Optimal Asset Location and Allocation with Taxable and Tax-Deferred Investing

Robert M. Dammon

Carnegie Mellon University

Chester S. Spatt

Carnegie Mellon University

Harold H. Zhang*

University of North Carolina

First Draft: November 1, 1999

Current Draft: November 1, 2002

* Robert M. Dammon and Chester S. Spatt are from the Graduate School of Industrial Administration at Carnegie Mellon University and Harold H. Zhang is from the Kenan-Flagler Business School at the University of North Carolina at Chapel Hill. We thank Gene Amromin, Peter Bossaerts, Jerome Detemple, Doug Fore, Richard Green, Robert McDonald (the editor), Peter Schotman, two anonymous referees, and seminar participants at the California Institute of Technology, London Business School, the University of Pittsburgh, the University of Southern California, the University of Utah, Washington University in Saint Louis, the 2000 Western Finance Association Meetings at Sun Valley, the Stanford Institute for Economic Policy Research Asset Location Conference, and the Center for Economic Policy Research Summer Symposium at Gerzensee, Switzerland for helpful comments. The financial support provided by the Teachers Insurance Annuity Association–College Retirement Equities Fund is gratefully acknowledged.

Abstract

We investigate optimal intertemporal asset allocation and location decisions for investors with both taxable and tax-deferred investment opportunities. With unrestricted borrowing, investors optimally allocate their entire tax-deferred wealth to taxable bonds and combine either borrowing or lending with equity in the taxable account to achieve their optimal overall risk exposure. When prohibited from borrowing, a mix of stocks and bonds may be held in the tax-deferred account, but only if the taxable account is allocated entirely to equity. In this case, the fraction of total wealth allocated to equity is inversely related to the fraction of total wealth held in the tax-deferred account. We also examine the effects of liquidity shocks on optimal asset location and allocation.

A central problem confronting investors in practice is how to optimally invest the funds held in their taxable and tax-deferred savings accounts. The problem involves making both an optimal *asset allocation* decision (i.e., deciding how much of each asset to hold) and an optimal *asset location* decision (i.e., deciding which assets to hold in the taxable and tax-deferred accounts). Investors would like to make these decisions to reduce the tax burden of owning financial assets, while maintaining an optimally diversified portfolio over time. While only limited guidance is available to investors grappling with this problem, the decision is crucial to the total wealth accumulation and welfare of investors over their lifetimes.

This paper examines the joint determination of the asset allocation decisions for both the taxable and tax-deferred savings accounts in the presence of taxes on investment income and capital gains. In an earlier paper without a tax-deferred account, Dammon, Spatt and Zhang (2001a) show how an investor's optimal asset allocation depends upon the investor's initial asset holdings, embedded capital gain, and age. By incorporating a tax-deferred account into the analysis, we investigate how the opportunity for tax-deferred investing influences the investor's asset allocation within the taxable account and how this allocation interacts with the optimal allocation within the tax-deferred account. This is in striking contrast to the standard approach to financial planning, which ignores the interaction between these two accounts and fails to consider how the investor's age, embedded capital gains, and available wealth levels in the taxable and tax-deferred accounts influence the asset allocation and location decisions.

The ability to invest on a tax-deferred basis is valuable to investors because it allows them to earn the pre-tax return on assets. However, because assets differ in terms of the tax liabilities they create for investors, the value of tax-deferred investing will depend upon which assets are held in the tax-deferred account. For example, it is well understood that because the interest on municipal bonds is exempt from taxation, they should not be held in the tax-deferred account. Our analysis of the optimal asset location policy focuses mainly on taxable bonds and equity. We show that there is a strong locational preference for holding taxable bonds in the tax-deferred account and equity in the taxable account. This preference reflects the higher tax burden of bonds relative to equity. When held in the taxable account, equity generates less ordinary income than taxable bonds, provides the investor with a valuable tax-timing option to realize capital losses and defer capital gains, and allows the investor to avoid the payment of the tax on capital gains altogether at the time of death. Our analysis also examines the circumstances under which equity ownership arises in the tax-deferred account, despite its greater attractiveness in the taxable account.

When investors have unrestricted borrowing opportunities in their taxable accounts, the optimal asset location policy involves allocating the entire tax-deferred account to taxable bonds. Investors then combine either borrowing or lending with investment in equity in the taxable account to achieve their optimal overall risk exposure. This optimal asset location policy holds even when capital gains and losses are taxed on an accrual basis (i.e., no deferral option). The optimal asset location policy with unrestricted borrowing opportunities follows directly from the arbitrage arguments made by Black (1980) and Tepper (1981) in analyzing the optimal investment policy for corporations with defined-benefit pension plans.[1] Despite the preference for holding equity in the taxable account, our numerical analysis indicates that the proportion of total wealth allocated to equity is relatively insensitive to the split of wealth between the taxable and tax-deferred accounts when investors have the ability to borrow.

When investors are prohibited from borrowing, the optimal asset location policy is slightly more complicated. Although investors still have a preference for holding taxable bonds in the tax-deferred account, it may not be optimal to allocate the entire tax-deferred account to taxable bonds if doing so causes the overall portfolio to be overweighted in bonds. In this case, investors may hold a mix of stocks and bonds in their tax-deferred accounts, but only if they hold an all-equity portfolio in their taxable accounts. Investors do not hold a mix of stocks and bonds in both the taxable and tax-deferred accounts simultaneously. We numerically investigate the optimal asset location and allocation decisions in the presence of borrowing constraints by incorporating a tax-deferred investment account into the intertemporal consumption-investment model developed by Dammon, Spatt, and Zhang (2001a).[2] We illustrate how the relative wealth levels in the taxable and tax-deferred accounts influence the optimal asset location and allocation decisions. We find that equity spills over into the tax-deferred account only for investors with sufficiently high levels of tax-deferred wealth. However, because equity is less valuable when held in the tax-deferred account, the proportion of total wealth allocated to equity is lower for investors with higher levels of tax-deferred wealth. The model also allows us to examine the effects of age and liquidity shocks on the optimal asset allocation and location decisions.

In two recent papers, Shoven (1999) and Shoven and Sialm (2002) examine the optimal asset location decision in a model with taxable and tax-deferred savings accounts, but without intertemporal consumption, savings, or portfolio rebalancing decisions. They argue that because actively-managed equity mutual funds distribute a large fraction of their capital gains each year, it can be optimal for highly-taxed investors to locate them in the tax-deferred account and hold tax-exempt bonds in the taxable account. Poterba, Shoven, and Sialm (2000) document this empirically using the historical returns on actively-managed equity mutual funds over the 1962-98 period. We compare the performance of this strategy to the alternative strategy of holding *tax-efficient* equity (e.g., individual stocks, index funds, or exchange-traded funds) in the taxable account and taxable bonds in the tax-deferred account. Using arbitrage arguments, we show that the strategy of holding an actively-managed equity mutual fund in the tax-deferred account and tax-exempt bonds in the taxable account can be optimal only if the actively-managed mutual fund is (1) highly tax-inefficient and (2) substantially outperforms similar tax-efficient equity investments (net of transaction costs and fees) on a risk-adjusted basis. Given the well-documented underperformance of actively-managed equity mutual funds, we argue that investors are better off holding tax-efficient equity investments, locating them in the taxable account, and holding taxable bonds in the tax-deferred account. Of course, highly-taxed investors who wish to hold a mix of stocks and bonds in their taxable account may still find tax-exempt bonds to be a better alternative than taxable bonds.

In principle, with most (or all) of the taxable account allocated to equity, the investor may face liquidity problems if the value of equity declines substantially. This can give the investor an incentive to hold some bonds in the taxable account to reduce the risk of needing to liquidate a portion of the tax-deferred account to finance consumption. In a recent paper, Huang (2000) uses our model to examine the effects of exogenous liquidity shocks on the optimal asset location decision. She shows that when investors face a *known* future liquidity shock that is proportional to total wealth it can be optimal to hedge by shifting some bond holdings from the tax-deferred account to the taxable account. Our own analysis of this type of liquidity shock indicates that the hedging demand for taxable bonds is non-zero only for investors with taxable wealth slightly below the magnitude of the shock and only in the years immediately preceeding the occurrence of the shock. Moreover, the magnitude of the hedging demand for taxable bonds is small relative to the overall holdings of taxable bonds. We also examine the effect of *random* liquidity shocks to consumption that occur with a constant probability over time. We find that even for a relatively high annual probability of experiencing a large shock to consumption investors do not hold taxable bonds in their taxable accounts for hedging reasons. However, investors do reduce the annual contributions to their tax-deferred accounts when the wealth in their taxable accounts is far below the magnitude of the potential shock.

The results we derive on the optimal location of assets between the taxable and tax-deferred accounts are in sharp contrast to the financial advice that investors typically receive. Financial advisors commonly recommend that investors hold a mix of stocks and bonds in both their taxable and tax-deferred accounts, with some financial advisors recommending that investors tilt their tax-deferred accounts toward equity. The asset location decisions made in practice mirror these recommendations, with many investors holding equity in a tax-deferred account and bonds in a taxable account. Poterba and Samwick (1999) report that 48.3 percent of investors who own taxable bonds in taxable accounts also own equity in tax-deferred accounts and that 41.6 percent of investors who own equity in tax-deferred accounts also own taxable bonds in taxable accounts. They also document that 53.1 percent of the owners of tax-exempt bonds also owned equity in tax-deferred accounts and that 11.3 percent of the owners of equity in tax-deferred accounts also owned tax-exempt bonds. Bergstresser and Poterba (2002) report similar findings and document that a large proportion of investors have substantially more equity in their tax-deferred accounts than in their taxable accounts. We discuss this "asset location puzzle" and investigate the welfare costs of locating assets suboptimally between the taxable and tax-deferred accounts. We estimate that the welfare costs of locating assets suboptimally can exceed 20 percent for young investors. We also examine the welfare benefits of tax-deferred investing as a function of the investor's age and level of retirement account wealth.

The paper is organized as follows. In Section I we derive some general theoretical results regarding optimal asset location using basic arbitrage arguments. In Section II we present our numerical analysis of the optimal asset location and allocation decisions, focusing on the case in which there are restrictions on borrowing and short sales. We also conduct a welfare analysis of the optimal asset location policy and investigate the effect of exogenous liquidity shocks on the optimal asset location decision. Section III concludes the paper.

I. Optimal Asset Location

In this section we use arbitrage arguments to derive results on the optimal location of asset holdings. Our approach is similar to the arbitrage approaches used by Black (1980) and Tepper (1981) to analyze corporate pension policy and by Huang (2000) to analyze the asset location decision. The arbitrage approach involves making a risk-preserving change in the location of asset holdings to determine whether the total after-tax return on the investor's portfolio can be improved. The objective is to identify the asset location policy that produces the highest risk-adjusted after-tax return for the investor.

We initially assume that investors are *forced* to realize all capital gains and losses each year (i.e., no deferral option) and have *unrestricted* borrowing and short-sale opportunities in their taxable accounts. (We later relax these assumptions to see what effect they have on the optimal location decision.) We also assume that the tax rate on ordinary income (dividends and interest), τ_d, is higher than the tax rate on capital gains and losses, τ_g. Under these conditions we show that investors prefer to allocate their entire tax-deferred wealth to the asset with the *highest yield.* Investors then adjust the asset holdings in their taxable accounts, borrowing or selling short if necessary, to achieve an optimal overall risk exposure. For our purposes, we define *yield* as the fraction of total asset value (price) that is distributed to the investor as dividends or interest.

We define the random pre-tax return on asset i as $\tilde{r}_i = (1 + d_i)(1 + \tilde{g}_i) - 1$, where d_i denotes the constant pre-tax yield on asset i and $\tilde{g}_i$ denotes the random pre-tax capital gain return on asset i. For the riskless taxable bond (asset 0), we assume that $\tilde{g}_0 = 0$ and $d_0 = r$. Consider an investor in this environment who has positive holdings of both the riskless taxable bond and risky asset i in the tax-deferred account. For this investor a shift of one after-tax dollar from asset i to the taxable bond in the tax-deferred account, offset by a shift of x_i dollars from the riskless taxable bond (either through an outright sale or through borrowing) to asset i in the taxable account, leads to the following change in the after-tax cash flows on the investor's overall portfolio:

$$\Delta C_i = r - [(1+\tilde{g}_i)(1+d_i) - 1] + x_i\{[(1+\tilde{g}_i)(1+d_i(1-\tau_d)) - \tilde{g}_i\tau_g - 1] - r(1-\tau_d)\} \quad (1)$$

Letting $x_i = (1+d_i)/[1+d_i(1-\tau_d)-\tau_g]$, it is easily shown that for all values of $\tilde{g}_i$,

$$\Delta C_i = x_i \left[\frac{(r-d_i)(\tau_d-\tau_g)}{1+d_i}\right] \quad (2)$$

Since ΔC_i is independent of $\tilde{g}_i$, it represents a *risk-free* after-tax cash flow that can be generated by shifting the location of asset holdings. If $\Delta C_i > 0$, then the investor is strictly better off holding taxable bonds in the tax-deferred account and asset i in the taxable account. If $\Delta C_i < 0$, then the investor is strictly better off holding taxable bonds in the taxable account and asset i in the tax-deferred account. To determine the tax benefit of shifting one after-tax dollar from risky asset i to risky asset j in the tax-deferred account, with an offsetting adjustment in the taxable account, one simply needs to compute the difference $(\Delta C_i - \Delta C_j)$.[3] Only if $\Delta C_i = 0$ for all i is the investor indifferent to the location of his asset holdings.

