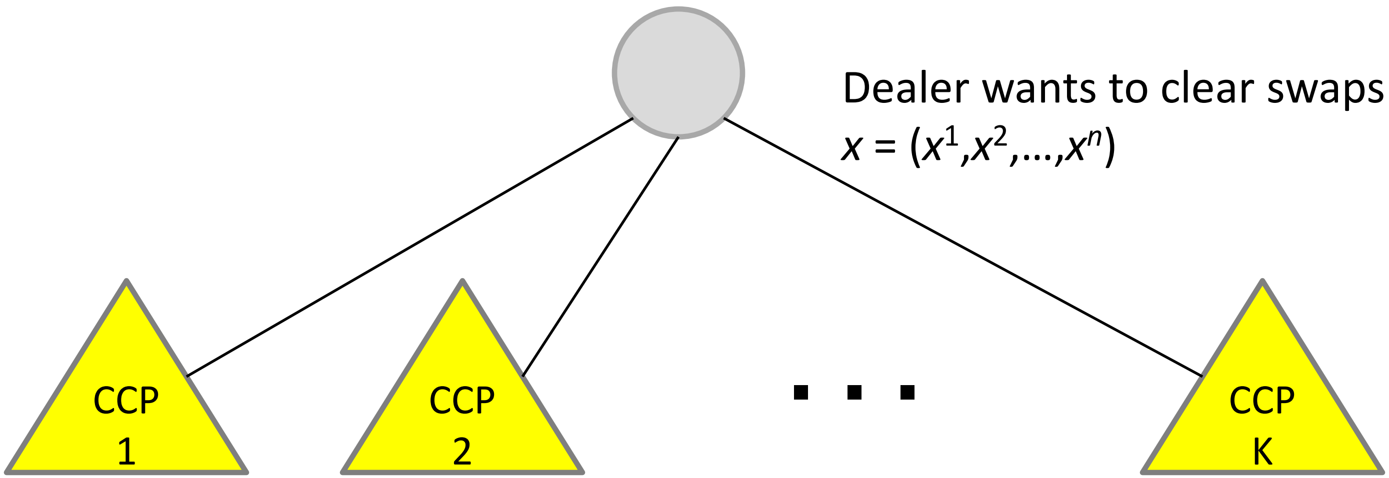
Then $f(\lambda x) = \lambda^\alpha f(x), \quad \lambda > 0$



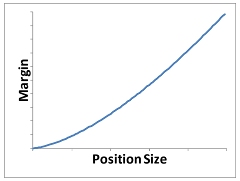
# Superlinear Margin

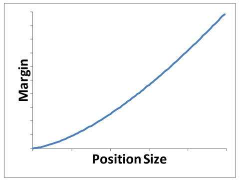
$n$ types of swaps cleared by $K$ CCPs

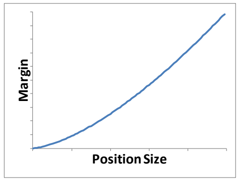
Dealer wants to clear swaps of size $x = (x^1, x^2, \ldots, x^n)$




Allocation: $x_1$ $x_2$ $x_K$




How should the dealer allocate the position to minimize total margin?

# The Dealer's Margin Minimization Problem

- Suppose all CCPs apply margin function $f$
- Dealer's problem:

$$\min_{x_1, x_2, \ldots, x_K} \sum_{i=1}^{K} f(x_i) \quad \text{subject to } x_1 + x_2 + \cdots + x_K = x$$

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**Proposition:** (a) If $f$ is

(i) Subadditive: $f(x+y) \leq f(x) + f(y)$

(ii) Positively homogeneous: $f(\lambda x) = \lambda f(x)$, $\forall \lambda \geq 0$

(as in the case of standard deviation) then clearing everything through one CCP is optimal, as is any allocation of the form $x_i = k_i x$, $k_i \geq 0$, $k_1 + k_2 + \cdots + k_K = 1$.
(b) If $f$ is strictly convex, then the unique optimum is an equal allocation

$$x_i = x/K, \quad i = 1, \ldots, K.$$

# Margin Requirement Through Price Impact

- Consider a scalar position of size $x$ cleared in a market with $K$ CCPs
- Suppose the margin function is given by

$$f(x) = \underbrace{x}_{\text{Size of position}} \underbrace{F(x)}_{\text{Price impact of liquidation}}$$

