Quantum Computation

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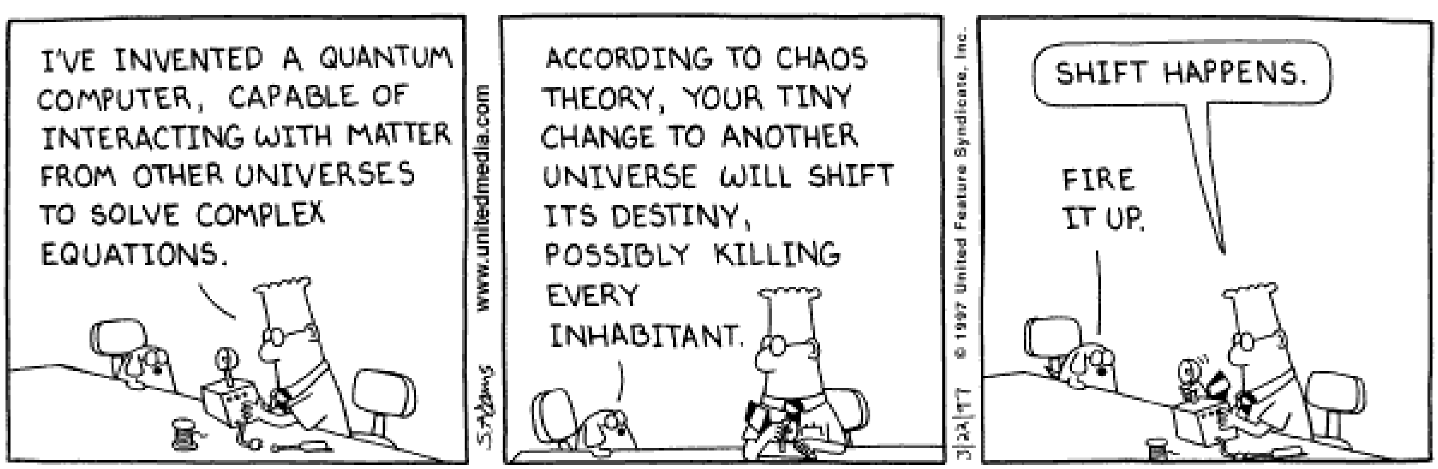


What does it mean to compute?

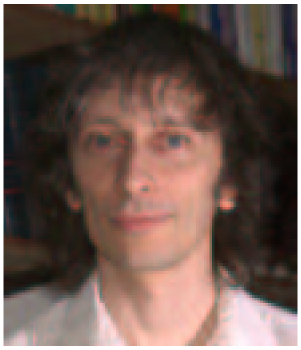
Q: Is there a general algorithm to determine whether a mathematical conjecture is true or false?


Church-Turing:
NO!

Church-Turing thesis: Any algorithmic process can be simulated on a Turing machine.

Ad hoc empirical justification.

Strong Church-Turing thesis: Any algorithmic process can be efficiently simulated on a probabilistic Turing machine.



Deutsch:

Can we justify C-T thesis using laws of physics?

Quantum mechanics seems to be very hard to simulate on a classical computer.

Might it be that computers exploiting quantum mechanics are not efficiently simulatable on a probabilistic Turing machine?

(Violation of strong C-T thesis!)

Might it be that such a computer can solve some computational problems faster than a probabilistic Turing machine?

Candidate universal computer:
quantum computer

Quantum circuit model

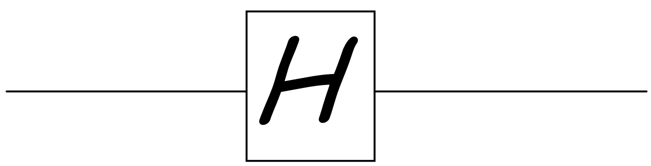
Classical	Quantum
Unit: bit	Unit: qubit
1. Prepare n bit input	1. Prepare n qubit input
2. Logic gates	2. Reversible quantum logic gates
3. Readout value of bits	3. Readout partial information about qubits

Single qubit quantum logic gates

Pauli gates:

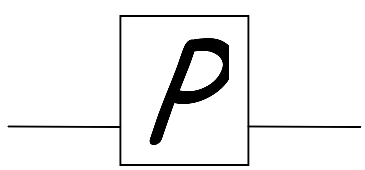
$$X=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix};\quad Y=\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix};\quad Z=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$