Since x_i is strictly positive, the sign of ΔC_i depends upon the sign of $(r-d_i)(\tau_d-\tau_g)$. If $\tau_d = \tau_g$, then $\Delta C_i = 0$ for all i and the investor is indifferent to the location of his asset holdings. This indifference result is independent of the expected returns and yields on assets and only requires that the total returns on all assets be taxed identically each year. When $\tau_d > \tau_g$, the sign of ΔC_i depends upon the sign of $(r-d_i)$, with the value of ΔC_i monotonically *decreasing* in d_i. Thus, when $\tau_d > \tau_g$ the investor prefers to allocate his entire tax-deferred wealth to the asset with the *highest yield*, with all other assets held in the taxable account.[4]

If the riskless taxable interest rate exceeds the yields on all other assets (i.e., $d_i < r$ for all i), the investor optimally holds only riskless taxable bonds in the tax-deferred account. After allocating his entire tax-deferred wealth to the asset with the highest yield, the investor then adjusts the asset holdings in the taxable account, borrowing or selling short if necessary, to achieve the desired overall risk exposure. This asset location policy provides the investor with the highest level of tax-arbitrage profits without affecting the risk profile of his overall portfolio. The optimal asset location policy also is independent of the joint distribution of asset returns and investors' preferences.

It is widely believed that because actively-managed mutual funds distribute significant capital gains each year it can be tax efficient to hold these funds (to the extent that they are held at all) in a tax-deferred capacity. Similarly, it is believed that an investor who engages in active trading should do so in a tax-deferred account to avoid the payment of capital gains taxes. Our analysis of the optimal asset location policy sheds some light on this issue. Recall that our analysis is based upon the assumption that investors are *forced* to realize all capital gains and losses each year (i.e., no deferral option). Yet, despite the inability to defer capital gains, our analysis indicates that it is still optimal to locate the asset with the *highest yield* in the tax-deferred account provided $\tau_d > \tau_g$.[5] Thus, even though actively-managed mutual funds distribute most, or even all, of their capital gains each year, they should not be held in the tax-deferred account if the riskless taxable bond has a higher yield. Only in the extreme case in which the actively-managed mutual fund distributes 100 percent of its capital gains each year, with all gains realized *short term* so that $\tau_g = \tau_d$, would the investor be *indifferent* to holding the actively-managed mutual fund or riskless taxable bond in the tax-deferred account.[6]

According to the above analysis, locating equity (or equity mutual funds) in the tax-deferred account and taxable bonds in the taxable account can be optimal only if equity has a higher yield than taxable bonds. In a recent paper, Shoven and Sialm (2002) show that when investors can invest in *tax-exempt* bonds it may be optimal to locate equity in the tax-deferred account, even though the interest rate on taxable bonds exceeds the yield on equity (i.e., $r > d$). Using our framework, the incremental riskless after-tax cash flow due to shifting one after-tax dollar from taxable bonds to equity in the tax-deferred account and shifting $x = (1+d)/[1+d(1-\tau_d)-\tau_g]$ dollars from equity to tax-exempt bonds in the taxable account is:

$$\Delta C = x\left[r(\tau_d - \tau_m) - \frac{(r-d)(\tau_d - \tau_g)}{1+d}\right], \tag{3}$$

where τ_m is the implicit tax rate reflected in the yield differential between riskless taxable bonds and riskless tax-exempt bonds.[7] If $\tau_g > \tau_d - [r(\tau_d - \tau_m)(1+d)]/(r-d)$, then $\Delta C > 0$ and it is optimal for the investor to hold equity in the tax-deferred account and tax-exempt bonds in the taxable account. Assuming $r = 6\%$, $d = 2\%$, $\tau_d = 36\%$, and $\tau_m = 25\%$,[8] $\Delta C > 0$ for all $\tau_g > 19.17\%$. For a tax-inefficient equity mutual fund that realizes 75% of its capital gains each year, two-thirds of which are short term and one-third of which are long term, the effective capital gains tax rate is $\tau_g = 23\%$. In contrast, for a tax-efficient index fund that realizes only 15% of its capital gains each year, 5% of which are short term and 95% of which are long term, the effective capital gains tax rate is only $\tau_g = 3.12\%$.[9] Clearly it can be optimal to hold equity in the tax-deferred account (instead of the taxable account), but only if the form of the equity holding is *highly* tax inefficient.

While the above analysis is instructive, it does not directly answer the question as to whether it is better to hold a tax-efficient index fund in the taxable account and taxable bonds in the tax-deferred account, or to hold an actively-managed (tax-inefficient) equity mutual fund in the tax-deferred account and tax-exempt bonds in the taxable account. Assume that the two equity funds have identical dividend yields and are perfectly correlated on a pre-tax basis. Let τ_{gi} denote the effective capital gains tax rate on the tax-efficient index fund. It is straightforward to show that a shift of one after-tax dollar from taxable bonds to the actively-managed mutual fund (asset j) in the tax-deferred account, offset by a shift of $x_{ij} = (1 + d_j)/[1 + d_j(1 - \tau_d) - \tau_{gi}]$ dollars from the index fund (asset i) to tax-exempt bonds in the taxable account, produces the following *riskless* after-tax cash flow on the investor's overall portfolio:[10]

$$\Delta C_{ij} = [d_j + \alpha_j(1 + d_j) - r] - x_{ij}[d_j(1 - \tau_d) - r(1 - \tau_m)], \qquad (4)$$

where $\alpha_j = (\tilde{g}_j - \tilde{g}_i)$ is the *riskless* pre-tax capital gain return differential between the actively-managed mutual fund and the tax-efficient index fund. Thus, we can interpret $\alpha_j(1 + d_j)$ as the *certainty-equivalent* pre-tax abnormal return (net of transaction costs and fees) on the actively-managed mutual fund. Using the tax rates, dividend yields, and interest rates from above, the value of ΔC_{ij} is strictly positive provided $\alpha_j(1 + d_j) > 0.00654$. This implies that it is optimal to hold the actively-managed mutual fund in the tax-deferred account and municipal bonds in the taxable account (instead of taxable bonds in the tax-deferred account and the tax-efficient index fund in the taxable account) only if the actively-managed mutual fund generates a *certainty-equivalent* pre-tax abnormal return (net of transaction costs and fees) of at least 65.4 basis points per year.[11] Moreover, since it is not uncommon for actively-managed mutual funds to have expense ratios that are 100 basis points or more above that of a passive index fund, a *certainty-equivalent* pre-tax abnormal return (before transaction costs and fees) of 165 basis points or more may be necessary before it is beneficial to hold the actively-managed equity mutual fund in the tax-deferred account. Given the well-documented under-performance of actively-managed mutual funds, investors are more likely to benefit from holding income-producing taxable bonds in their tax-deferred accounts and tax-efficient equity investments (e.g., individual stocks, index funds, and exchange-traded funds) in their taxable accounts.[12]

With unrestricted borrowing and short-sale opportunities, the investor optimally allocates his entire tax-deferred wealth to the asset with the highest yield (typically taxable bonds) and either borrows or sells short in the taxable account to achieve an optimal overall risk exposure. If the investor faces restrictions on borrowing or selling short, then the optimal asset location policy is more complicated. In this case, the investor shifts his tax-deferred wealth into the asset with the highest yield until offsetting adjustments in the taxable account are no longer possible because of the borrowing or short-sale restrictions.[13] The investor then begins to allocate his remaining tax-deferred wealth to the asset with the next highest yield until the restrictions again bind. The process continues with successively lower yielding assets until the investor's tax-deferred wealth has been completely allocated. Thus, with borrowing and short-sale constraints, the investor may hold a mix of taxable bonds and equity in the tax-deferred account, but only if the taxable account is invested entirely in assets with lower yields.

While the optimal asset location policy maximizes the tax efficiency of the investor's overall portfolio, it also increases the risk of the taxable portfolio relative to the tax-deferred portfolio. With restrictions on borrowing and short sales, this shift in risk between the taxable and tax-deferred accounts may become important for some investors. For example, an investor with relatively little taxable wealth (relative to tax-deferred wealth) may wish to control the risk of his taxable portfolio to guarantee a minimum level of consumption. For such an investor, the assets held in the taxable and tax-deferred accounts may not conform to the optimal location policy we just described. We investigate the implications of liquidity needs for optimal asset location in Section II.E.

While the above analysis highlights the importance of tax efficiency, it has largely ignored the benefits of optimal tax timing. In practice, investors are not forced to realize capital gains and losses each year, but have the ability to optimally time these realizations. With the ability to realize losses and defer gains, holding equity (or other risky assets) in the taxable account can further increase tax efficiency. Not only can investors exploit the tax-timing option by realizing losses and deferring gains, but because of the reset (or step-up) provision at death the embedded capital gain tax liability can be completely avoided through deferral. Thus, even in situations where equity (or other risky asset) generates higher ordinary income than a riskless taxable bond, the value of the tax-timing option may still be high enough to overcome the disadvantage of the higher yield. Although the optimal asset location policy is difficult to derive analytically in the presence of tax-timing options, it is intuitive that assets with relatively lower yields and higher volatilities (typically individual stocks, index funds, and exchange-traded funds) will be held in the taxable account. However, since yields tend to increase with risk for some assets (e.g., taxable bonds), it is unclear which asset is most appropriate for the tax-deferred account. Depending upon the tradeoff between yield and volatility, low-risk government bonds or high-yield corporate bonds may be found in the tax-deferred account. Nevertheless, in the absence of liquidity needs, the investor should not hold a mix of taxable bonds and equity in both the taxable and tax-deferred accounts simultaneously.

II. Numerical Analysis of Optimal Asset Allocation and Location

Section I focused on the investor's optimal asset location policy. In this section we investigate numerically how the optimal asset *allocation* decision interacts with the optimal asset *location* decision. Because the interaction between asset allocation and asset location is most pronounced when the investor faces borrowing and short-sale constraints, we focus our numerical analysis on this case. The model is briefly discussed in subsection A. In subsection B we numerically solve for the optimal decision rules as a function of the state variables. We conduct a simulation analysis of the optimal decisions over an investor's lifetime in subsection C. A welfare analysis is presented in subsection D. Finally, in subsection E we investigate the implications of liquidity needs on the optimal asset location policy.

A. The Model

Our model builds upon the specification in Dammon, Spatt and Zhang (2001a) by incorporating a tax-deferred (retirement) savings account together with a taxable savings account into an intertemporal model of optimal consumption and portfolio choice. Since the model itself is not the main contribution or focus of the paper, we restrict our discussion in this section to the important features of the model and refer the interested reader to the Appendix for the details. The model assumes that the investor makes decisions annually starting at age 20 and lives for at most another 80 years (until age 100). The investor's annual mortality rates are calibrated to match those for the U.S. population. This allows us to directly consider the impact of the investor's age (and increasing mortality) upon the optimal location and allocation decisions.

Investors in the economy derive utility from consuming a single consumption good. We assume that investors receive annual endowment income prior to retirement at age 65. Although investors do not make an endogenous labor-leisure choice in our model, we interpret the endowment income as nonfinancial (or labor) income. Throughout the analysis we assume that pre-tax nonfinancial income is a constant fraction, l, of the investor's contemporaneous total wealth (taxable plus tax-deferred wealth) prior to retirement. This assumption is needed in our numerical analysis to keep the problem homogeneous in wealth and to limit the number of state variables. Because investors are assumed to receive nonfinanical income throughout their working years, young investors with significant future nonfinancial income will adjust the risk of their portfolios by holding slightly more equity (as a proportion of total financial wealth) than without nonfinancial income. Finally, the existence of nonfinancial income makes it less likely that the investor will encounter liquidity problems in financing consumption.

Investors can trade two assets in the financial markets: a riskless taxable one-period bond (equivalent to a taxable money market account) and a risky stock index.[14] No transaction costs are incurred for trading these assets. The pre-tax nominal return on the taxable bond is denoted r and is assumed to be constant over time. The pre-tax nominal return on the risky stock index is $\tilde{r}_s = (1 + d)(1 + \tilde{g}) - 1$, where d is the constant dividend yield and $\tilde{g}$ is the random pre-tax capital gain return. To derive numerical solutions, we assume that $\tilde{g}$ follows a binomial process with a constant mean and variance.