- We will assume $F(0)=0$ and $f$ increasing and strictly convex

# Why The Right Model Yields The Wrong Margin

- The dealer optimally sends $x/K$ to each CCP
- Each CCP collects margin equal to

$$f(x/K) = (x/K)F(x/K)$$

- But the total market impact if the dealer fails will be $F(x)$ so each CCP should collect margin equal to

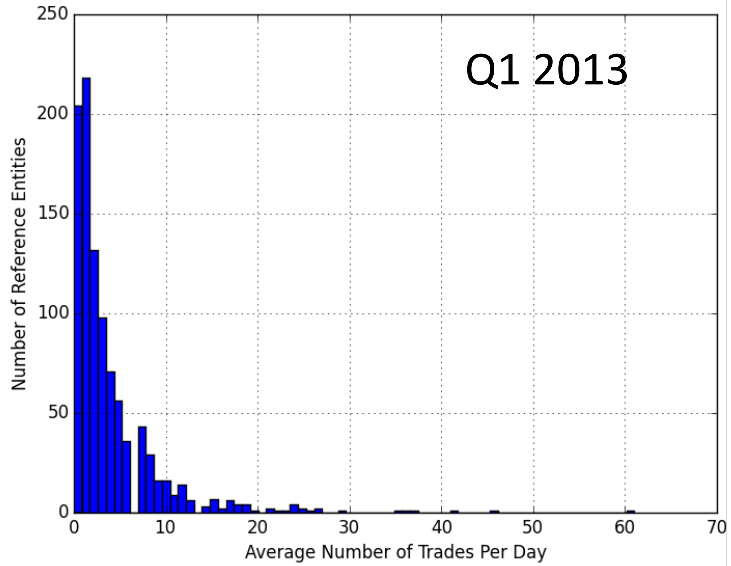
$$(x/K)F(x)$$

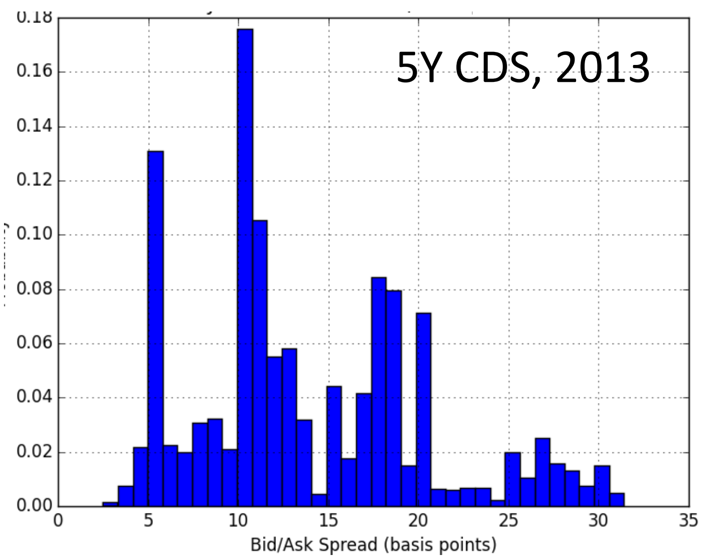
- In other words, each CCP needs to replace the "true" margin function $f$ with the "wrong" margin function

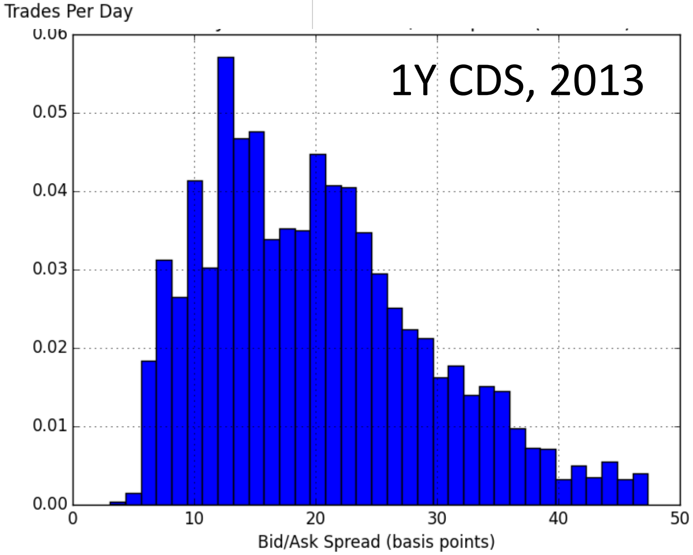
$$g(x) = xF(Kx)$$

In order to end up with the right level of margin

# Is Liquidity An Issue?



# CDS Margin Methodology:  Liquidity Charges

- ICE Clear Credit:
  - "Positions that exceed selected thresholds are subject to additional, exponentially increasing, initial margin requirements."

