

Grade 12 Advanced Functions and Introductory Calculus
Final Examination



Wednesday, January 29, 2003
1:00–3:00
2 hours

NAME: _____
(Print)

Check that you have all 7 pages of this exam.
Put your name on every sheet.
Answer all questions in the space provided.
Calculators are permitted.
Neat and complete solutions are required for full marks. Good luck!

1. Determine the remainder when $(x^3 + 5x^2 + 2x - 1)$ is divided by $(x + 2)$.

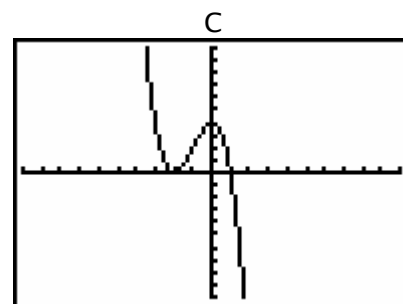
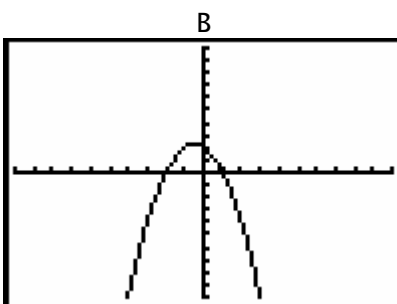
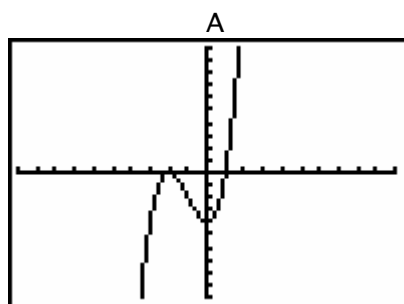
K

2. When $x^3 - 3kx^2 + x + 5$ is divided by $(x - 2)$, the remainder is 9.
Find the value of k.

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3. Consider $f(x) = -(x - 1)(x + 2)^2$.
Circle the graph of $f(x)$. Explain your reasoning.

K



4. Consider $f(x) = 2x^3 + 3x^2 - 11x - 6$.

A a) Factor $2x^3 + 3x^2 - 11x - 6$ completely.

A b) Use your answer from a) to find the x-intercepts of $f(x)$.

A c) Using only the x-intercepts and your knowledge of dominant terms, sketch $f(x)$.



5. Consider the graph of $h(x)$.

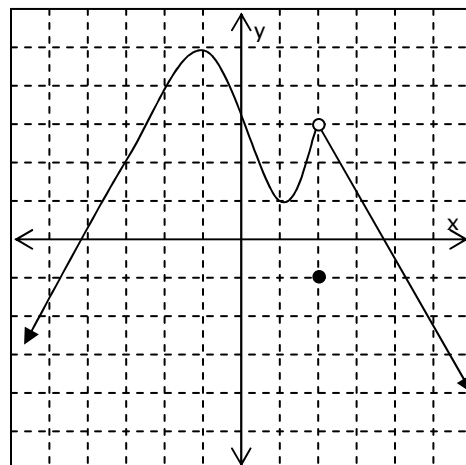
K a) Determine $h(2)$.

K b) Determine $\lim_{x \rightarrow 2^-} h(x)$.

K c) Determine $\lim_{x \rightarrow 2^+} h(x)$.

K d) Determine $\lim_{x \rightarrow 2} h(x)$.

K e) Is $h(x)$ continuous? Explain.



6. Evaluate the following limits.

K ,K a) $\lim_{x \rightarrow 1} \frac{x^3 + 1}{x^2 + 1}$

b) $\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9}$

7. Find the derivative of $f(x) = 2x^2 - 1$ using first principles.

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8. Differentiate and simplify.

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a) $y = \frac{3x - 1}{2x^2 + 5}$

b) $f(x) = \sqrt{3x^3 + 2}$

K ,K

c) $y = xe^{3x-1}$

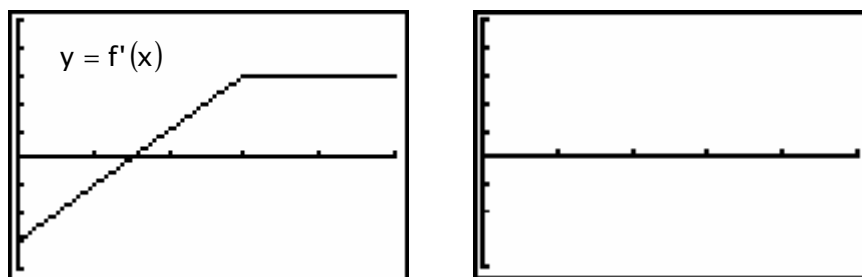
d) $f(x) = \ln(x^3 - 3)$

9. At what point(s) on the curve $y = x + \frac{4}{x}$ is the slope of the tangent -3 ?

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10. The following graph represents the first derivative of a function.