Investors can hold financial assets in two different types of accounts: a taxable account and a tax-deferred retirement account. We assume that investors are not allowed to borrow or sell short in either account. Nominal dividend and interest payments generated from the financial assets held in the taxable account are taxed at the ordinary tax rate of τ_d. *Realized* capital gains (and losses) on stock held in the taxable account are taxed (rebated) at a constant rate of τ_g. All *unrealized* capital gains and losses remain untaxed. To calculate the nominal capital gain, we assume that the tax basis is equal to the weighted average purchase price of all shares held by the investor at the time of sale. This modeling approach was first introduced by Dammon, Spatt and Zhang (2001a) and facilitates our numerical analysis by limiting the number of state variables. The assumption that there is a single risky asset and the use of the average basis rule cause the value of the tax-timing option on equity to be understated and induces the investor to hold less equity than would be the case with multiple risky assets and separate tax bases for each asset purchase.[15]

The treatment of the investor's retirement account is broadly consistent with practice. Prior to retirement, the investor is assumed to contribute a constant fraction k of his pre-tax nonfinancial income to a retirement account each year. The investor allocates his tax-deferred wealth to the taxable bond and the risky stock index and is allowed to rebalance his portfolio holdings in the retirement account without paying capital gains taxes or transaction costs. Nominal dividends, interest, and capital gains generated from the financial assets held in the retirement account are not subject to immediate taxation, but are tax deferred. After retirement, the investor is required to withdraw the fraction h_t of the remaining tax-deferred wealth at age t, where h_t is the inverse of the investor's remaining life expectancy at age t.[16] We assume that the investor contributes the maximum to the retirement account during his working years and withdraws the minimum from the retirement account during his retirement years. Although investors in practice are allowed to withdraw funds from their retirement accounts prior to age $59\frac{1}{2}$ with a 10% penalty, we assume that the investor is not allowed to withdraw funds from the tax-deferred account prior to retirement.[17] The mandatory withdrawals from the retirement account are fully taxed as ordinary income at the rate τ_d. In essence, the investor owns the fraction $(1 - \tau_d)$ of his retirement account balance and has the opportunity to earn pre-tax returns on these funds until they are withdrawn.

The investor's problem is to maximize the discounted expected utility of lifetime consumption, given his initial endowment of assets and wealth, subject to an intertemporal budget constraint. Since the investor has a positive probability of death at each date, the treatment of terminal wealth is important. We assume that at the time of death the asset holdings in the taxable account are liquidated without incurring a capital gains tax. This is consistent with the *reset (or step-up) provision* of the current U.S. tax code, which requires the tax bases of all inherited assets to be costlessly reset to the current market prices at the time of the investor's death. We also assume that the assets held in the investor's retirement account are liquidated at the time of death and the proceeds taxed as ordinary income. At the time of death, the investor's total wealth is liquidated and distributed as a bequest to his beneficiary. For simplicity, we assume that the investor derives utility from his bequest equal to the utility his beneficiary would derive if the bequest was used to purchase an annuity contract that provided a constant amount of *real* consumption for H periods. Higher values for H indicate a stronger bequest motive for the investor.

The liquidation value of the investor's asset holdings (i.e., the after-tax value of the retirement account plus the pre-tax value of the taxable account) serves as our measure of total wealth at each date. To eliminate total wealth as a state variable, we assume that the investor has constant relative risk averse preferences. After normalizing by total wealth, the investor's intertemporal consumption and portfolio problem involves the following control (choice) variables: the consumption-wealth ratio, c_t, the fraction of taxable wealth allocated to equity, f_t, the fraction of taxable wealth allocated to riskless taxable bonds, b_t, and the fraction of tax-deferred wealth allocated to equity, θ_t. Given f_t and b_t, the fraction of the investor's taxable portfolio allocated to equity is $f_t/(f_t+b_t)$.[18] The relevant state variables for the normalized optimization problem are: the incoming proportion of equity in the taxable account, s_t, the basis-price ratio on the incoming equity holdings, p^*_{t-1}, the fraction of the investor's incoming total wealth that is held in the retirement account, y_t, and the investor's age, t. Because investors are allowed to rebalance their retirement account portfolios without incurring any transaction costs or taxes, the incoming asset holdings in this account are not relevant state variables for the investor's decision problem.

B. Numerical Solutions for the Optimal Policies

The *base-case* parameter values for our numerical analysis are summarized in Table I and discussed below. We assume that the nominal pre-tax interest rate on the riskless taxable bond is $r = 6\%$ per year, the nominal dividend yield on the stock index is $d = 2\%$ per year, and the annual inflation rate is $i = 3.5\%$. Inflation is relevant in our model because taxes are levied on nominal quantities. The nominal annual capital gain return on the stock index is assumed to follow a binomial process with a constant mean and standard deviation of $\bar{g} = 9\%$ and $\sigma = 20\%$, respectively. We assume that the tax rate on dividends and interest is $\tau_d = 36\%$ and the tax rate on *realized* capital gains and losses is $\tau_g = 20\%$. Because the pre-tax expected return on the stock index is given by $\bar{r}_s = (1 + \bar{g})(1 + d) - 1$, the annual pre-tax equity risk premium (above the riskless interest rate) is 5.18 percent. While this equity risk premium is relatively low compared to the historical average equity risk premium of about 8 percent, Fama and French (2002) and others have argued that the expected future equity risk premium should be substantially lower than the historical average. For reasonable levels of risk aversion, the lower equity risk premium also ensures that the investor's optimal portfolio will consist of less than 100 percent equity.

The investor is assumed to have power utility with an annual subjective discount factor of $\beta = 0.96$ and a risk aversion parameter of $\gamma = 3.0$. We set $H = 20$ in the bequest function, indicating that the investor values the bequest as though it provided a 20-year annuity of constant real consumption for his beneficiary.[19] We assume that pre-tax nonfinancial income is a constant $l = 15\%$ of the investor's total beginning-of-period wealth prior to age 65 and $l = 0\%$ thereafter. Before retirement at age 65, the investor is assumed to invest $k = 20\%$ of pre-tax nonfinancial income in the tax-deferred retirement account each year. This contribution rate is the maximum allowed in defined-contribution plans under current U.S. tax law for self-employed individuals. Although withdrawals from tax-deferred retirement accounts can be deferred until age $70\frac{1}{2}$ under current IRS rules, we assume that the investor is forced to begin withdrawing funds from the retirement account at age 65 in accordance with the withdrawal schedule h_t.

Figure 1 shows the optimal equity proportions for the taxable account (top panel), the tax-deferred retirement account (middle panel), and the overall portfolio (bottom panel). These optimal equity proportions are shown as a function of the investor's age and the fraction of beginning-of-period total wealth held in the retirement account. A two-dimensional representation of these optimal equity proportions are shown in Figure 2 for age 35 (top panel) and age 75 (bottom panel). Figures 1 and 2 are constructed using the base-case parameter values and assuming that the basis-price ratio is $p^* = 1.0$. With a basis-price ratio of $p^* = 1.0$ the investor has neither a gain nor a loss on his existing equity holdings and can rebalance his portfolio in the taxable account without incurring a tax cost. We refer to these optimal equity proportions as the *zero-gain optimum* equity holdings.

As the fraction of wealth in the retirement account increases, the optimal equity proportion in the taxable account increases. This reflects the preference for holding taxable bonds in the retirement account, thus making it necessary for the investor to increase the proportion of equity in the taxable account in order to maintain an optimal overall portfolio mix. Note, however, that because of the prohibition on borrowing the proportion of equity in the taxable account is bounded above by 100 percent. The top panels of Figures 1 and 2 show that during his working years, the investor allocates 100 percent of the taxable account to equity whenever the tax-deferred wealth exceeds about 30 percent of total wealth. In contrast, with *unrestricted borrowing* (not shown), the optimal holding of equity in the taxable account would continue to increase beyond 100 percent as the investor borrows to invest in equity in the taxable account. For example, with 50 percent of total wealth held in the retirement account, investors in their working years hold approximately 175% of their taxable wealth in equity when they are allowed to borrow.

Figures 1 (middle panel) and 2 show that the optimal equity proportion in the tax-deferred account can be non-zero when investors are prohibited from borrowing. Note, however, that the investor does not hold equity in the tax-deferred account until the tax-deferred wealth exceeds approximately 40 percent of total wealth. The reason the investor does not add equity to the retirement account as soon as he is constrained in the taxable account is because equity is much less valuable when held in the retirement account. For this reason, the investor holds equity in the retirement account only when the level of tax-deferred wealth is sufficiently high and refrains from holding a mix of stocks and bonds in both the taxable and tax-deferred accounts simultaneously. In fact, over a range of tax-deferred wealth the investor holds an all-equity portfolio in the taxable account and an all-bond portfolio in the retirement account. The optimal proportion of tax-deferred wealth allocated to equity is lower for elderly investors because the mandatory distributions from the retirement account allow for more equity to be held in the taxable account. These findings are in contrast to the *unrestricted borrowing* case in which the investor always allocates his entire tax-deferred wealth to taxable bonds (as shown in Section I).

Figures 1 (bottom panel) and 2 show the optimal proportion of *total wealth* allocated to equity for the no-borrowing case. The figures illustrate that optimal asset allocation for an investor's overall portfolio depends upon the split of wealth between the taxable and tax-deferred accounts. In fact, the optimal overall equity proportion (weakly) declines as the fraction of total wealth held in the retirement account increases. The optimal overall equity proportion is relatively flat in the level of tax-deferred wealth as long as the investor holds less than 100 percent equity in the taxable account. For young investors, the optimal overall holding of equity is approximately 68% at low levels of tax-deferred wealth. Once the investor is constrained in his taxable account, however, the optimal overall equity proportion begins to decline. This reflects the investor's reluctance to substitute equity for taxable bonds in the retirement account. At sufficiently high levels of tax-deferred wealth, the investor begins to add equity to the retirement account to avoid becoming too underweighted in equity. This causes the optimal overall equity proportion to level off once again. For young investors, the optimal overall equity proportion is slightly less than 50% at high levels of tax-deferred wealth. In contrast, when the investor has *unrestricted borrowing* opportunities (not shown), the optimal overall equity proportion is relatively constant across all levels of tax-deferred wealth.[20] For young investors, the optimal overall equity proportion is approximately 70% with *unrestricted borrowing.*

The effect of age on the overall equity proportion is also shown in the bottom panel of Figure 1. While the overall equity proportion remains relatively constant at young ages, there is a slight decline between the ages of 60 and 65. This drop in the optimal overall equity proportion at these ages reflects the anticipated loss of the relatively low-risk nonfinancial income after retirement. This effect is less pronounced at high levels of retirement wealth, where the exposure to equity has already been reduced at young ages because of the restrictions on borrowing. At late ages, the optimal overall equity proportion *increases* slightly with age. This reflects the higher value of equity for elderly investors who, because of their higher mortality rates, benefit the most from the forgiveness of capital gains taxes at death. This behavior is similar to that derived by Dammon, Spatt, and Zhang (2001a) in a model without a retirement account.

Figures 1 and 2 were constructed assuming that the basis-price ratio for equity held in the taxable account was $p^* = 1.0$. With an embedded capital loss (i.e., $p^* > 1.0$), the investor optimally sells his entire equity holding in the taxable account to benefit from the tax rebate and immediately rebalances to the *zero-gain optimum* equity holdings shown in Figure 1. With an embedded capital gain (i.e., $p^* < 1.0$), the investor's optimal equity holdings will differ from those shown in Figure 1. If the investor's taxable account is initially *underweighted* in equity, his equity holdings following optimal rebalancing will be slightly lower than the *zero-gain optimum* equity holdings. This is because the averaging rule used to compute the tax basis reduces the value of holding additional equity when existing shares have an embedded capital gain. The smaller the embedded capital gain, the closer will be the optimal equity holdings to the *zero-gain optimum.* If the investor's taxable account is initially *overweighted* in equity, there exists a tradeoff between the diversification benefits and tax costs of rebalancing. The willingness of the investor to realize embedded capital gains to rebalance his portfolio will depend upon a number of factors. Smaller embedded capital gains, larger deviations from the *zero-gain optimum* equity holdings, lower mortality risk, and lower levels of future nonfinancial income will increase the amount of rebalancing that is optimal. However, because rebalancing is costly in this case, the optimal equity holdings are higher than the *zero-gain optimum* equity holdings.

Our results illustrate a preference for holding equity in the taxable account and taxable bonds in the tax-deferred account to the extent possible. We also have shown how the investor's overall asset allocation depends upon age, the basis-price ratio, and the split of wealth between the taxable and tax-deferred accounts.[21] In the next section, we investigate the time-series profile of the investor's optimal consumption and portfolio allocation decisions using simulation analysis.

C. Simulation Analysis

Given the investor's optimal consumption and investment policies defined on the state space, we can obtain time-series profiles of optimal consumption and portfolio allocations by simulating the capital gain return on the risky stock index. Using our base-case parameter values, the simulation begins for an investor at age 20 with an initial basis-price ratio of $p^* = 1$. The investor is prohibited from borrowing and is assumed to have no tax-deferred wealth at age 20. Table II shows the age profiles for the optimal consumption, portfolio holdings, and level of retirement wealth. The values reported in the table are *averages* at each age taken across 5,000 simulation trials.

Table II shows that the investor's optimal consumption-wealth ratio slowly falls as the investor ages during his working years and then slowly rises as he ages during his retirement years. The decline in the investor's optimal consumption-wealth ratio during his working years reflects the anticipated loss of nonfinancial income after retirement. The increase in the investor's consumption-wealth ratio during his retirement years reflects the bequest motive. With $H = 20$, the bequest provides the investor the same utility as his beneficiary would receive from consuming a 20-year annuity stream. Hence, as the investor ages (and mortality risk increases) he increases his optimal consumption-wealth ratio in an attempt to equate the expected marginal utility of his own consumption with that of his beneficiary. With a stronger bequest motive ($H = \infty$), the investor's optimal consumption-wealth ratio declines with age during retirement.