- CME Group:
  - "The liquidity risk requirement is designed to capture the liquidity and concentration premium during liquidation of the credit portfolio of a defaulted member
  - For large positions, this loss scales super-linearly by the number of days liquidation will take at a constant unwinding rate, therefore by the position size"

- LCH.Clearnet
  - "Liquidity charge:  In order to take into account the actual cost of liquidating a portfolio, bid-ask spreads need to be covered. Therefore, a specific charge is added, to model the cost of transaction, which increases for positions in excess of a given size."

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- Full disclosure: I serve on the risk committee of a swaps CCP

# What If The CCPs Have Different Models?

- We simplify to two CCPs
- We allow vector positions
- CCP $i$ believes the true price impact for vector position $x$ is $G_i(x)$
- CCP $i$ charges margin as if the price impact were $F_i(x)$
- In other words, it charges $x^\mathsf{T} F_i(x)$

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- CCP $i$ charges margin as if the price impact were $F_i(x)$
- In other words, it charges $x^\top F_i(x)$

- A dealer trading $x$ minimizes margin by solving

$$\min_{x_1, x_2} x_1^\top F_1(x_1) + x_2^\top F_2(x_2) \quad \text{subject to } x_1 + x_2 = x$$

- CCPs want to set margin charges to end up with enough margin after the dealer optimizes

# Equilibrium

Given price impact beliefs $G_1, G_2$ for the two CCPs, an equilibrium is defined by

- Allocation functions $x_i : \mathbb{R}^n \to \mathbb{R}^n$, $i = 1, 2$
- Price impact functions $F_i : \mathbb{R}^n \to \mathbb{R}^n$, $i = 1, 2$,

  with $F_i(0) = 0$ and $x \mapsto x^\top F_i(x)$ strictly convex

satisfying

- $(x_1(x), x_2(x))$ solves the dealer's allocation problem for all $x$
- $x_i^\top F_i(x_i) \geq x_i^\top G_i(x)$ (Sufficient margin condition)

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- $x_i^\top F_i(x_i) = x_i^\top G_i(x)$ (Sufficient margin condition)

We assume a competitive market in which CCPs cannot collect excess margin

# Linear Price Impact

- Specialize to the case of linear price impact

$$G_i(x) = G_i x, \quad G_i \in \mathbb{R}^{n \times n}$$

- Further suppose that

$$F_i(x) = F_i x, \quad F_i \in \mathbb{R}^{n \times n}$$

- In other words, CCP margin charges are quadratic,

$$x \mapsto x^\top F_i x$$

- We assume that the matrices $G_i$ and $F_i$ are symmetric and positive definite

# Digression on Linear Price Impact

- This is a multivariate Kyle (1985) model
  - In the usual, scalar Kyle model, price impact is linear, transaction cost is quadratic
- Do price impacts across different swaps make sense?
- Yes
  - CDS for firms in the same sector
  - 1-year and 5-year CDS for the same firm
  - Different series of the same index (the London Whale trade)
  - Also for interest rate swaps

- Cross-asset impacts are very difficult to estimate. Could be based on correlations in returns, but we are interested in impact at dealer's default

# Equilibrium With Linear Price Impact

**Theorem.** A necessary and sufficient condition for an equilibrium is that the CCPs have common beliefs on market impact, meaning $G_1 = G_2 \equiv G$.

In this case, all equilibria are determined by matrices $F_1, F_2$ satisfying

$$G^{-1} = F_1^{-1} + F_2^{-1}$$

CCPs need to agree on "true" price impact but not on the margin they charge

# Discussion

$$G^{-1} = F_1^{-1} + F_2^{-1}$$

- Special case: $F_i = 2G$, charge twice your belief and get half the volume

- More generally, we can have

$$F_1 = \frac{G}{\alpha}, \quad F_2 = \frac{G}{(1-\alpha)}, \quad \alpha \in (0,1).$$

The CCP that sets the margin lower gets more of the volume and needs to correct less for hidden illiquidity