A Sketch the graph of a possible original function on the grid provided.



11. The flu is spreading through a local high school.

At time t days after the beginning of the epidemic, there are $P(t)$ students sick where

$$P(t) = 30t - 2t^2, \quad 0 \leq t \leq 15.$$

A a) How many students have the flu when $t = 2$?

A b) At what rate is the flu spreading when $t = 2$?

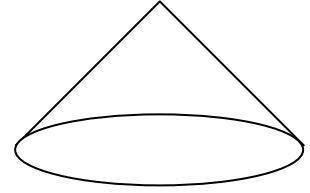
A c) How many students are sick when the flu is spreading at the rate of 10 students per day?

12. Find the slope of the tangent line to $x^2 - 2y^2 = xy$ at the point $(2,1)$.

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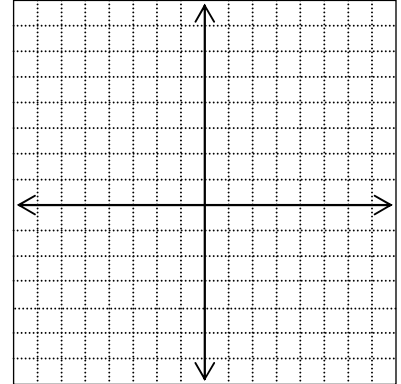
13. Sand is being dumped on a pile in such a way that it always forms a cone whose radius is equal to twice its height. If the sand is being dumped at a rate of $30 \text{ m}^3/\text{hour}$, at what rate is the height of the pile increasing when there are $288\pi \text{ m}^3$ of sand in the pile?

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14. Consider $f(x) = \frac{3x}{x-4}$.

- K
- a) State the equation of any vertical asymptotes. Determine the behaviour of the curve near its vertical asymptote(s). Sketch $f(x)$ near its vertical asymptote(s) on the grid below.

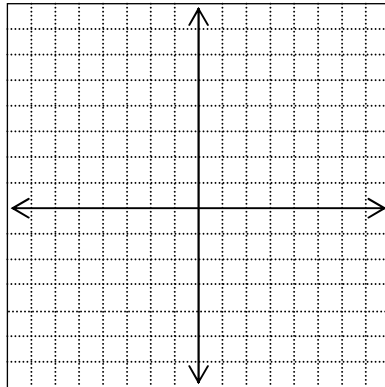


- K
- b) State the equation of any horizontal asymptotes. Determine the behaviour of the curve near its horizontal asymptote. Sketch $f(x)$ near its horizontal asymptote on the same grid.

15. Sketch a possible function that could be described by this table.

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Interval	$(-\infty, -3)$	$(-3, 0)$	$(0, 4)$	$(4, +\infty)$
$f(x)$	Increasing	Decreasing	Increasing	Increasing
Concavity	Upward	Upward	Upward	Downward



16. “Mathe-magics” novelty store sells 200 supernatural solid gold statues of Pythagoras for \$350 each. A statistical survey indicates to the store manager that for each \$10 discount offered to the buyers, the number of statues sold will increase by 20 statues a week. How large a discount should the store offer to maximize its revenue?

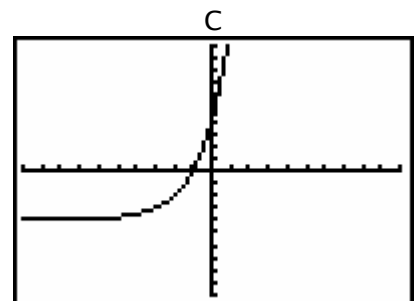
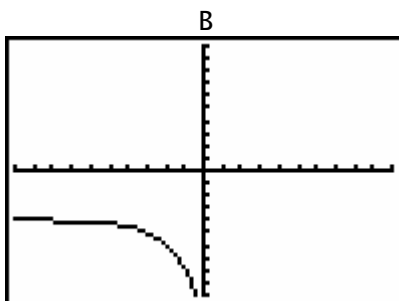
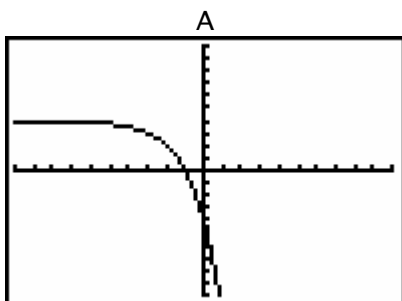
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17. Consider $f(x) = -2^{x+3} - 1$.

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Circle the graph of $f(x)$. Explain your reasoning.



18. Solve.

K ,K

a) $3^{x-5} = 60$

b) $\log_2(x+2) + \log_2(x) = 3$

19. A culture grows in such a way that its mass (in grams) at any time t (in hours) is given by:

$$M(t) = 60t - e^{0.5t} \text{ where } 1 \leq t \leq 13$$

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a) How fast is the mass growing when $t = 4$?

A

b) What is the maximum mass of this culture?

GOOD LUCK NEXT SEMESTER!