The investor contributes $k = 20\%$ of pre-tax nonfinancial income to the retirement account each year. Given the high levels of consumption, these retirement contributions represent the bulk of the investor's overall savings during his working years. As a result, the fraction of total wealth held in the retirement account increases rapidly at young ages. The fraction of total wealth held in the retirement account reaches its maximum of 45% at age 55, well before the investor reaches retirement age.[22] The decline in the fraction of total wealth held in the retirement account in the years prior to retirement reflects two things: (1) the lower fraction of total after-tax savings allocated to the retirement account at these ages and (2) the relatively lower average return earned on the assets (primarily taxable bonds) held in the retirement account. Because of the mandatory distributions from the tax-deferred account during his retirement years, the investor's after-tax retirement account declines rapidly after age 65. By the time the investor reaches age 80 the after-tax wealth in the retirement account is only slightly more than 12 percent of total wealth on average.

The last six columns of Table II provide information about the investor's optimal lifetime portfolio choices. Because of the relatively high equity risk premium the investor's overall demand for equity is extremely high and frequently exceeds the available resources in the taxable account. As Table II indicates, a large proportion of simulation trials result in an all-equity portfolio in the taxable account at some point during the investor's lifetime. Most (but not all) of these cases result in some positive holdings of equity in the retirement account. Since equity is less valuable when held in the retirement account, the overall holding of equity declines in these cases. While the investor has limited opportunities at early ages to realize losses on equity held in the taxable account, the profile of the average basis-price ratio in Table II indicates that he quickly becomes locked in to a capital gain. During his retirement years, the average proportion of tax-deferred wealth allocated to equity declines rapidly as the investor begins to liquidate the equity holdings in the retirement account to fund his mandatory distributions. As a proportion of total wealth, the investor's equity holdings continue to increase during his retirement years reflecting the high average return on equity and the investor's reluctance to sell equity with embedded capital gains.[23]

The results of the simulation analysis illustrate the time-series properties of the investor's optimal consumption, portfolio allocation, and asset location decisions. In the next section, we investigate the welfare benefits from following the optimal asset allocation and location policies and illustrate the welfare benefits of tax-deferred investing.

D. Alternative Investment Policies and Welfare Analysis

Because many individuals in practice hold a mix of bonds and stocks in both their taxable and tax-deferred accounts, and in some cases actually tilt their retirement accounts toward equity, we want to examine the utility costs of following these suboptimal policies. We examine two alternative investment strategies. In the first, investors hold the same mix of bonds and stocks in both their taxable and tax-deferred accounts. In the second, investors first allocate their holdings of equity to the retirement account before holding equity in their taxable accounts. Given our earlier analysis, this latter policy is the worst possible policy in terms of asset location. For both of the alternative investment policies we allow the investor to choose the optimal mix of stocks and bonds as a function of the state variables. We then conduct a welfare analysis by computing the amount of additional wealth (allocated entirely to the taxable account) that is needed to equate the investor's total expected utility under the suboptimal location policy to that under the optimal location policy. This allows us to quantify the cost of ignoring the optimal location of securities across the taxable and tax-deferred accounts.

The welfare analysis is conducted using our base-case parameter values, with one important exception. To avoid the difficulties associated with accurately measuring the utility costs when nonfinancial income is proportional to wealth, we assume that nonfinancial income is zero (i.e., $l = 0$).[24] With nonfinancial income equal to zero the investor is also prohibited from making additional future contributions to the retirement account. These changes allow us to focus on the welfare costs associated with the suboptimal location policies when applied to the pre-existing wealth levels. The optimal asset location decision is unaffected by the elimination of nonfinancial income and retirement account contributions. Moreover, the optimal equity holdings in the absence of nonfinancial income are similar to those shown in Figure 1 (without the decline in equity holdings as the investor approaches retirement).

The top panel of Figure 3 shows the utility costs for an investor who is forced to hold the same portfolio mix in both the taxable and tax-deferred accounts. The middle panel of Figure 3 shows the utility costs for an investor who is not allowed to hold equity in the taxable account unless he first allocates 100 percent of his tax-deferred wealth to equity. The utility costs are shown as a function of the investor's age and level of tax-deferred wealth. The figures are drawn for a basis-price ratio of $p^* = 1.0$. Since there is no embedded capital gain or loss on the investor's portfolio, the initial equity proportion in the taxable account has no effect on the optimal decision rules or utility costs.

The utility costs depicted in these two figures exhibit a strong age effect. Other things being equal, younger investors bear a higher utility cost by deviating from the optimal policy because they have longer horizons over which the suboptimal policy is in effect. The utility costs are also hump-shaped in the level of tax-deferred wealth at young and middle ages. Intuitively, the utility costs of being constrained to follow the suboptimal policy is highest when the investor's wealth is split relatively evenly across the taxable and tax-deferred accounts and smallest when concentrated in one or the other of these two accounts.[25]

In the top panel of Figure 3, the utility costs are generally less than 5 percent across all ages, with the exception that young investors with moderate levels of retirement account wealth have slightly higher utility costs. The utility costs depicted in the middle panel of Figure 3 are higher than those depicted in the top panel. This is to be expected since locating equity in the tax-deferred account before locating it in the taxable account is the worst possible policy for asset location. For young investors with moderate levels of retirement account wealth the utility costs are nearly 15 percent. These utility costs would be even higher if the investor contributed additional funds to the retirement account over time. The overall impression given by these two figures is that the benefits from optimally locating equity in the taxable account and bonds in the tax-deferred account can be large, especially for young and middle-aged investors.

Our model also can be used to measure the welfare benefits of tax-deferred investing. Using the base-case parameters (without nonfinancial income), and an assumed basis-price ratio of $p^* = 1.0$, the bottom panel of Figure 3 shows the shadow prices for tax-deferred wealth. The shadow price measures the amount of taxable wealth the investor is willing to give up in order to receive one additional dollar of tax-deferred wealth. The shadow prices depicted in Figure 3 are all greater than 1.0, indicating that it is beneficial for the investor to have a higher fraction of total wealth in the retirement account.[26] The shadow prices are highest for young investors, who have longer horizons over which to benefit from tax-deferred investing, and for investors with lower levels of tax-deferred wealth, who can efficiently allocate their tax-deferred wealth to taxable bonds.[27] Since investors are forced to liquidate their retirement accounts during their retirement years, the shadow prices for additional tax-deferred wealth decline rapidly after age 65.

E. Effects of Liquidity on Optimal Asset Location

The optimal location policy involves holding equity in the taxable account and taxable bonds in the retirement account to the extent possible. This optimal location policy does not alter the risk exposure of the investor's overall portfolio, but it does alter the separate risk exposures of the taxable and tax-deferred accounts. In principle, an investor with most of his wealth in the tax-deferred account may wish to reduce the risk of his taxable portfolio by holding some (taxable or tax-deferred) bonds to ensure that he can finance a minimum level of consumption during his working years. Huang (2000) was the first to suggest that liquidity needs may alter the optimal asset location policy. However, despite the assumption that the investor cannot borrow or withdraw funds from the retirement account prior to retirement, holding bonds in the taxable account for liquidity reasons does not arise in our model. This is because the investor's optimal consumption and savings plans can be financed solely from the financial and nonfinancial income received each year.

To generate hedging demand for bonds in the taxable account, it is necessary to introduce an exogenous liquidity shock into our model. This can be done in a number of different ways. We choose to analyze the effects of liquidity using an extension of our model in which investors face an exogenous shock to consumption. The analysis is conducted using our base-case parameter values. However, unlike our earlier analysis, we allow investors to endogenously determine their optimal contributions to the retirement account during their working years (subject to a contribution limit of $k = 20\%$) and their optimal withdrawals from the retirement account during their retirement years (subject to the minimum withdrawal schedule). Investors are also allowed to withdraw funds from their retirement accounts prior to retirement, but must pay ordinary income taxes and a 10 percent penalty on the early withdrawals. Because the taxable account is the preferred location for equity even without the benefits of optimal tax timing, we simplify our analysis by assuming that all capital gains and losses are realized each year and taxed at the rate of $\tau_g = 20\%$. This assumption allows us to drop the incoming equity proportion in the taxable account and the basis-price ratio as state variables.

We first analyze a situation in which the investor faces a *known* "consumption gulp" of 50 percent of total wealth at age 30. This is similar to the approach taken by Huang (2000) in analyzing the effect of liquidity on optimal asset location.[28] The purchase of a home or college tuition are examples of the type of consumption expenditures we have in mind.[29] Figure 4 illustrates the optimal holding of equity in the taxable account (top panel) and in the tax-deferred account (middle panel) as a function of the investor's age and the level of tax-deferred wealth. Except for a few years prior to age 30, the investor does not hold bonds in the taxable account unless the tax-deferred account is entirely invested in bonds. At age 30 the investor liquidates a portion of the tax-deferred account only if the wealth in the taxable account is insufficient to finance the exogenous "consumption gulp". This occurs for tax-deferred wealth greater than 50 percent of total wealth. As a consequence, for high levels of tax-deferred wealth the investor at age 30 completely liquidates the wealth in the taxable account and holds all financial assets in the tax-deferred account. For tax-deferred wealth less than 50 percent of total wealth, the investor at age 30 finances the entire "consumption gulp" by liquidating a portion of the taxable portfolio. The investor then optimally allocates the remaining wealth in the taxable account between stocks and bonds, with some bonds held in the taxable account only if the tax-deferred account is entirely invested in bonds. Thus, the asset location decisions made at age 30 and beyond are completely consistent with the desire to hold equity in the taxable account and taxable bonds in the retirement account to the extent possible.

Between ages 25 and 30, the investor holds a mix of stocks and bonds in both the taxable and tax-deferred accounts for tax-deferred wealth in the range of 50 to 65 percent of total wealth. This reflects the investor's desire to reduce the risk of his taxable portfolio when the wealth in the taxable account is slightly less than the wealth needed to finance the upcoming "consumption gulp".[30] The demand for bonds in the taxable account in this case is driven entirely by liquidity considerations. Note, however, that the optimal holding of bonds in the taxable account is small relative to the holding of bonds in the tax-deferred account. Investors with tax-deferred wealth less than 50 percent, or greater than 65 percent, of total wealth do not hedge their liquidity risk and, therefore, hold bonds in their taxable account only if their tax-deferred wealth is entirely invested in bonds. The bottom panel of Figure 4 shows that investors with substantial tax-deferred wealth also reduce (or eliminate) the contributions to their retirement accounts prior to age 30 to increase the wealth in their taxable accounts and avoid paying the penalty to withdraw the funds at age 30.[31] The higher the level of tax-deferred wealth the earlier the investor reduces retirement contributions.

The above analysis assumes that the timing of the "consumption gulp" is known with certainty. Another approach to analyzing the effects of liquidity is to assume that the investor faces a constant per-period probability of receiving a shock to consumption that requires him to consume 50 percent of his total wealth.[32] The consumption shocks are assumed to occur with a 10 percent probability each period, are independent over time, and uncorrelated with asset returns. Figure 5 illustrates the investor's optimal equity holdings in the taxable account (top panel) and the tax-deferred account (middle panel), as well as the optimal contributions to the retirement account during his working years (bottom panel). Unlike the previous case with a known "consumption gulp" at age 30, the optimal asset location policy exhibited in Figure 5 shows no sign of hedging liquidity risk in the taxable account. For all ages and levels of tax-deferred wealth, the investor optimally holds bonds in the taxable account only if the entire tax-deferred wealth is invested in bonds. This is the same optimal asset location policy we presented in Section II.B. in the absence of liquidity shocks. Intuitively, the value of holding bonds in the tax-deferred account is so high that investors with less than 50 percent of their total wealth in the taxable account are willing to bear the risk of incurring a 10 percent penalty to withdraw the incremental funds from their retirement accounts to meet any unforseen consumption needs. To the extent that the investor hedges liquidity risks, he does so by reducing the contributions to the retirement account as shown in the bottom panel of Figure 5. The level of tax-deferred wealth at which the investor stops contributing to the retirement account is higher at later ages.

III. Summary and Conclusion

In this paper we analyzed the optimal dynamic asset allocation and location decisions for an investor with both taxable and tax-deferred investment opportunities. Our results indicate that investors have a strong preference for locating taxable bonds in the tax-deferred retirement account and equity in the taxable account. This preference reflects the higher tax burden of taxable bonds relative to equity. When investors can borrow without restrictions in their taxable accounts, it is optimal for them to invest their entire retirement account wealth in taxable bonds and either borrow or lend in the taxable account to achieve an optimal overall portfolio mix. Moreover, the opportunity to invest in tax-exempt bonds does not alter the optimal asset location policy provided equity can be held in a relatively tax-efficient form (e.g., individual stocks, index mutual funds, or exchange-traded funds).

When investors are prohibited from borrowing, the optimal asset location policy depends upon whether the investors face liquidity shocks to consumption. In the absence of liquidity shocks, investors may optimally hold a mix of stocks and bonds in the tax-deferred account, but only if they invest entirely in equity in the taxable account. In the presence of liquidity shocks, investors with insufficient resources in the taxable account to meet the potential need for liquidity tend to reduce their future contributions to the tax-deferred account. Whether investors also adjust the location of their bond holdings to reduce the risk of the taxable portfolio depends upon the magnitude and likelihood of the liquidity shocks. We find that the probability and magnitude of the liquidity shocks need to be rather large in order to induce investors to deviate from the tax-minimizing location policy.