Hadamard gate:



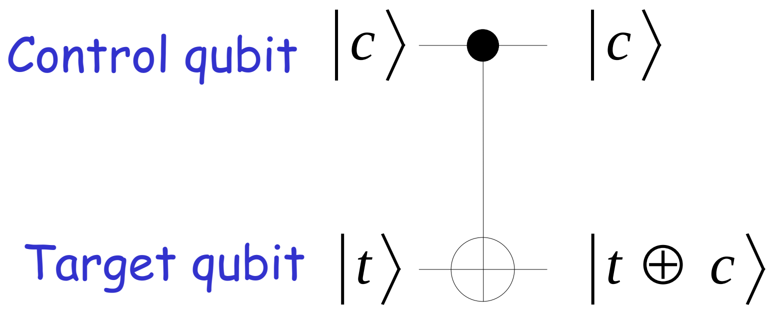
$$H|0\rangle=\frac{|0\rangle+|1\rangle}{\sqrt{2}};\quad H|1\rangle=\frac{|0\rangle-|1\rangle}{\sqrt{2}};\quad H=\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

Phase gate:



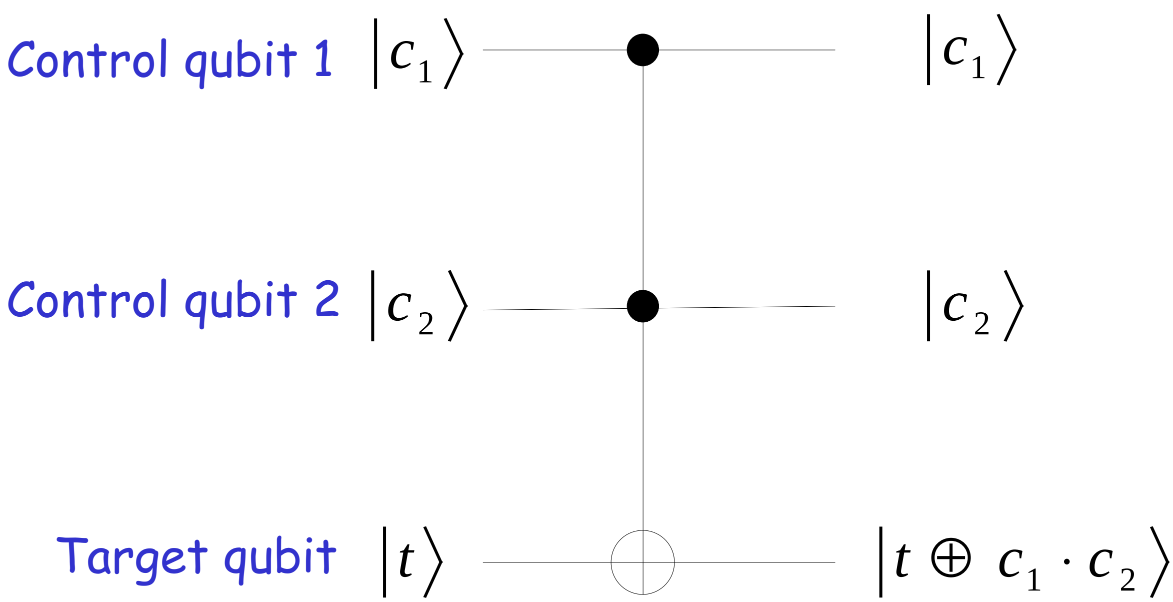
$$P|0\rangle=|0\rangle;\quad P|1\rangle=i|1\rangle$$

Controlled not gate:



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Toffoli gate



Universal Logic Gates

Suppose U is an arbitrary unitary transformation on n qubits.