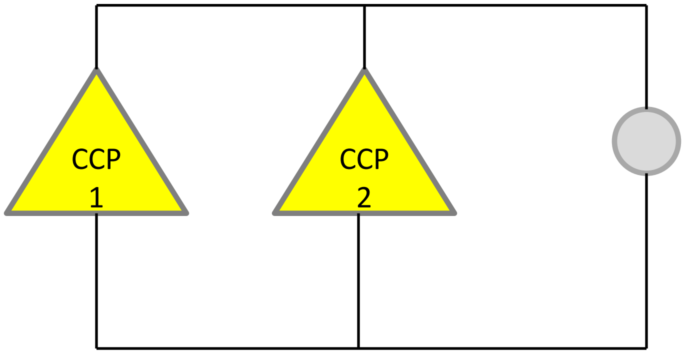
# Parallel Sum of Matrices

- The operation

$$(F_1^{-1} + F_2^{-1})^{-1}$$

is called the *parallel sum* of matrices (Anderson and Duffin 1969)

- It yields the *effective margin* in the market, so our condition states that the effective margin needs to equal the CCPs' share view on the margin required




Margin requirements combine like resistors connected in parallel:

resistance ~ price impact per unit traded
current ~ size of trade
voltage ~ price impact of trade

# If They Disagree: A Race to the Bottom

- Consider the scalar case with price impact views $G_1 < G_2$
- Suppose, initially, they charge according to their views, $F_i = G_i$.
- A dealer trading $x$ minimizes margin by setting

$$x_1 = \frac{F_2}{F_1 + F_2} x, \quad x_2 = \frac{F_1}{F_1 + F_2} x$$

- CCPs update their charges to have enough margin:

$$\hat{F}_1 x_1^2 = x_1 (G_1 x), \quad \hat{F}_2 x_2^2 = x_2 (G_2 x)$$

- This yields

$$\frac{\hat{F}_2}{\hat{F}_1} = \left(\frac{G_2}{G_1}\right)\left(\frac{F_2}{F_1}\right) \to \infty, \qquad x_1 \to x, \quad x_2 \to 0$$

- The CCP that estimates a higher liquidation cost gets driven out

# Equilibrium With Non-Participation

- We expand the strategy space for each CCP, allowing it to decide whether to clear certain types of swaps (as opposed to just setting margin levels)
- This partitions the set of swap types into three groups:
  - Cleared only by CCP 1
  - Cleared by both
  - Cleared only by CCP2
- We partition vectors and matrices in accordance with this decomposition
- We remove any swap types not cleared by either CCP

# Equilibrium With Non-Participation

**Theorem.** An equilibrium exists if and only if the CCPs' price impact views have a common block diagonal structure

$$G_i = \begin{pmatrix} G_i(1,1) & & \\ & G_i(2,2) & \\ & & G_i(3,3) \end{pmatrix}, \quad i = 1, 2,$$

with $G_1(2,2) = G_2(2,2) \equiv G(2,2)$. In this case, all equilibria are determined by matrices $F_1$, $F_2$,

$$F_1 = \begin{pmatrix} G_1(1,1) & \\ & F_1(2,2) \end{pmatrix}, \quad F_2 = \begin{pmatrix} F_2(2,2) & \\ & G_2(3,3) \end{pmatrix},$$

satisfying

$$G(2,2)^{-1} = F_1(2,2)^{-1} + F_2(2,2)^{-1}$$

CCPs need to

- agree on "true" price impact for swaps they both clear
- clear anything that impacts anything they clear

# Adding Uncertainty

Previously we had

- $(x_1(x), x_2(x))$ solves the dealer's allocation problem for all $x$
- $x_i^\top F_i x_i = x_i^\top G_i x$ (Sufficient margin condition)

Now we add (uncorrelated, zero mean) uncertainty to

- Each CCP's inference about total position size: $x + \epsilon_i$
- Each CCP's views on price impact: $G_i$ stochastic, uncorrelated with $\epsilon_i$

Equilibrium condition becomes

$$x_i^\top F_i x_i = x_i^\top \mathbb{E}[G_i(x+\epsilon)] = x_i^\top \mathbb{E}[G_i] x$$

and results go through replacing $G_i$ with $\mathbb{E}[G_i]$

# What Can We Say With Nonlinear Price Impact?

- For the scalar case, we have a general characterization of equilibrium, but it is difficult to apply

- Example:

If common view of price impact is

$$G(x) = cx^{\beta}, \quad \beta > 0,$$

then we get an equilibrium with $F_i(x) = b_i x^{\beta}$, $i = 1, 2$, for any $b_1, b_2$ satisfying

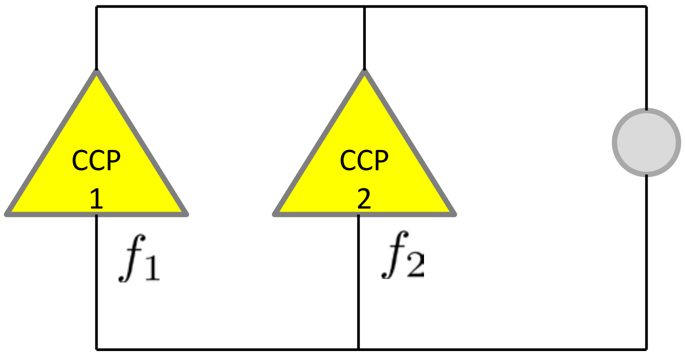
$$b_1^{-1/\beta} + b_2^{-1/\beta} = c^{-1/\beta}$$

- Similarity with linear case is not accidental. Both are consequences of *effective margin*

# Effective Margin

The effective margin requirement for the market is the inf-convolution of the individual margin requirements:

$$
\begin{aligned}
f_{\text{eff}}(x) &= \min_{x_1} \{f_1(x_1) + f_2(x - x_1)\} \\
&= (f_1 \Box f_2)(x).
\end{aligned}
$$



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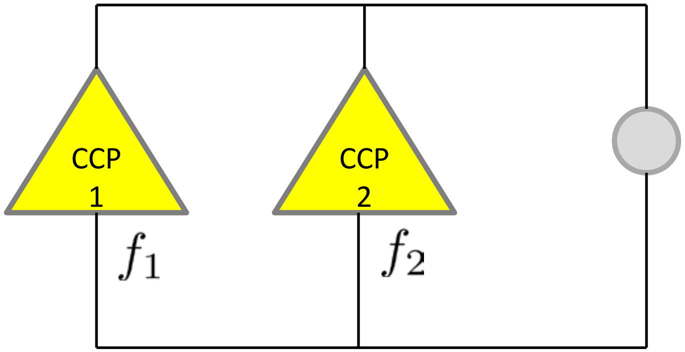


$$\begin{aligned} f_{\text{eff}}(x) &= \min_{x_1} \{f_1(x_1) + f_2(x - x_1)\} \\ &= (f_1 \square f_2)(x). \end{aligned}$$

For proper convex functions (Rockafellar 1973, Thm. 16.4)

$$(f_1 \square f_2)^* = (f_1^* + f_2^*)$$

where $f^*$ is the conjugate of $f$, $f^*(y) = \sup_x\{x^\top y - f(x)\}$. In the strictly convex quadratic case

$$f(x) = x^\top F x, \quad f^*(x) = x^\top F^{-1} x$$

and the effective margin is given by

$$x^\top (F_1^{-1} + F_2^{-1})^{-1} x$$

# Equilibrium With Nonlinear Price Impact

**Theorem:** [Scalar case, nonlinear impact]

(i) If the CCPs have common beliefs $G_1 = G_2 = G$, then an equilibrium exists. All equilibria result in proportional allocations $x_1 = \alpha x$ and $x_2 = (1 - \alpha)x$, for some $\alpha \in (0, 1)$.

(ii) If an equilibrium with proportional allocations exists, then the CCPs have common beliefs $G_1 = G_2 = G$.

(iii) In any equilibrium with common beliefs, $f_{\text{eff}} = g$, meaning that the effective margin equals the shared view on required margin and

$$g = (f_1^* + f_2^*)^*$$

where $g(x) = xG(x)$.

# Back to the Real World: Implications

- CCPs need to consider liquidation cost/price impact in setting margin
  - This requires superlinear margin

- Because superlinear margin creates an incentive for dealers to spread positions, CCPs need to account for what they don't see in setting margin
  - Margin needs to be higher than what the "right" model says
  - Good backtesting is bad
  - CCPs and/or dealers need to share information about trades at other CCPs

- To avoid a race to the bottom, CCPs need shared information about "true" liquidation cost. Potential solutions:
  - Firm commitments to buy (short puts) from dealers as part of their guarantee fund contributions
  - Fed and CFTC recently called for standard stress tests for CCPs. Add impact of other CCPs to these stress tests

# Thank You