Our analysis points to an important "asset location puzzle" in which the asset location decisions observed in practice deviate substantially from the predictions of our model. Poterba and Samwick (1999), Bergstresser and Poterba (2002), and Ameriks and Zeldes (2000) document that investors in practice commonly hold a mix of bonds and stocks in both their taxable and tax-deferred accounts, and in many instances tilt their tax-deferred investments toward equity. While liquidity considerations may help explain some of the observed behavior, we do not believe that liquidity concerns alone can fully account for the magnitude of the deviations that are observed in practice, especially for investors who can borrow and for elderly investors who have unrestricted access to their retirement savings. Our welfare analysis suggests that many investors would benefit considerably from shifting the location of their asset holdings to more closely conform to the tax-efficient policies derived in this paper.

REFERENCES

Ameriks, J. and S. Zeldes, 2000, How Do Household Portfolio Shares Vary with Age?, unpublished manuscript, Columbia University.

Amromin, G., 2001, Portfolio Allocation Choices in Taxable and Tax-Deferred Accounts: An Empirical Analysis of Tax Efficiency, unpublished manuscript, University of Chicago.

Bergstresser, D. and J. Poterba, 2002, Asset Allocation and Asset Location Decisions: Evidence from the Survey of Consumer Finances, unpublished manuscript, Massachusetts Institute of Technology.

Black, F., 1980, The Tax Consequences of Long-Run Pension Policy, *Financial Analysts Journal*, July–August, 21-28.

Carhart, M., 1997, On Persistence in Mutual Fund Performance, *Journal of Finance* 52, 57-82.

Constantinides, G., 1983, Capital Market Equilibrium With Personal Taxes, *Econometrica* 51, 611-636.

Constantinides, G., 1984, Optimal Stock Trading with Personal Taxes: Implications for Prices and the Abnormal January Returns, *Journal of Financial Economics* 13, 65-89.

Dammon, R., K. Dunn and C. Spatt, 1989, A Reexamination of the Value of Tax Options, *Review of Financial Studies* 2, 341-372.

Dammon, R. and C. Spatt, 1996, The Optimal Trading and Pricing of Securities with Asymmetric Capital Gains Taxes and Transaction Costs, *Review of Financial Studies* 9, 921-952.

Dammon, R., C. Spatt and H. Zhang, 1999, Optimal Asset Allocation with Taxable and Tax-Deferred Investing, unpublished manuscript, Carnegie Mellon University.

Dammon, R., C. Spatt and H. Zhang, 2001a, Optimal Consumption and Investment with Capital Gains Taxes, *Review of Financial Studies* 14, 583-616.

Dammon, R., C. Spatt and H. Zhang, 2001b, Diversification and Capital Gains Taxes with Multiple Risky Assets, unpublished manuscript, Carnegie Mellon University and University of North Carolina.

Fama, E. and K. French, 2002, The Equity Premium, *Journal of Finance* 57, 637-659.

Green, R., 1993, A Simple Model of the Taxable and Tax-Exempt Yield Curves, *Review of Financial Studies* 6, 233-264.

Gruber, M., 1996, Another Puzzle: The Growth in Actively Managed Mutual Funds, *Journal of Finance* 51, 783-810.

Huang, J., 2000, Taxable or Tax-Deferred Account? Portfolio Decision with Multiple Investment Goals, unpublished manuscript, Massachusetts Institute of Technology.

Poterba, J. and A. Samwick, 1999, Taxation and Household Portfolio Composition: Evidence from Tax Reforms in the 1980s and 1990s, *Journal of Public Economics*, forthcoming.

Poterba, J., J. Shoven, and C. Sialm, 2000, Asset Location for Retirement Savers, National Bureau of Economic Research Working Paper No. 7991, November.

Reichenstein, W., Summer 2001, Asset Allocation and Asset Location Decisions Revisted, *Journal of Wealth Management*, 16-26.

Shoven, J., 1999, The Location and Allocation of Assets in Pension and Conventional Savings Accounts, National Bureau of Economic Research Working Paper No. 7007, March.

Shoven, J. and C. Sialm, 2002, Asset Location for Tax-Deferred and Conventional Savings Accounts, *Journal of Public Economics*, forthcoming.

Tepper, I., 1981, Taxation and Corporate Pension Policy, *Journal of Finance* 36, 1-13.

Williams, J., 1985, Trading and Valuing Depreciable Assets, *Journal of Financial Economics* 14, 283-308.

Appendix: Derivation of the Model

Our model builds upon the model developed in Dammon, Spatt, and Zhang (2001a) by incorporating tax-deferred investing with taxable investing. The investor's intertemporal consumption-investment problem can be stated as follows:

$$\max_{C_t, n_t, B_t, \theta_t} E\left\{\sum_{t=0}^{T} \beta^t \left[F(t)u\left(\frac{C_t}{(1+i)^t}\right) + [F(t-1) - F(t)] \sum_{k=t+1}^{t+H} \beta^{k-t} u\left(\frac{A_H \overline{W}_t}{(1+i)^t}\right)\right]\right\} \tag{A1}$$

s.t.

$$\overline{W}_t = W_t + Y_t(1-\tau_d), \; t = 0, \cdots, T, \tag{A2}$$

$$W_t = L_t(1-\tau_d) + n_{t-1}[1 + (1-\tau_d)d]P_t + B_{t-1}[1 + (1-\tau_d)r], \quad t = 0, \cdots, T, \tag{A3}$$

$$Y_t = W^r_{t-1}[\theta_{t-1}(1+g_t)(1+d) + (1-\theta_{t-1})(1+r)], \quad t = 1, \cdots, T, \tag{A4}$$

$$C_t = \overline{W}_t - \tau_g G_t - n_t P_t - B_t - W^r_t(1-\tau_d), \quad t = 0, \cdots, T-1, \tag{A5}$$

$$W^r_t = Y_t + kL_t, \quad t = 0, \cdots, J-1, \tag{A6}$$

$$W^r_t = Y_t(1-h_t), \quad t = J, \cdots, T-1, \tag{A7}$$

$$C_t \geq 0, \; n_t \geq 0, \quad B_t \geq 0, \; 0 \leq \theta_t \leq 1, \quad t = 0, \cdots, T-1, \tag{A8}$$

$$n_T = 0, \quad B_T = 0, \quad W^r_T = 0, \tag{A9}$$

where t denotes time (or age), F_t is the probability of living through period t, $u(\cdot)$ denotes the investor's utility function, β is the subjective discount factor for utility, C_t is nominal consumption, n_t is the number of shares of stock held in the taxable account, B_t is the amount invested in bonds in the taxable account, θ_t is the fraction of tax-deferred wealth allocated to equity, $\overline{W}_t$ is total wealth, W_t is the wealth in the taxable account after payment of ordinary income taxes but prior to the payment of capital gains taxes, Y_t is the pre-tax wealth in the tax-deferred account before contributions (withdrawals) in period t, W^r_t is the wealth in the tax-deferred account after contributions (withdrawals) in period t, L_t is the pre-tax nonfinancial income, kL_t is the contribution to the retirement account in period t, h_tY_t is the withdrawal from the retirement account in period t, P_t is the per share stock price, d is the nominal dividend yield, r is the nominal riskless interest rate, g_t is the nominal pre-tax capital gain return, i is the constant rate of inflation, G_t is the total realized capital gain in period t, τ_d is the ordinary tax rate, and τ_g is the capital gains tax rate. The initial portfolio holdings, n_{-1} and B_{-1}, initial nonfinancial income, L_0, and initial tax-deferred wealth, Y_0, are assumed to be non-negative. The value of $F(t)$ in equation (A1) is given by:

$$F(t) = exp(-\sum_{j=0}^{t} \lambda_j), \tag{A10}$$

where $\lambda_j > 0$ is the single-period hazard rate for period j with $\lambda_T = \infty$.

The expression inside the square brackets in Eq. (A1) is the investor's probability-weighted utility at date t. The first term measures the utility of consumption in period t weighted by the probability of living through period t, while the second term is the utility of the investor's bequest weighted by the probability of dying in period t. As written, the bequest provides the investor with a constant level of real consumption for a period of H years, where $A_H = \frac{r^*(1+r^*)^H}{(1+r^*)^H - 1}$ is the H–period annuity factor and $r^* = [(1-\tau_d)r - i]/(1+i)$ is the real after-tax interest rate.

Equation (A2) defines the investor's total wealth as the sum of his taxable wealth and the fraction $(1-\tau_d)$ of his retirement account balance. Equations (A3) and (A4) define the wealth in the taxable and tax-deferred accounts, respectively. Equation (A5) is the investor's intertemporal budget constraint. The realized capital gain (loss) in equation (A5) is given by:

$$G_t = \{I(P^*_{t-1} > P_t)n_{t-1} + [1 - I(P^*_{t-1} > P_t)]\max(n_{t-1} - n_t, 0)\}(P_t - P^*_{t-1}), \tag{A11}$$

where P^*_{t-1} is the investor's tax basis on shares held at the beginning of period t and $I(P^*_{t-1} > P_t)$ is an indicator function that takes the value of one if there is an embedded capital loss (i.e., $P^*_{t-1} > P_t$) and zero otherwise. Following Dammon, Spatt, and Zhang (2001a), we assume that the tax basis is the weighted average purchase price for shares held in the taxable account. The nominal tax basis follows the law of motion:

$$P^*_t = \begin{cases} \frac{n_{t-1}P^*_{t-1} + \max(n_t - n_{t-1}, 0)P_t}{n_{t-1} + \max(n_t - n_{t-1}, 0)}, & \text{if } P^*_{t-1} < P_t \\ P_t, & \text{if } P^*_{t-1} \geq P_t. \end{cases} \tag{A12}$$

This formulation takes into account that, in the absence of transaction costs, the investor optimally sells his entire holding of equity to realize a tax loss when $P^*_{t-1} \geq P_t$ and immediately repurchases equity to rebalance his portfolio. In this case, the tax basis of the newly acquired shares is the current market price, P_t. Also, when the investor has an embedded capital gain on existing shares (i.e., $P^*_{t-1} < P_t$), the tax basis is unchanged unless the investor purchases additional shares in period t (i.e., $n_t > n_{t-1}$).

Equations (A6) and (A7) impose constraints on the contributions to, and withdrawals from, the retirement account. Equation (A8) requires consumption to be non-negative and prohibits short sales and borrowing in the taxable and tax-deferred accounts. If investors are allowed to borrow or sell short in the taxable account, the non-negativity constraints on B_t and n_t are relaxed. Finally, equation (A9) requires the investor to liquidate his holdings at date T.

We assume that the investor's preferences can be expressed as follows:

$$u\left[\frac{C_t}{(1+i)^t}\right] = \frac{\left[\frac{C_t}{(1+i)^t}\right]^{1-\gamma}}{1-\gamma} \tag{A13}$$

where γ is the investor's relative risk aversion coefficient. Note that the summation appearing in the second term of the objective function can be rewritten as follows:

$$\sum_{k=t+1}^{t+H} \beta^{k-t} u\left[\frac{A_H \overline{W}_t}{(1+i)^t}\right] = \frac{\beta(1-\beta^H)\left[\frac{A_H \overline{W}_t}{(1+i)^t}\right]^{1-\gamma}}{(1-\beta)(1-\gamma)}.$$

Letting X_t denote the vector of state variables at date t, we can write the Bellman equation for the above maximization problem as follows:

$$V(X_t) = \max_{C_t, n_t, B_t, \theta_t} \left\{ \frac{e^{-\lambda_t}\left[\frac{C_t}{(1+i)^t}\right]^{1-\gamma}}{1-\gamma} + \frac{(1-e^{-\lambda_t})\beta(1-\beta^H)\left[\frac{A_H \overline{W}_t}{(1+i)^t}\right]^{1-\gamma}}{(1-\beta)(1-\gamma)} + e^{-\lambda_t}\beta E_t\left[V(X_{t+1})\right] \right\} \tag{A14}$$

for $t = 0, \cdots, T-1$, subject to equations (A2)-(A9). The sufficient state variables for the investor's problem at date t is denoted by the following vector:

$$X_t = [P_t, P^*_{t-1}, n_{t-1}, W_t, Y_t, L_t]'. \tag{A15}$$

We simplify the investor's optimization problem by normalizing by the investor's total wealth, $\overline{W}_t$, and assuming that the investor's nonfinancial income is a constant fraction of total wealth, $l = L_t/\overline{W}_t$. Let $s_t = n_{t-1}P_t/W_t$ be the beginning-of-period equity proportion in the taxable account, $f_t = n_tP_t/W_t$ be the fraction of taxable wealth allocated to equity after trading at date t, $b_t = B_t/W_t$ be the fraction of taxable wealth allocated to taxable bonds after trading at date t, $y_t = Y_t(1-\tau_d)/\overline{W}_t$ be the fraction of the investor's beginning-of-period total wealth that is held in the retirement account before trading at date t, $w_t^r = W_t^r/\overline{W}_t$ be the fraction of the investor's beginning-of-period wealth that is held in the retirement account after trading at date t, and $p_{t-1}^* = P_{t-1}^*/P_t$ be the investor's tax basis–price ratio on the initial equity holdings in the taxable account. Then, the gross nominal rate of return on the investor's taxable portfolio from period t to $t+1$, after the tax on dividends and interest, but prior to the payment of capital gain taxes, is:

$$R_{t+1} = \frac{f_t[1+(1-\tau_d)d](1+g_{t+1}) + [1+(1-\tau_d)r]b_t}{f_t+b_t} \tag{A16}$$

and the gross rate of return on the investor's tax-deferred portfolio from period t to $t+1$ is:

$$R_{t+1}^r = \theta_t(1+d)(1+g_{t+1}) + (1-\theta_t)(1+r) \tag{A17}$$

Using this notation, equation (A2) can be written as the following linear dynamic wealth equation

$$\overline{W}_{t+1} = \left[R_{t+1}(f_t+b_t)\frac{1-y_t}{1-l(1-\tau_d)} + R_{t+1}^r\frac{w_t^r(1-\tau_d)}{1-l(1-\tau_d)}\right]\overline{W}_t. \tag{A18}$$