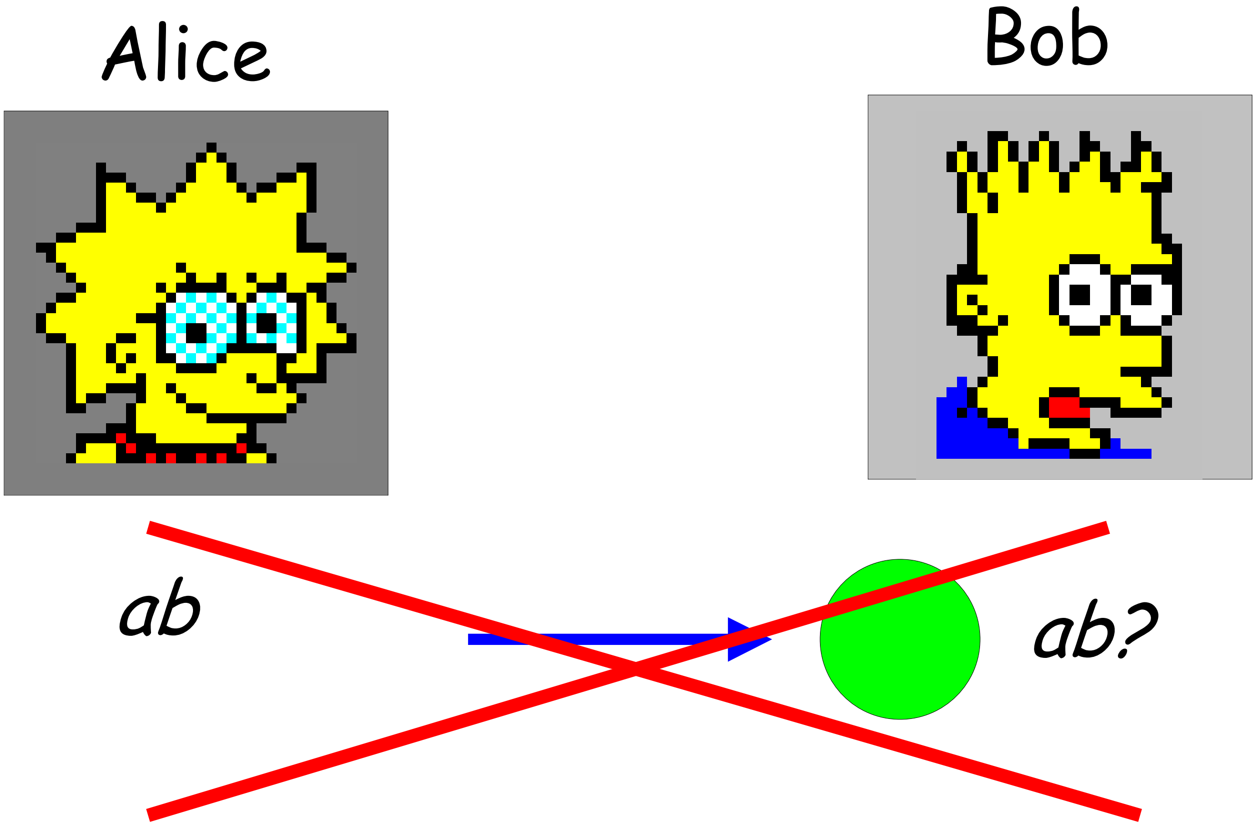
U can be approximated arbitrarily well by a sequence of Hadamard gates, phase gates, controlled not gates, and Toffoli gates.

These gates may all be performed fault-tolerantly, so in principle noise is not a problem.

Quantum circuit model

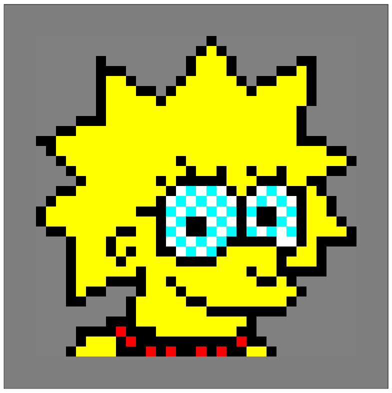
1. Prepare state $|0,0,\ldots,0\rangle$
2. Apply circuit of Hadamard, phase, controlled not and Toffoli gates.
3. Measure in the computational basis.