Similarly, the intertemporal budget constraint in equation (A5) can be written as follows:

$$c_t = 1 - \tau_g\delta_t(1-y_t) - (f_t+b_t)(1-y_t) - w_t^r(1-\tau_d) \tag{A19}$$

where $c_t = C_t/\overline{W}_t$ is the consumption–wealth ratio for period t,

$$\delta_t = G_t/W_t = \{I(p_{t-1}^* > 1)s_t + [1 - I(p_{t-1}^* > 1)]\max(s_t - f_t, 0)\}(1 - p_{t-1}^*) \tag{A20}$$

is the fraction of beginning-of-period wealth that is taxable as *realized* capital gains in period t, and

p^*_{t-1} is given by

$$p^*_{t-1} = \begin{cases} \frac{[s_{t-1}p^*_{t-2}+\max(f_{t-1}-s_{t-1},0)]/(1+g_t)}{s_{t-1}+\max(f_{t-1}-s_{t-1},0)}, & \text{if } p^*_{t-2} < 1 \\ \frac{1}{1+g_t}, & \text{if } p^*_{t-2} \geq 1. \end{cases} \tag{A21}$$

The linearity of the dynamic wealth equation and the assumption of constant relative risk averse preferences ensure that our model has the property that the consumption and portfolio decision rules, $\{c_t, f_t, b_t, \theta_t\}$, are independent of total wealth, $\overline{W}_t$. Furthermore, with the above normalization, the relevant state variables for the investor's problem become $x_t = \{s_t, p^*_{t-1}, y_t\}$. Defining $v(x_t) = V(X_t)/[\overline{W}_t/(1+i)^t]^{1-\gamma}$ to be the normalized value function and $\overline{w}_{t+1} = \overline{W}_{t+1}/[\overline{W}_t(1+i)]$ to be one plus the *real* growth rate in wealth from period t to period $t+1$, the investor's problem can be restated as follows:

$$v(x_t) = \max_{c_t, f_t, b_t, \theta_t} \left\{ \frac{e^{-\lambda_t} c_t^{1-\gamma}}{1-\gamma} + \frac{(1-e^{-\lambda_t})\beta(1-\beta^H)A_H^{1-\gamma}}{(1-\beta)(1-\gamma)} + e^{-\lambda_t}\beta E_t\left[v(x_{t+1})\overline{w}_{t+1}^{1-\gamma}\right]\right\}, \quad t = 0, \cdots, T-1, \tag{A22}$$

s.t.

$$\overline{w}_{t+1} = \left(\frac{R_{t+1}}{1+i}\right)(f_t + b_t)\frac{1-y_t}{1-l(1-\tau_d)} + \left(\frac{R^r_{t+1}}{1+i}\right)\frac{w^r_t(1-\tau_d)}{1-l(1-\tau_d)}, \quad t = 0, \cdots, T-1, \tag{A23}$$

$$w^r_t = \frac{y_t}{1-\tau_d} + kl, \quad t = 0, \cdots, J-1, \tag{A24}$$

$$w^r_t = \frac{y_t}{1-\tau_d}(1-h_t), \quad t = J, \cdots, T-1, \tag{A25}$$

$$c_t \geq 0, \quad f_t \geq 0, \quad 0 \leq \theta_t \leq 1 \tag{A26}$$

where c_t is given by Eq. (A19), δ_t is given by Eq. (A20), and p^*_{t-1} is given by Eq. (A21).

The above problem can be solved numerically using backward recursion. To do this, we discretize the lagged endogenous state variables, $x_t = \{s_t, p^*_{t-1}, y_t\}$, into a grid of (101 x 101 x 21) over the following ranges: $s_t \in [0, 0.999]$, $p^*_{t-1} \in [0, 1.1]$, and $y_t \in [0, 0.8]$. At the terminal date T, the investor's value function takes the known value:

$$v_T = \frac{\beta(1-\beta^H)A_H^{1-\gamma}}{(1-\beta)(1-\gamma)} \tag{A27}$$

at all points in the state space. The value function at date T is then used to solve for the optimal decision rules for all points on the grid at date $T-1$. The procedure is repeated recursively for each time period until the solution for date $t = 0$ is found. Tri-linear interpolation is used to calculate the value function for points in the state space that lie between the grid points.

Footnotes

1. Black (1980) and Tepper (1981) show that it is tax-efficient for corporations to fully fund their pension plans, borrowing on corporate account if necessary, and to invest the pension plan assets entirely in taxable bonds. The implications of the Black-Tepper results for the optimal asset location policy were first discussed in an earlier version of this paper (Dammon, Spatt, and Zhang (1999)). Huang (2000) uses the Black-Tepper arbitrage arguments to formally derive the optimal asset location policy with unrestricted borrowing.

2. The intertemporal consumption-investment model incorporates many realistic features of the actual tax code, including the taxation of capital gains upon realization and the forgiveness of the tax on embedded capital gains at the time of death. The impact of optimal tax timing on the realization and trading behavior of investors is also studied by Constantinides (1983, 1984), Dammon, Dunn and Spatt (1989), Dammon and Spatt (1996), and Williams (1985). In contrast to this earlier work, however, the Dammon, Spatt and Zhang (2001a) model incorporates an optimal intertemporal portfolio decision, which involves a tradeoff between the diversification benefits and the tax costs of trading.

3. To keep the overall risk of the investor's portfolio constant, the shift of one after-tax dollar from asset i to asset j in the tax-deferred account requires a change in the taxable account of x_i dollars of asset i, $-x_j$ dollars of asset j, and $(x_j - x_i)$ dollars of the riskless taxable bond.

4. Huang (2000) also demonstrates that investors are indifferent to the location of their asset holdings when the total return on all assets are taxed at the same rate. When $\tau_d > \tau_g$, our results differ from those in Huang (2000). She shows that when capital losses are taxed at τ_d and capital gains are taxed at τ_g the asset with the highest *payout ratio* (i.e., the proportion of the total return that is taxable as ordinary income) is optimally held in the tax-deferred account. In practice, however, portfolio offset rules greatly diminish the benefits of differential long-term and short-term tax rates.

5. For mutual funds that distribute both long-term and short-term capital gains, the capital gains tax rate is a weighted average of the long-term and short-term tax rates, with the weights determined by the proportion of the total capital gain that is of each type. If all capital gains are

realized short-term each year, then $\tau_d = \tau_g$. Otherwise, $\tau_d > \tau_g$, even for the most active of mutual funds.

6. When capital gain tax rates are allowed to differ across assets, the riskless taxable bond will still be held in the tax-deferred account provided it has the highest yield (i.e., $d_i < r$ for all i). However, if the dividend yields on some assets exceed the riskless taxable interest rate, then the asset with the highest yield may *not* be held in the tax-deferred account. In this case, the values of ΔC_i in equation (2) for different assets will depend upon both the yield and asset-specific capital gains tax rate.

7. Equation (3) applies only for those investors who do not borrow in their taxable accounts. If investors do borrow, then the U.S. tax code disallows the tax deduction of the interest expense on the borrowing up to the level of the tax-exempt holdings. This has the effect of setting $\tau_d = \tau_m$ in equation (3) for each dollar of borrowing that is disallowed the tax deduction. The net effect is to lower the benefit of shifting equity into the tax-deferred account for investors that hold levered equity positions.

8. The implicit tax rate $\tau_m = 25\%$ is the 30-year average for long-term municpal bonds reported by Shoven and Sialm (2002). The implicit tax rate on short-term municipal bonds is typically closer to the statutory marginal tax rate for high-income investors. Green (1993) provides an equilibrium model of the municipal term structure that relies on clientele arguments and is broadly consistent with the empirical evidence.

9. The effective capital gains tax rate is based upon the assumption that long-term capital gains are taxed at 20%, short-term capital gains are taxed at 36%, and unrealized capital gains are untaxed by virtue of the fact that investors can defer the realization of capital gains until death, at which time the embedded tax liability is forgiven. The realization percentages used to calculate the effective capital gains tax rates for actively-managed and index mutual funds are broadly consistent with those reported in Shoven and Sialm (2002).

10. We are implicitly assuming that the financial markets are rich enough that a portfolio of securities can be constructed to match any risk and yield characteristics the investor desires. The assumption of perfect correlation implies that the return on asset j is of the following form:

$\tilde{g}_j = \gamma_j \tilde{g}_i + \alpha_j$. Without loss of generality, we assume that $\gamma_j = 1$ in our analysis.

11. If the dividend yields on both funds are zero, then the *certainty-equivalent* pre-tax abnormal return (net of transaction costs and fees) on the actively-managed mutual fund must exceed 135.5 basis points per year.

12. See Gruber (1996) and Carhart (1997) for empirical evidence on the relationship between performance and fees for active and passive mutual funds.

13. Our discussion here assumes that the tax rate on capital gains, τ_g, is identical across all risky assets. If not, then assets will be ranked on the basis of the values of $-\Delta C_i$ in equation (2) instead of yields.

14. We do not include tax-exempt bonds in our analysis because, as discussed in Section I, the existence of tax-exempt bonds does not alter the asset location decision when investors have the opportunity to invest in equity that is relatively tax efficient (e.g., exchange-traded funds, passive index mutual funds, or individual stocks). With tax-exempt bonds, the only change that would occur in our analysis is that high-tax bracket investors (those with $\tau_d > \tau_m$) would prefer to hold tax-exempt bonds instead of taxable bonds in the taxable account.

15. See Dammon, Spatt and Zhang (2001b) for an analysis of the optimal intertemporal portfolio problem in the presence of capital gains taxes with multiple risky assets, but without a tax-deferred account.

16. Recently, the IRS has adopted a minimum withdrawal schedule that is based upon the joint life expectancy of the individual and a hypothetical beneficiary. Consequently, our withdrawal rates are somewhat higher than those required by the new regulations issued by the IRS. Although we assume that the balance in the retirement account is subject to immediate taxation at the time of the investor's death, the recently adopted IRS regulations allow the beneficiary to withdraw the remaining funds according to his own life expectancy. Consequently, our analysis somewhat understates the potential benefits of tax-deferred investing.

17. While these assumptions may seem restrictive, our later welfare analysis shows that it is optimal for investors to maximize the funds available in their tax-deferred accounts in the absence of liquidity shocks. Moreover, in the absence of liquidity shocks, investors generate sufficient financial

and nonfinancial income during their working years to finance their optimal consumption without the need to liquidate their tax-deferred accounts. We relax these assumptions and allow the investor to optimize the contributions and withdrawals from the tax-deferred account, including withdrawals prior to retirement with penalty, when analyzing liquidity shocks in Section II.E.

18. $(f_t + b_t) < 1$ because taxable wealth is also allocated to consumption and the payment of taxes on realized capital gains.

19. We also calculated the optimal decision rules for higher and lower values of H. A weaker (stronger) bequest motive increases (reduces) the optimal consumption-wealth ratio, especially at late ages, but has relatively little effect on the investor's optimal portfolio holdings across the state space.

20. With unrestricted borrowing, the optimal overall equity proportion increases slightly in the level of tax-deferred wealth. Higher levels of tax-deferred wealth allow the investor to generate higher levels of riskless tax-arbitrage profits. By investing the incremental tax-deferred wealth in bonds, and borrowing in the taxable account to invest in equity, the investor is able to increase his total wealth without incurring additional risk. The investor responds to this risk-free increase in wealth by increasing slightly his overall exposure to equity.

21. Reichenstein (2001) uses a one-period mean-variance model to generate a series of numerical examples that illustrate the interaction between the asset location and asset allocation decisions. His model, however, does not consider the impacts of age, basis-price ratio, or the split of wealth between the taxable and tax-deferred accounts on the optimal decisions.

22. Across the 5,000 simulations, there is considerable variation in the magnitude of tax-deferred wealth. For example, at age 55 the minimum and maximum values of tax-deferred wealth are 13% and 81%, respectively, of total wealth. The timing of when tax-deferred wealth reaches its maximum is also somewhat variable, although it typically occurs between the ages of 50 and 60.

23. With unrestricted borrowing, the investor's optimal holding of equity in the taxable account is likely to exceed 100% at late ages. This is because the investor prefers to sell bonds and borrow to finance consumption rather than selling equity with embedded capital gains. At death, the investor's equity holdings are liquidated without payment of the capital gains tax, all borrowing is

repaid, and the remaining wealth is used to finance his bequest.

24. The increase in total wealth needed to compensate the investor for following the suboptimal location policy will understate the true welfare costs if future nonfinancial income is proportional to total wealth. This is because an increase in current total wealth also provides the investor with future benefits in the form of additional nonfinancial income.

25. Because the split of wealth between the taxable and tax-deferred accounts is changing over time, the utility costs depend upon more than just the investor's current distribution of wealth across these two accounts. In general, it will also depend upon the investor's age, anticipated future retirement account contributions and withdrawals, level of nonfinancial income, consumption plans, and asset returns. This explains why the level of retirement account wealth that produces the highest utility cost is not equal to 0.5 and why it is not the same across all ages.

26. Since consumption must be financed exclusively from the resources held in the taxable account, the shadow prices can be less than 1.0 for extremely high levels of tax-deferred wealth. We investigate the effects of liquidity needs in the next section.

27. For an investor at age 20 with zero tax-deferred wealth, the shadow price in Figure 3 is \$2.71. To understand the magnitude of this value, consider the following simple example. Investing one dollar of tax-deferred wealth in bonds earning a pre-tax return of 6 percent per year will grow to \$24.65 after 55 years (the life expectancy for a 20-year-old). To produce the same terminal value after 55 years, an investor would need to invest \$3.10 of taxable wealth in bonds earning an after-tax return of 3.84 percent per year.

28. Huang (2000) uses a variant of our model to analyze liquidity by assuming that the investor faces a one-time tax on total wealth at a known future date. The wealth tax is a dead-weight loss to the investor in her model. In contrast, the "consumption gulp" in our model serves as a constraint on the minimum level of consumption.

29. The amount of money spent to buy a house or to fund a child's college education is partially a function of the investor's wealth. If the "consumption gulp" is a fixed dollar amount, instead of a fixed percentage of total wealth, then two complications arise. First, it is necessary to introduce wealth as an additional continuous state variable. Second, it is necessary to define a penalty function

in the event that the investor's wealth is not sufficient to meet the required "consumption gulp." In this case, the severity of the penalty function will determine the extent to which the investor hedges in the taxable account.

30. The hedging demand for bonds in the taxable account is also present, although muted, when there is no penalty on early withdrawals from the retirement account. This is because the investor wants to avoid reducing the wealth in the tax-deferred account so that it can continue to earn the pre-tax return on assets.

31. At age 30, the investor withdraws funds from the retirement account and pays the 10 percent penalty only if the wealth in the taxable account is insufficient to finance the required "consumption gulp". Beyond age 30, the investor contributes the maximum during his working years and withdraws the minimum during his retirement years. Without a penalty on early withdrawals, the investor contributes the maximum to the retirement account each year throughout his working years.

32. This approach is similar in spirit to the random shocks to nonfinancial income analyzed by Amromin (2001). Despite the potential for a catastrophic loss of nonfinancial income for an extended period of time, he finds that the hedging demand for taxable bonds in the taxable account is still small relative to the overall holding of taxable bonds.

Figure legends

Figure 1. Optimal equity proportions as a function of retirement wealth and age. The figure shows the optimal equity proportions in the taxable account (top panel), retirement account (middle panel), and overall portfolio (bottom panel) as a function of age and the fraction of total wealth held in the retirement account. The basis-price ratio is set at $p^* = 1.0$.

Figure 2. Optimal equity proportions at ages 35 and 75. The figure shows the optimal equity proportions in the taxable account (dash-dotted line), tax-deferred account (dashed line), and overall portfolio (solid line) at age 35 (top panel) and age 75 (bottom panel). The optimal equity proportions are shown as a function of the fraction of total wealth held in the retirement account. The basis-price ratio is set at $p^* = 1.0$.

Figure 3. Utility costs and shadow prices. The top panel shows the utility costs of following the suboptimal policy of holding the same portfolio mix in both the taxable and tax-deferred accounts. The middle panel shows the utility costs of following the suboptimal policy of allocating equity to the tax-deferred account before allocating equity to the taxable account. The utility costs are measured as the percentage increase in total wealth (allocated entirely to the taxable account) needed to compensate the investor for following the suboptimal policy. The bottom panel shows the shadow prices for an additional dollar of tax-deferred wealth. The shadow price is the amount taxable wealth the investor is willing to pay to receive an additional after-tax dollar in his retirement account. The basis-price ratio is set at $p^* = 1.0$ and future nonfinancial income and retirement contributions are assumed to be zero.

Figure 4. Optimal equity proportions and retirement contributions with a known "consumption gulp." The figure shows the optimal equity proportions and retirement contributions for the case in which the investor has a known "consumption gulp" of 50 percent of total wealth at age 30. A 10 percent penalty is enforced on withdrawals from the retirement account prior to age 65. The optimal equity proportions and retirement contributions are shown as a function of age (prior to age 30) and the fraction of total wealth held in the retirement account. The top panel depicts the optimal equity proportion in the taxable account, the middle panel depicts the optimal equity proportion in the tax-deferred account, and the bottom panel depicts the optimal retirement

contributions (as a percentage of pre-tax non-financial income).

Figure 5. Optimal equity proportions and retirement contributions with a random "consumption gulp." The figure shows the optimal equity proportions and retirement contributions for the case in which the investor faces a 10 percent probability each period of receiving a shock to consumption requiring him to consume 50 percent of his total wealth. The consumption shocks are assumed to be independent of time and are uncorrelated with asset returns. A 10 percent penalty is enforced on withdrawals from the retirement account prior to age 65. The optimal equity proportions and retirement contributions are shown as a function of age (prior to age 65) and the fraction of total wealth held in the retirement account. The top panel depicts the optimal equity proportion in the taxable account, the middle panel depicts the optimal equity proportion in the tax-deferred account, and the bottom panel depicts the optimal retirement contributions (as a percentage of pre-tax non-financial income).

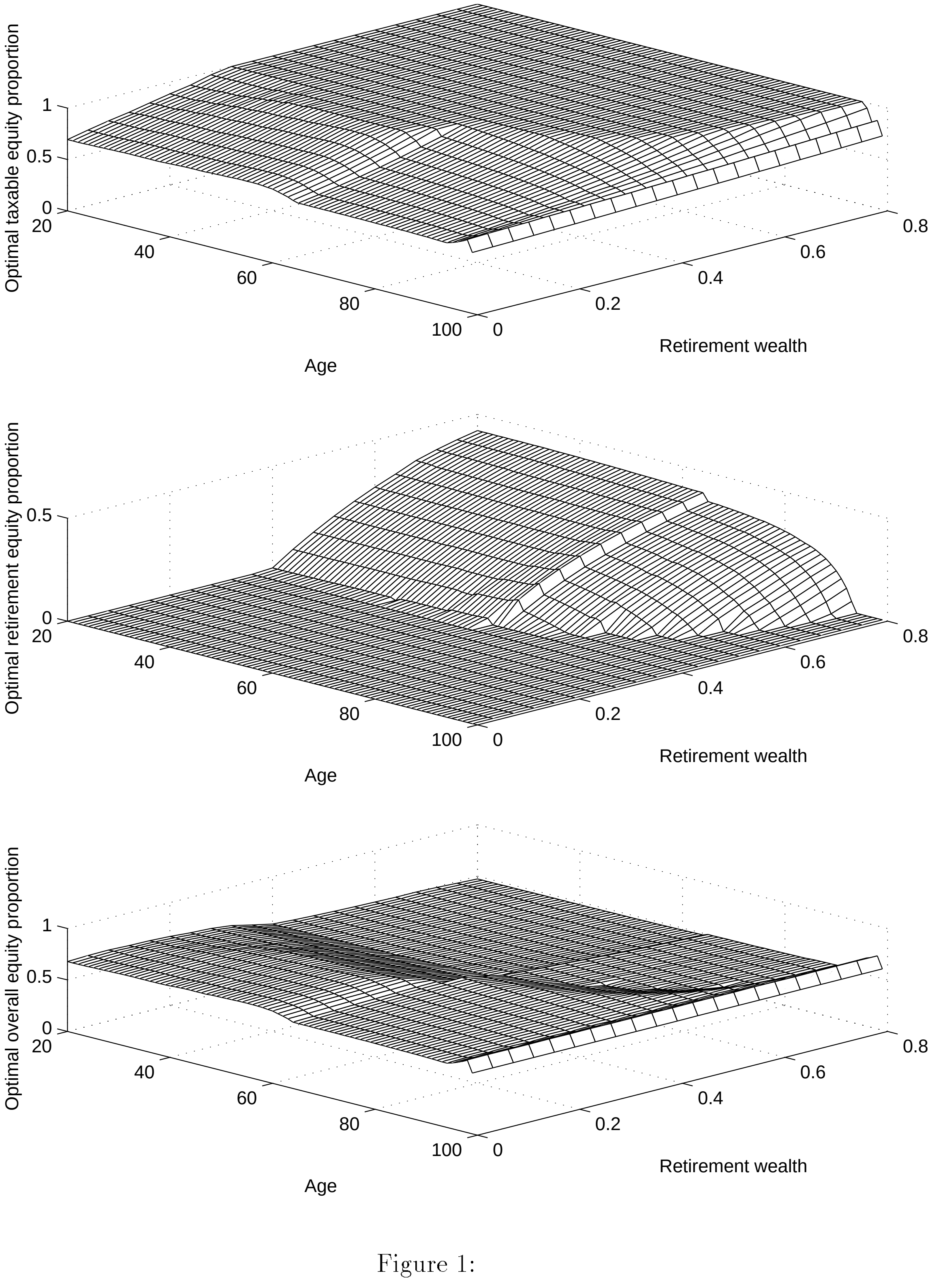

Optimal taxable equity proportion
1
0.5
0
20
40
60
80
100
Age
0
0.2
0.4
0.6
0.8
Retirement wealth
Optimal retirement equity proportion
0.5
0
20
40
60
80
100
Age
0
0.2
0.4
0.6
0.8
Retirement wealth
Optimal overall equity proportion
1
0.5
0
20
40
60
80
100
Age
0
0.2
0.4
0.6
0.8
Retirement wealth

Figure 1:

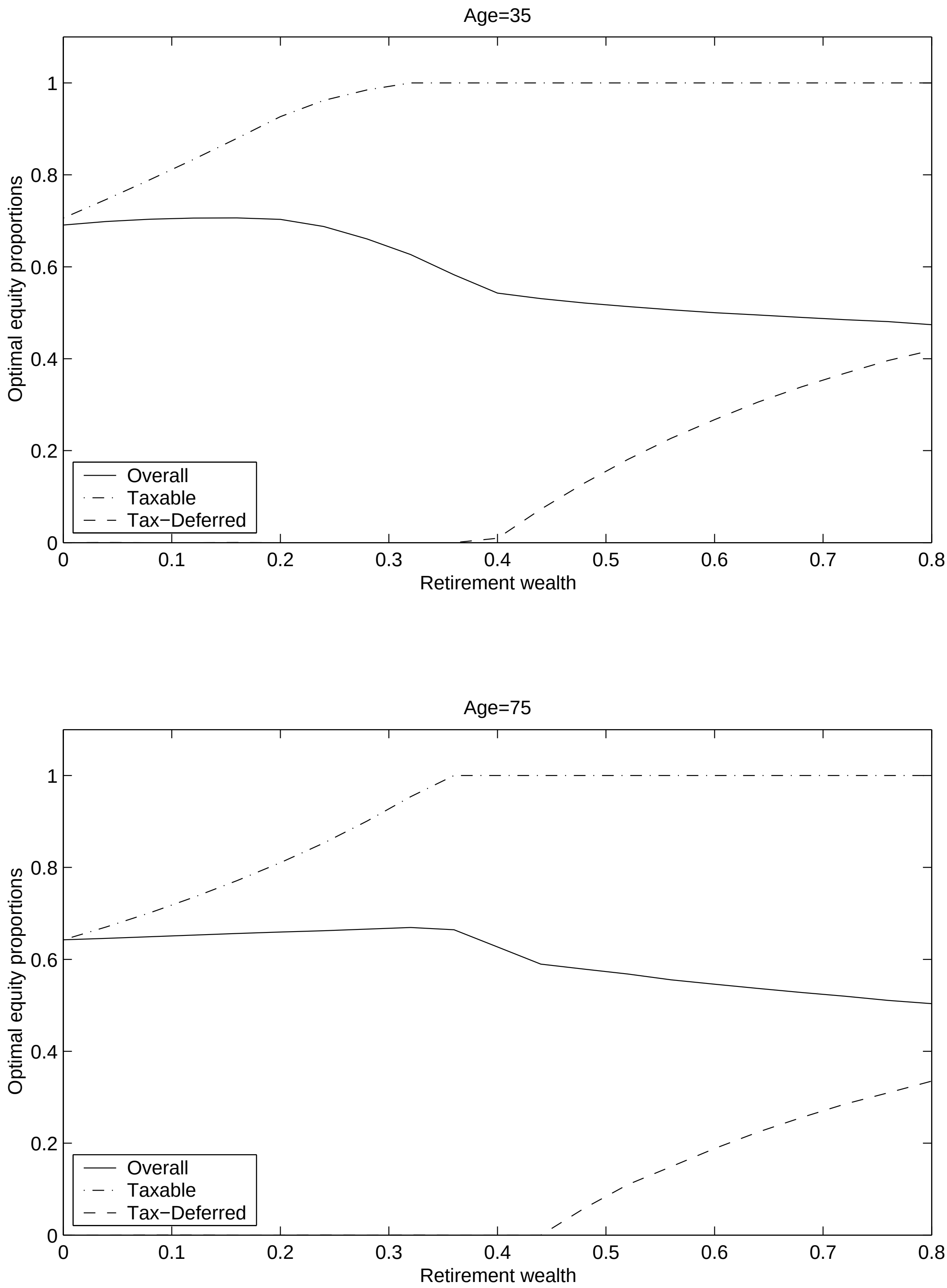

Age=35
Optimal equity proportions
Retirement wealth
Overall
Taxable
Tax−Deferred
0
0.2
0.4
0.6
0.8
1
0.1
0.3
0.5
0.7
Age=75
Optimal equity proportions
Retirement wealth
Overall
Taxable
Tax−Deferred