Superdense coding

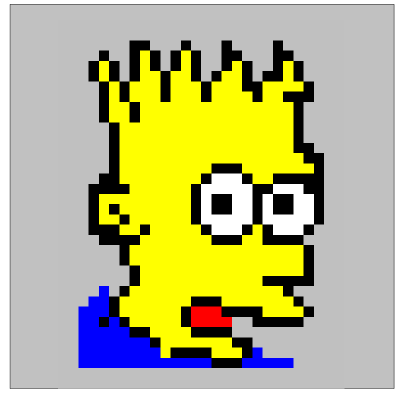


Superdense coding

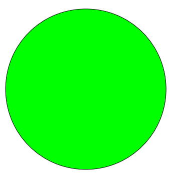
Alice

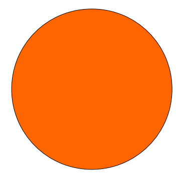


Bob



ab





Superdense coding

Alice

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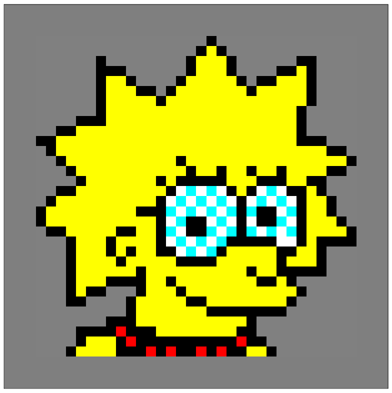

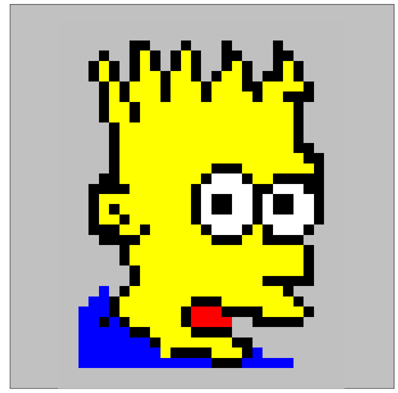

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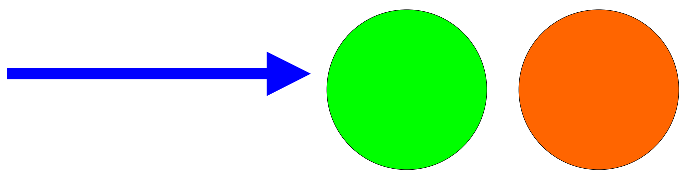
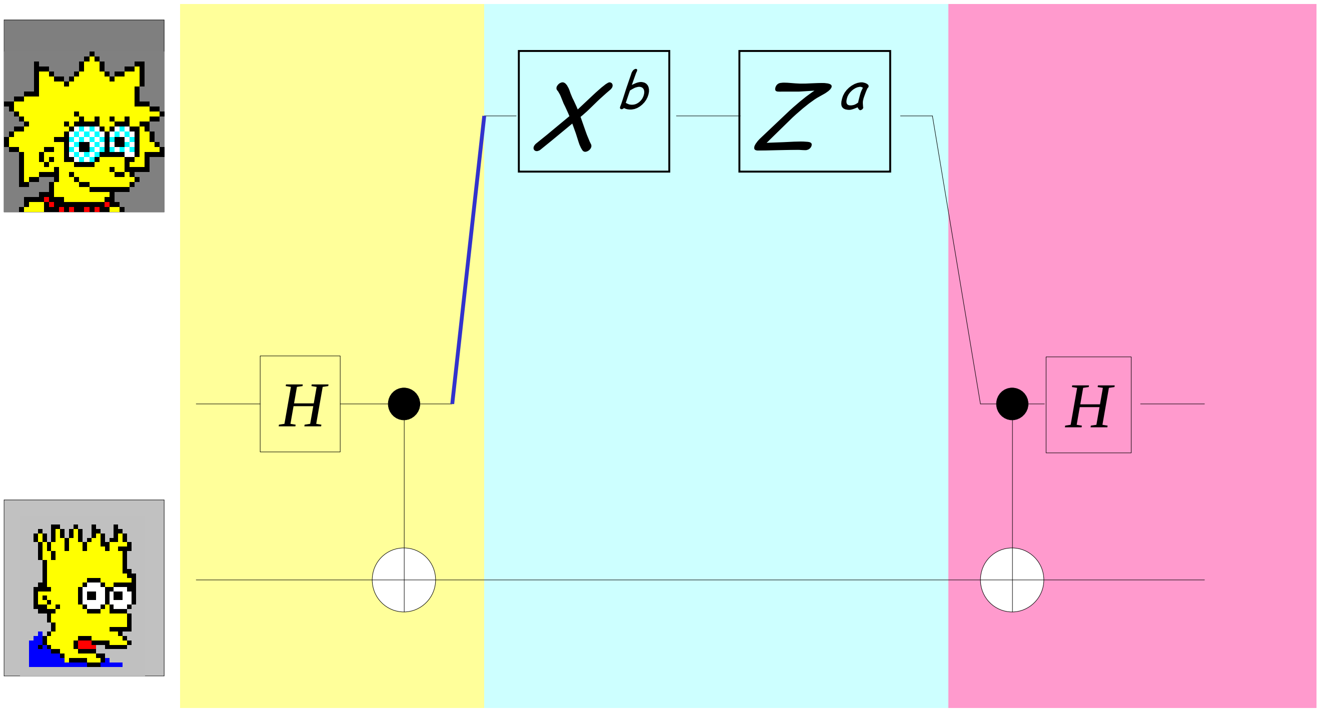


ab?

Superdense coding



$$00 \to 00 + 10 \to 00 + 11$$

$$00 + 11$$

$b = 0$: $00 + 11$ $b = 1$: $10 + 01$

$a = 0$: $00 + 11$ $a = 1$: $00 - 11$ $a = 0$: $10 + 01$ $a = 1$: $-10 + 01$

$$00 + 11 \quad 00 - 11 \quad 10 + 01 \quad -10 + 01$$

$$ab = 01: \; 10 + 01 \to 11 + 01 \to 01$$

Example: Deutsch's problem

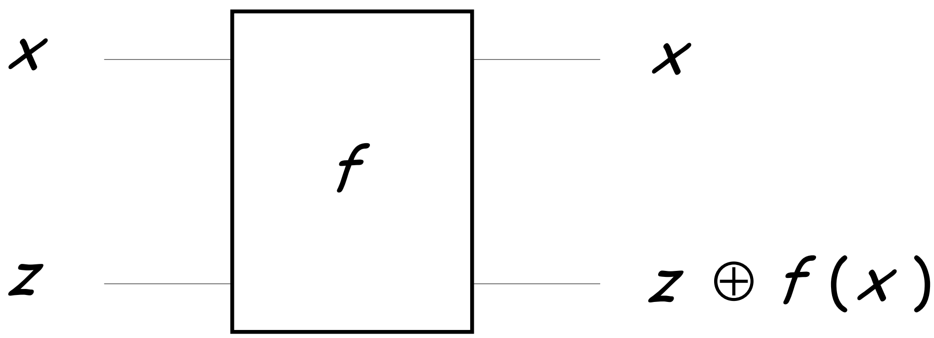
Given a black box computing a function $f:\{0,1\} \to \{0,1\}$

Our task is to determine whether f is constant or balanced?

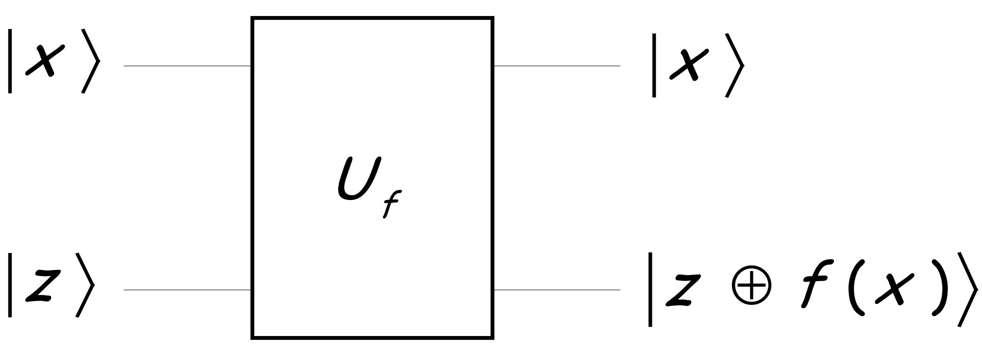
Classically we need to evaluate both $f(0)$ and $f(1)$.

Quantumly we need only use the black box for $f(\bullet)$ once!

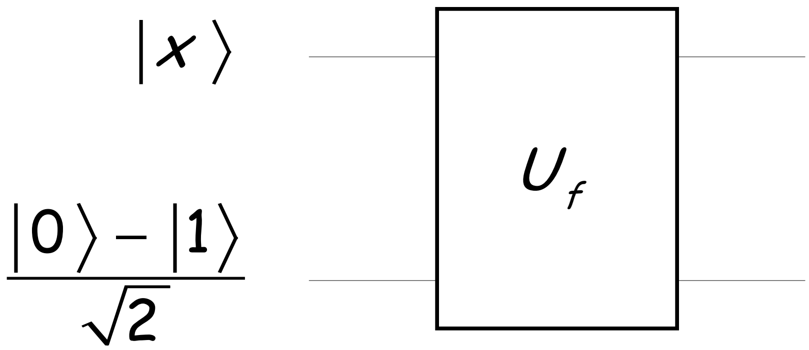
Classical black box



Quantum black box



Putting information in the phase



$f(x) = 0:$

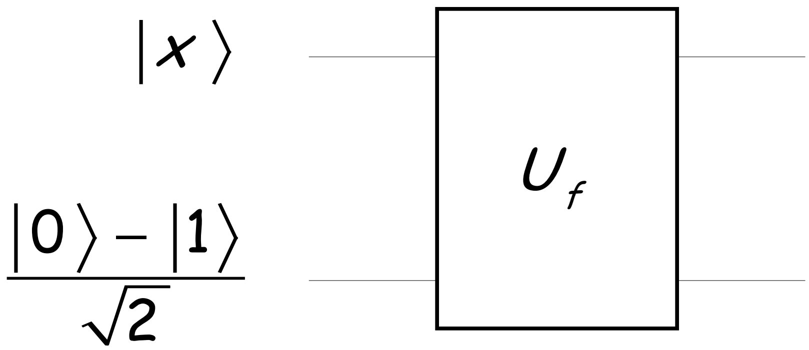
$$|x\rangle(|0\rangle - |1\rangle) \rightarrow |x\rangle(|0\rangle - |1\rangle)$$

$f(x) = 1:$

$$|x\rangle(|0\rangle - |1\rangle) \rightarrow |x\rangle(|1\rangle - |0\rangle)$$
$$= -|x\rangle(|0\rangle - |1\rangle)$$

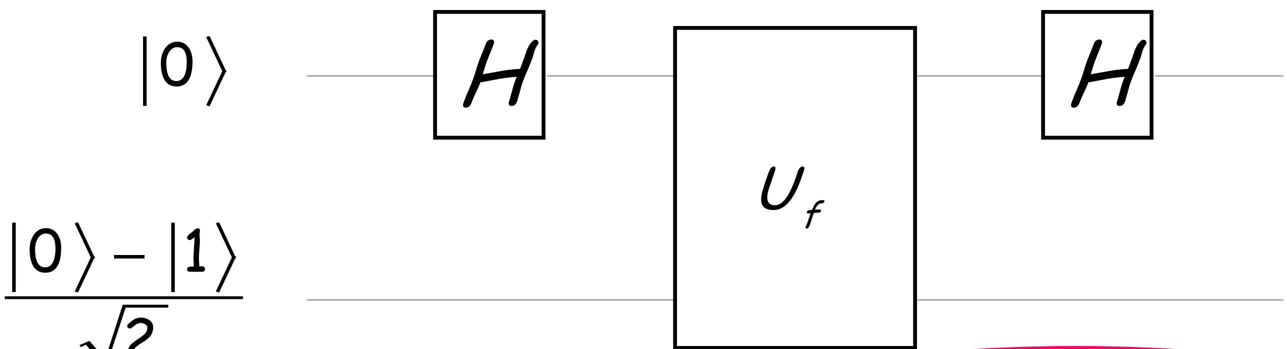
$$|x\rangle(|0\rangle - |1\rangle) \rightarrow (-1)^{f(x)}|x\rangle(|0\rangle - |1\rangle)$$

Putting information in the phase



$$|x\rangle \rightarrow (-1)^{f(x)}|x\rangle$$

Quantum algorithm for Deutsch's problem



Quantum parallelism

$$|0\rangle \to |0\rangle + |1\rangle$$

$$\to (-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle$$

$$\to (-1)^{f(0)}(|0\rangle + |1\rangle) + (-1)^{f(1)}(|0\rangle - |1\rangle)$$

$$= \left[(-1)^{f(0)} + (-1)^{f(1)}\right]|0\rangle + \left[(-1)^{f(0)} - (-1)^{f(1)}\right]|1\rangle$$

f const. $\Rightarrow$ all amplitude in $|0\rangle$.

f balan. $\Rightarrow$ all amplitude in $|1\rangle$.

[redacted]