Figure 2:

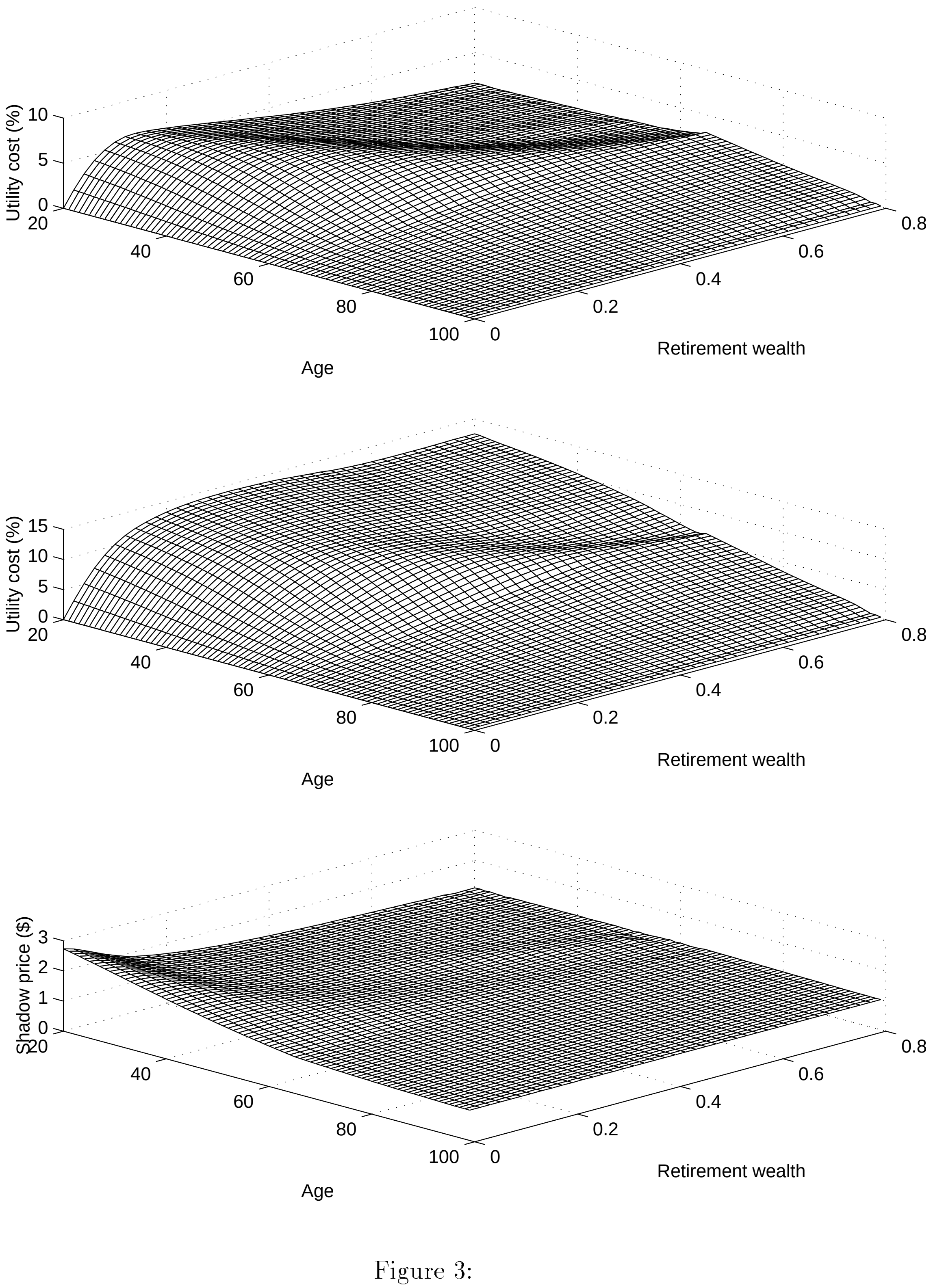

Utility cost (%)
10
5
0
20
40
60
80
100
Age
0
0.2
0.4
0.6
0.8
Retirement wealth
Utility cost (%)
15
10
5
0
20
40
60
80
100
Age
0
0.2
0.4
0.6
0.8
Retirement wealth
Shadow price ($)
3
2
1
0
20
40
60
80
100
Age
0
0.2
0.4
0.6
0.8
Retirement wealth

Figure 3:

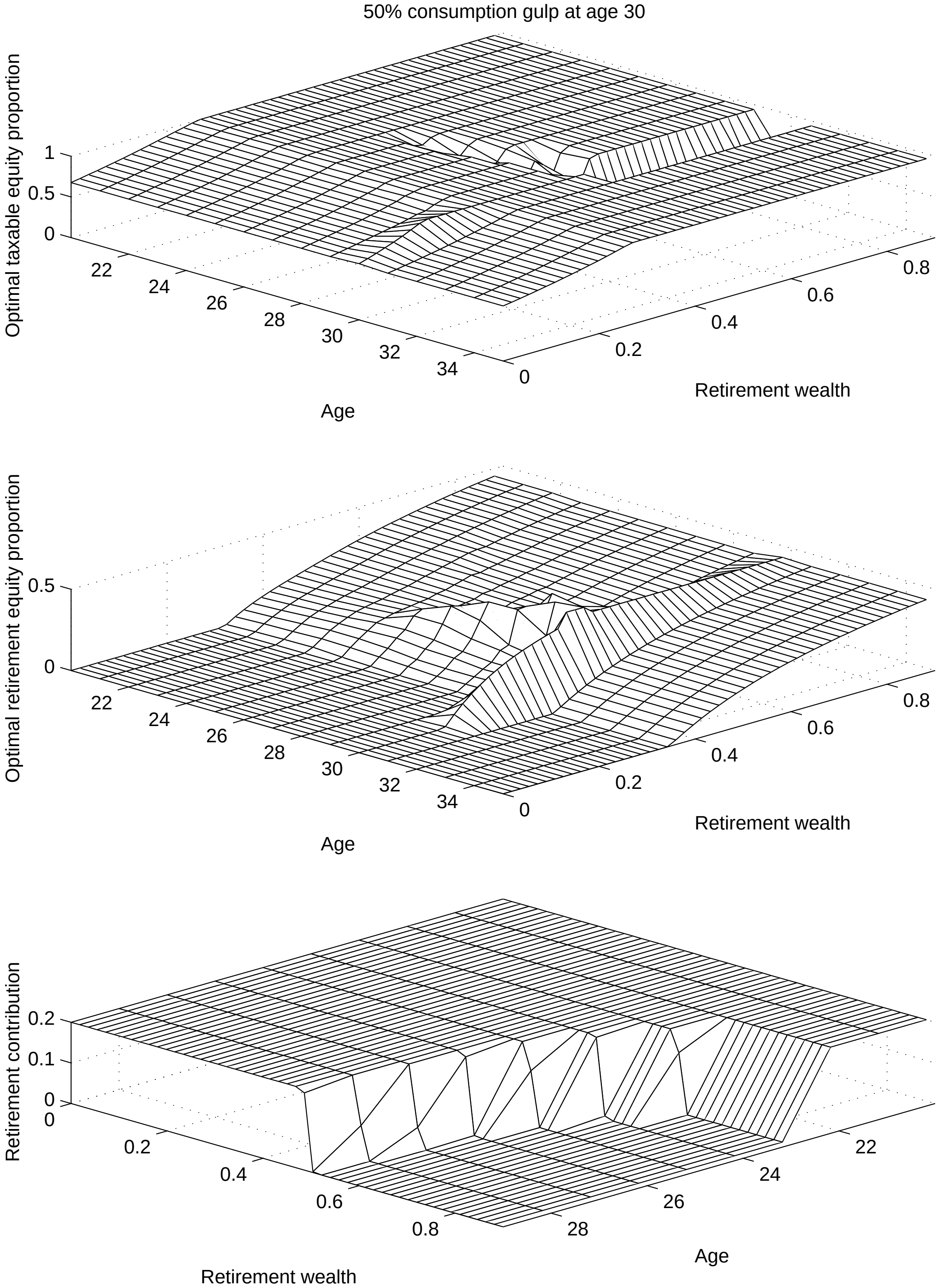
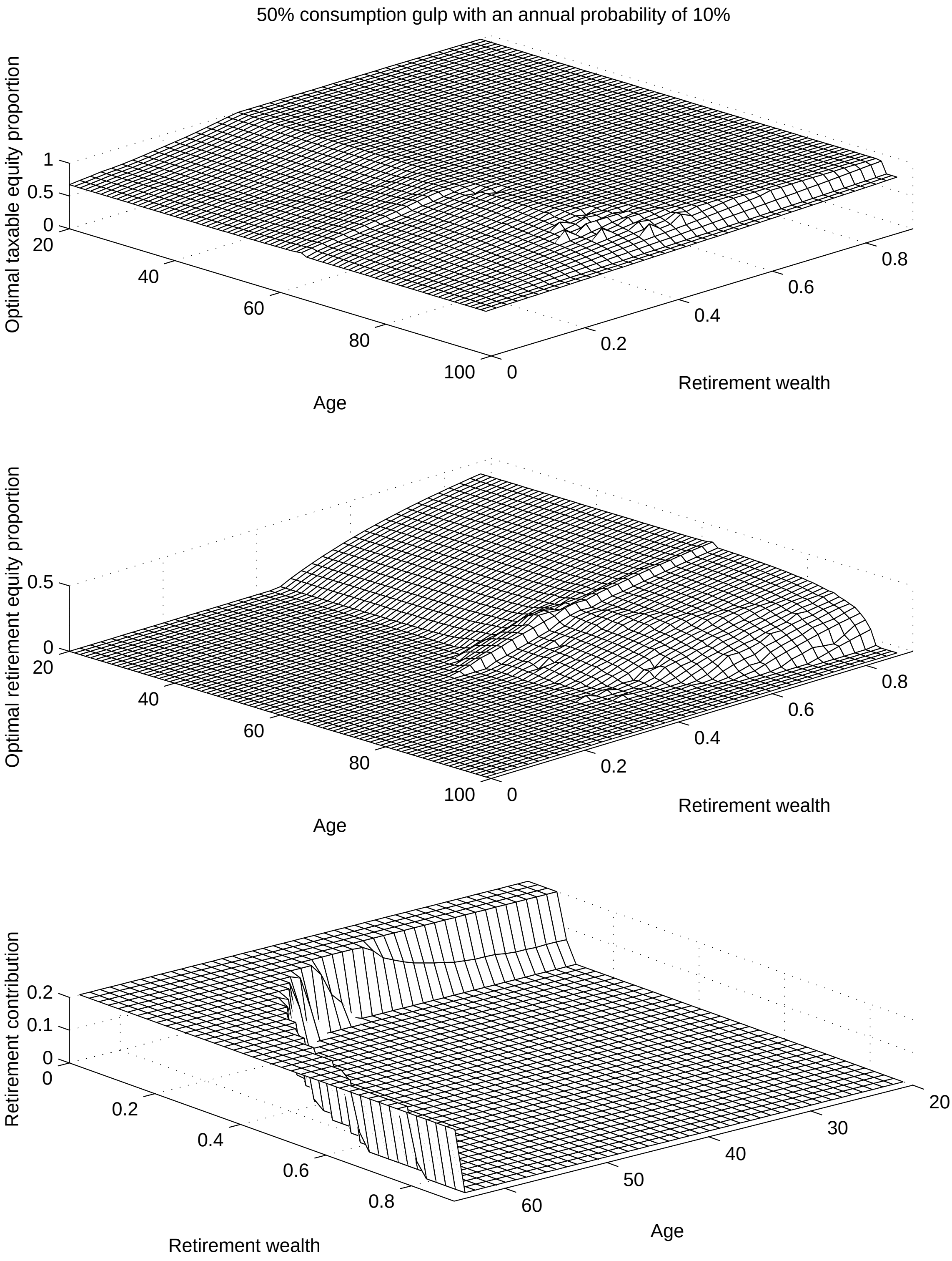

50% consumption gulp at age 30
Optimal taxable equity proportion
1
0.5
0
22
24
26
28
30
32
34
Age
0
0.2
0.4
0.6
0.8
Retirement wealth
Optimal retirement equity proportion
0.5
0
22
24
26
28
30
32
34
Age
0
0.2
0.4
0.6
0.8
Retirement wealth
Retirement contribution
0.2
0.1
0
0
0.2
0.4
0.6
0.8
Retirement wealth
22
24
26
28
Age

Figure 4:



50% consumption gulp with an annual probability of 10%
Optimal taxable equity proportion
1
0.5
0
20
40
60
80
100
Age
0
0.2
0.4
0.6
0.8
Retirement wealth
Optimal retirement equity proportion
0.5
0
20
40
60
80
100
Age
0
0.2
0.4
0.6
0.8
Retirement wealth
Retirement contribution
0.2
0.1
0
0
0.2
0.4
0.6
0.8
Retirement wealth
20
30
40
50
60
Age

Figure 5: