Quantum Error Correction II

Robust Quantum Information Processing

Manny

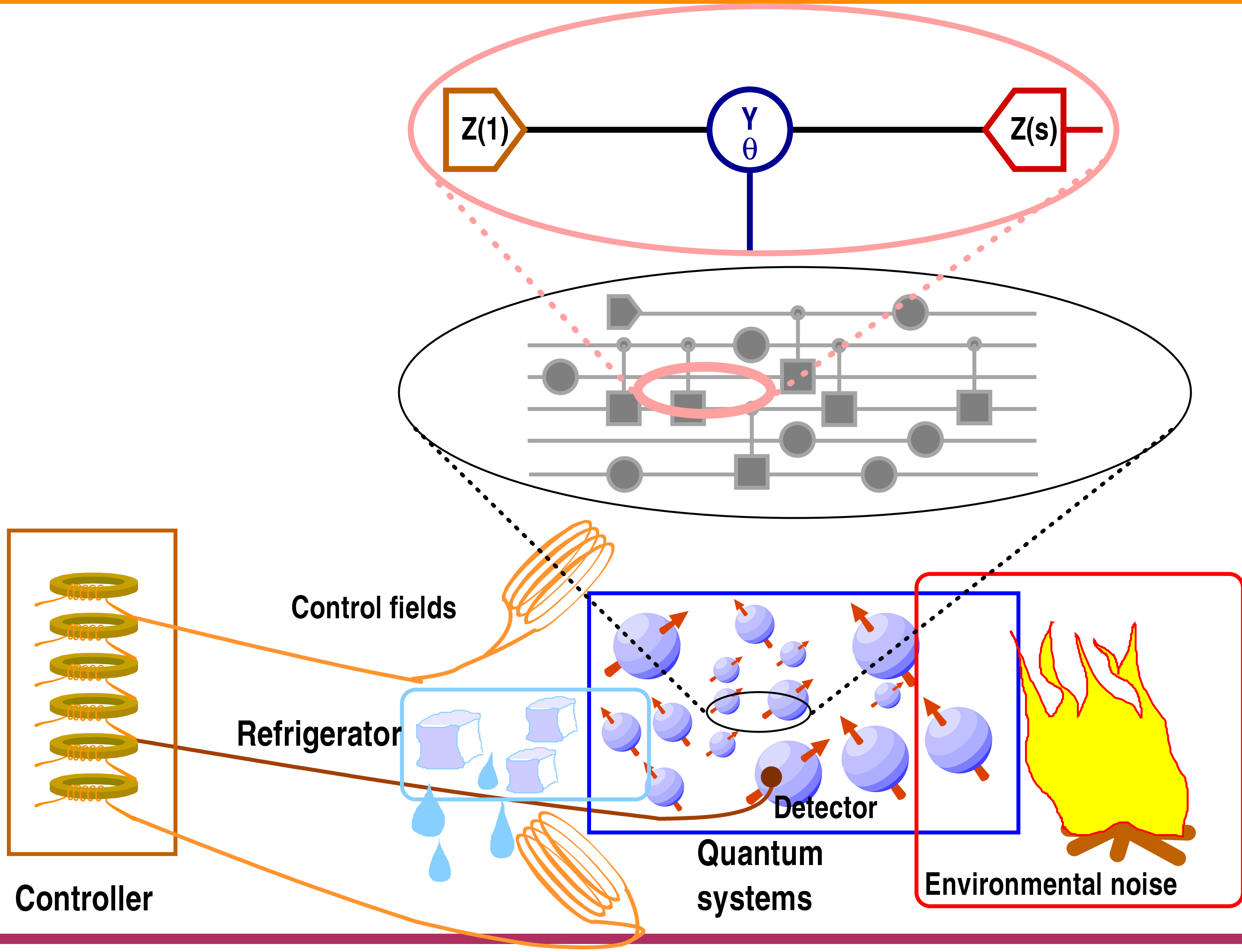
- Threshold theorems.
- Requirements for scalability.
- Physical noise models.
- Methods for scalability.

References

- Prehistory:
  - Quantum Zeno effect: Misra&Sudarshan 1977 [18].
  - Deutsch 1993, Barenco&*al.* 1996 [3].
- Discovery and Theory:
  - Shor 1995 [21], Steane 1995 [23].
  - Bennett&DiVincenzo&Smolin&Wootters 1996 [4], Knill&Laflamme 1996 [12].
  - Calderbank&Shor 1996 [6], Gottesman 1996 [8], Calderbank&Rains&Shor&Sloane 1997 [5].
- Fault tolerance and threshold accuracies:
  - Shor 1996 [22], Kitaev 1997 [11].
  - Aharonov&Ben-Or 1996 [1, 2], Knill&Laflamme&Zurek 1996 [15], Gottesman&Preskill 1997 [9, 20], Dür&Briegel&*al.* 1999 [7].
  - Gottesman 1997 [9], Gottesman&Chuang 1999 [10]
- Toward subsystems:
  - Quasi-particles . . .
  - Zanardi&Rasetti 1997 [27], Lidar&Chuang&Whaley 1998 [17].
  - Viola&Knill&Lloyd 1998 [25, 24, 26].
  - Knill&Laflamme 1996 [12], Knill&Laflamme&Viola 2000 [14].

General reference: (M)ike, Ch. 10. Nielsen&Chuang 2001 [19]

QIP System Overview



The Threshold Theorems

- The Accuracy Threshold Theorem:
  Assume the *requirements for scalable computing*. If the error per gate (including “no-op”) is less than a threshold, then it is possible to efficiently quantum compute arbitrarily accurately.

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  - Error Thresholds:
    - Worst case $> 10^{-6}$.
    - Estimate $> 10^{-4}$.
    - Communication $> 10^{-2}$.
    - Erasure $> 10^{-2}$.
    - Z-measurement $\geq .5$.

Shor 1996[22], Kitaev 1996[11], Aharonov&Ben-Or 1996[1], Knill& *al.* 1996[16].

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- Noise:
  - Sufficiently weak.
  - Quasi-independent.

Types of Noise

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  - Step-wise independent, discrete.
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    - Depolarization.
    - Dissipation, thermal relaxation.
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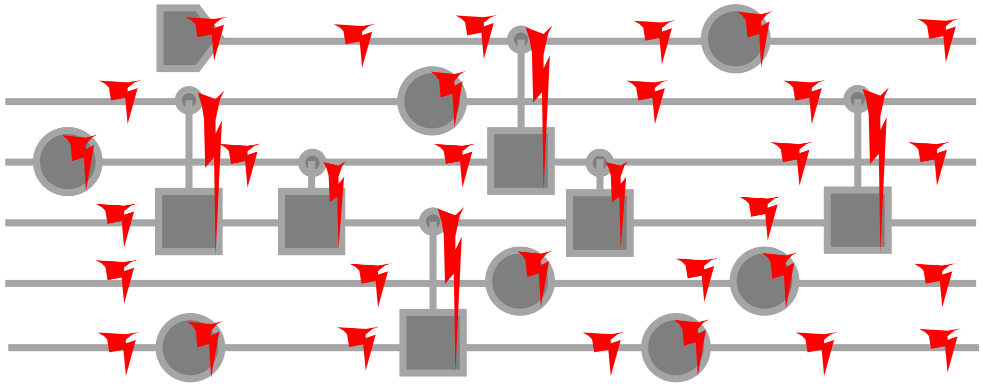
- By origin.
  - Thermal.
  - Control errors, faults.
    - Miscalibration.
    - Over/under-rotation.
  - Stray fields or inhomogeneity.

Noise Analysis



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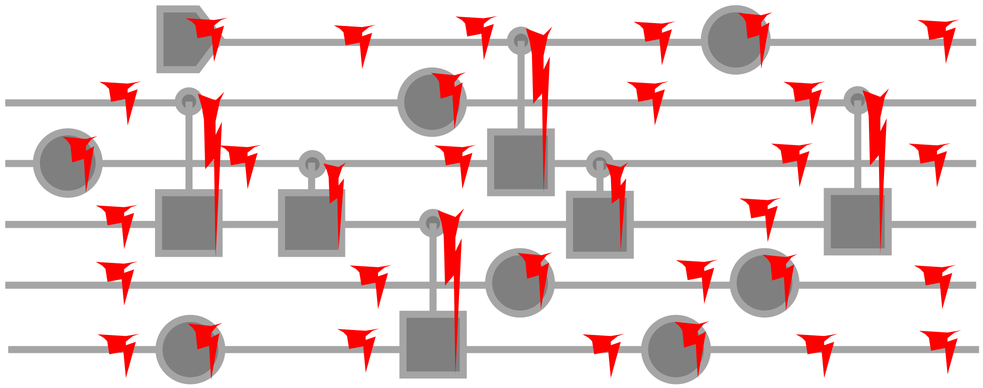

- Error locations:

  For each gate U_i including $U_i = \mathbf{noop}$:

$$U_{i\text{actual}} = E_i U_{i\text{intended}}$$

Locality: E_i acts on the same qubits as U_i.

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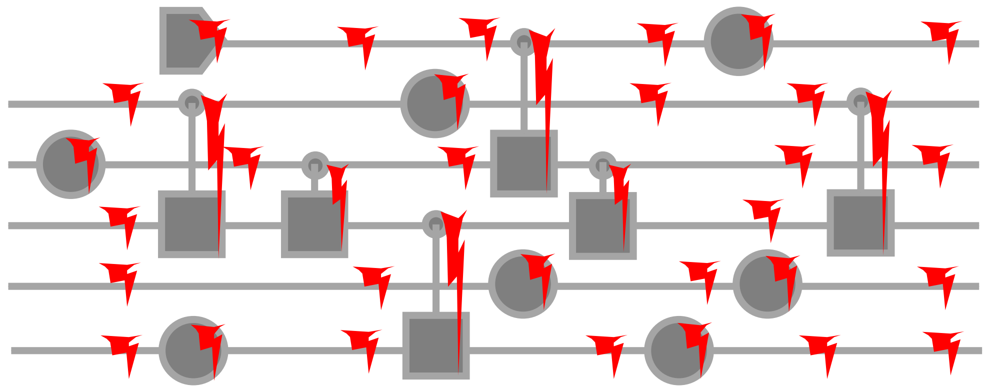

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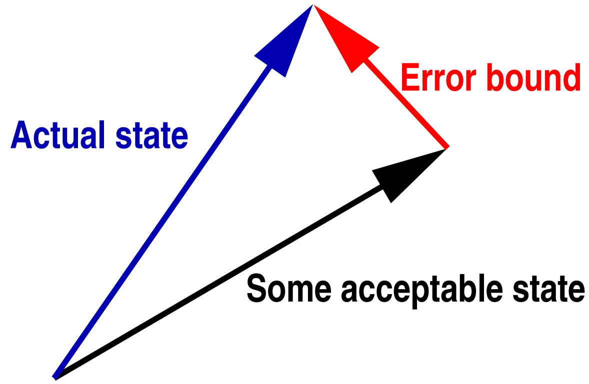
- Error expansion with "environment":

$$|\psi\rangle \rightarrow \sum_e |e\rangle_{\mathsf{E}}\ E_{e,n}U_n \cdots E_{e,2}U_2\ E_{e,1}U_1|\psi\rangle$$

The $|e\rangle_{\mathsf{E}}$ need not be orthogonal.

A Measure of Noise



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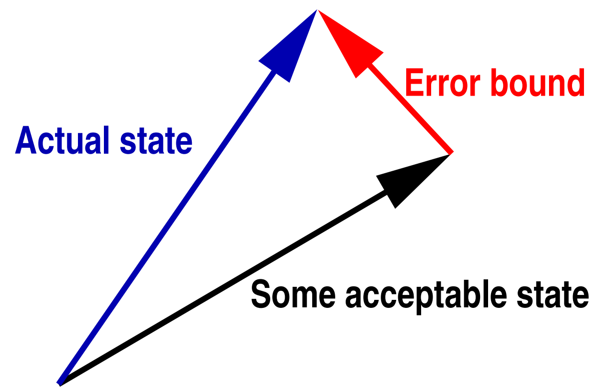

- Wavefunction:

r.h.s. states need not be normalized

$$|\text{output}\rangle = |\text{acceptable}\rangle + |\text{error}\rangle$$

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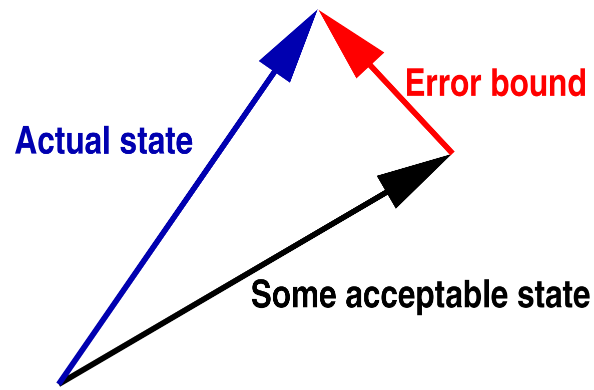

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- Density operator:

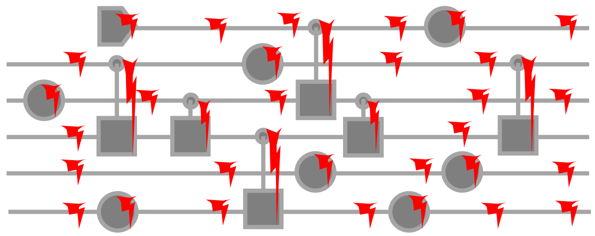
Error operator need not be a state.

$$\rho_{\text{out}} = \rho_{\text{acceptable}} + \rho_{\text{error}}$$

  - Error probability is bounded by $(\text{tr}|\rho_{\text{error}}| + |\text{tr}\rho_{\text{error}}|)/2$.

- Acceptable states need not be unique.

Quasi-independent Noise



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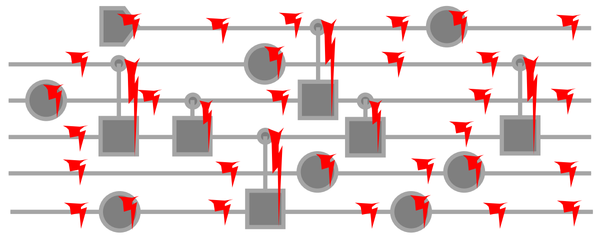

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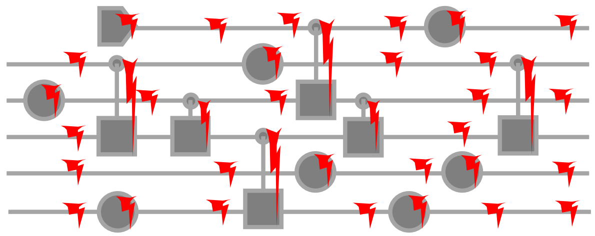

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- The *support* of error term e is
  the set of i such that $E_{e,i}$ is not $\mathbb{I}$.
- *Quasi-independent* with probability p if for each $I \subseteq \{1, \ldots, n\}$
  probability of errors with support $\supseteq I$ is $\leq p^{|I|}$.

Threshold: $p < 10^{-6}$ at worst.

Independent Noise Models

- Systematic models.
  - Only one environment state $|e\rangle_{\mathsf{E}}$.

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$$
\begin{aligned}
|\psi\rangle &\rightarrow |e\rangle_E\, V_n U_n \cdots V_2 U_2\, V_1 U_1 |\psi\rangle \\
&= |e\rangle_E\, (\mathbb{I} + E_n) U_n \cdots (\mathbb{I} + E_2) U_2\, (\mathbb{I} + E_1) U_1 |\psi\rangle
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$$|\psi\rangle \quad \rightarrow \quad (|0\rangle_{\mathsf{E}_n}\mathbb{I} + \ldots + |k_n\rangle_{\mathsf{E}_n} E_{n,k_n}) U_n \cdots$$
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  - Example: Independent thermal relaxation.

Simple Random Models

- Random bit flips $\mathcal{X}(p)$.
  - One qubit: $\sqrt{1-p}|0\rangle_E \mathbb{I} + \sqrt{p}|1\rangle_E \sigma_x$.

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- Depolarizing noise $\mathcal{P}(p)$.
  - One qubit:
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  - Other independent models can be "twirled" into this.

Threshold: $p < 10^{-4}$?

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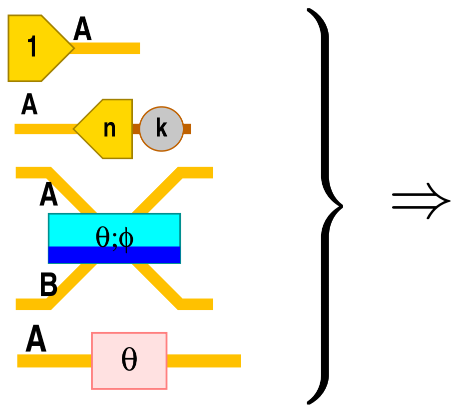
- Erasure/detected loss $\mathcal{L}(p)$.
  - $\sqrt{1-p}|0\rangle_{\mathrm{E}}\mathbb{I} + \sqrt{p}|1\rangle_{\mathrm{E}}\mathcal{P}(1)$.
  - E is accessible.

Threshold: $p < 10^{-2}$ at worst.

eLOQC Designer Errors

- eLOQC: Efficient linear optics quantum computation.



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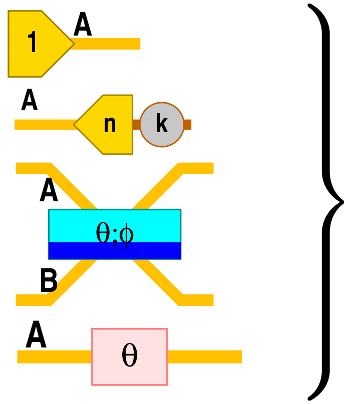

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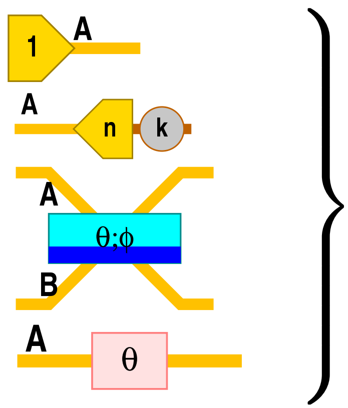

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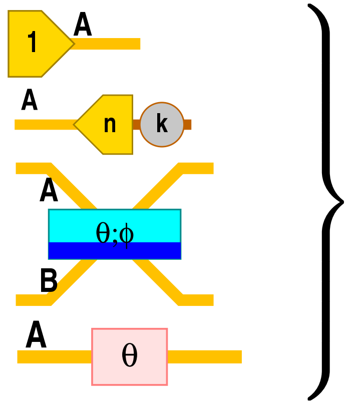

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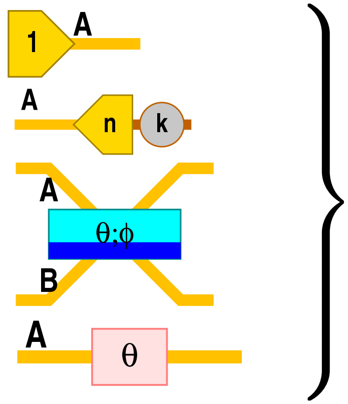

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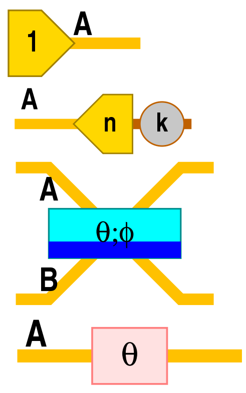

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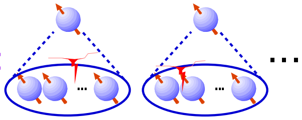
  - Elementary gates succeed, but...
  - ...with probability f: qubit measured.

Threshold: $f < .5$ at worst.

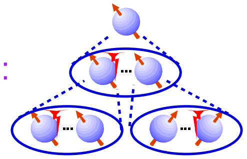
Knill&Laflamme&Milburn 2001 [13]

Methods for Scalability

- Error-correcting subsystems.

Go to: 

- Concatenation.

Go to: 

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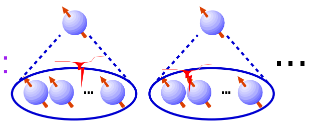
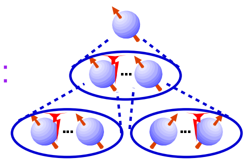
- Error-correcting subsystems. Go to: 

- Concatenation. Go to: 

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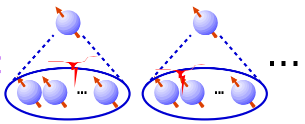
Go to: 

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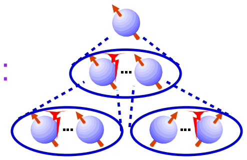
Go to: 

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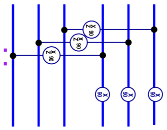
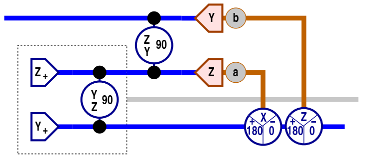
- Transversally encoded operations.

Go to: 

- Measurement or Operation

$=$ state preparation $+$ teleportation.

Go to: 

- Robust error detection and recovery.

Jump to: Conclusion

Methods for Scalability

- Error-correcting subsystems.

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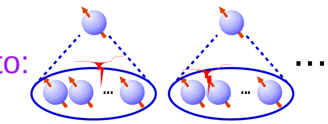

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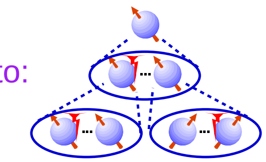

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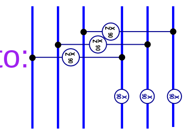

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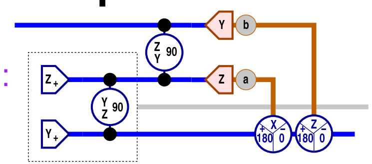

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$\Rightarrow$ accuracy thresholds.

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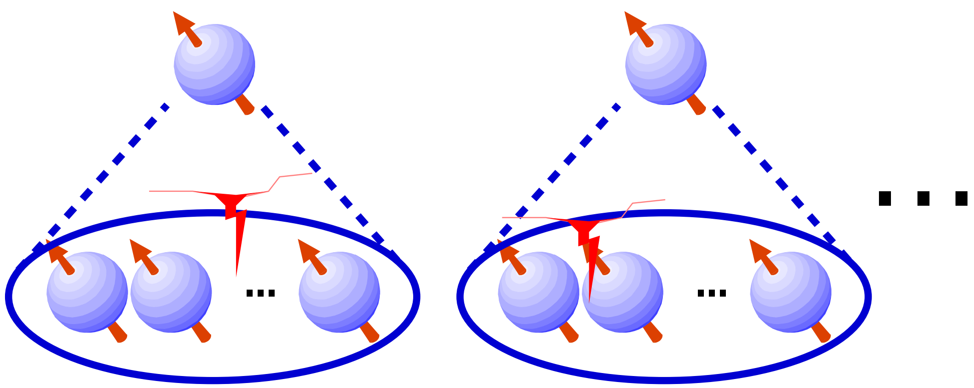
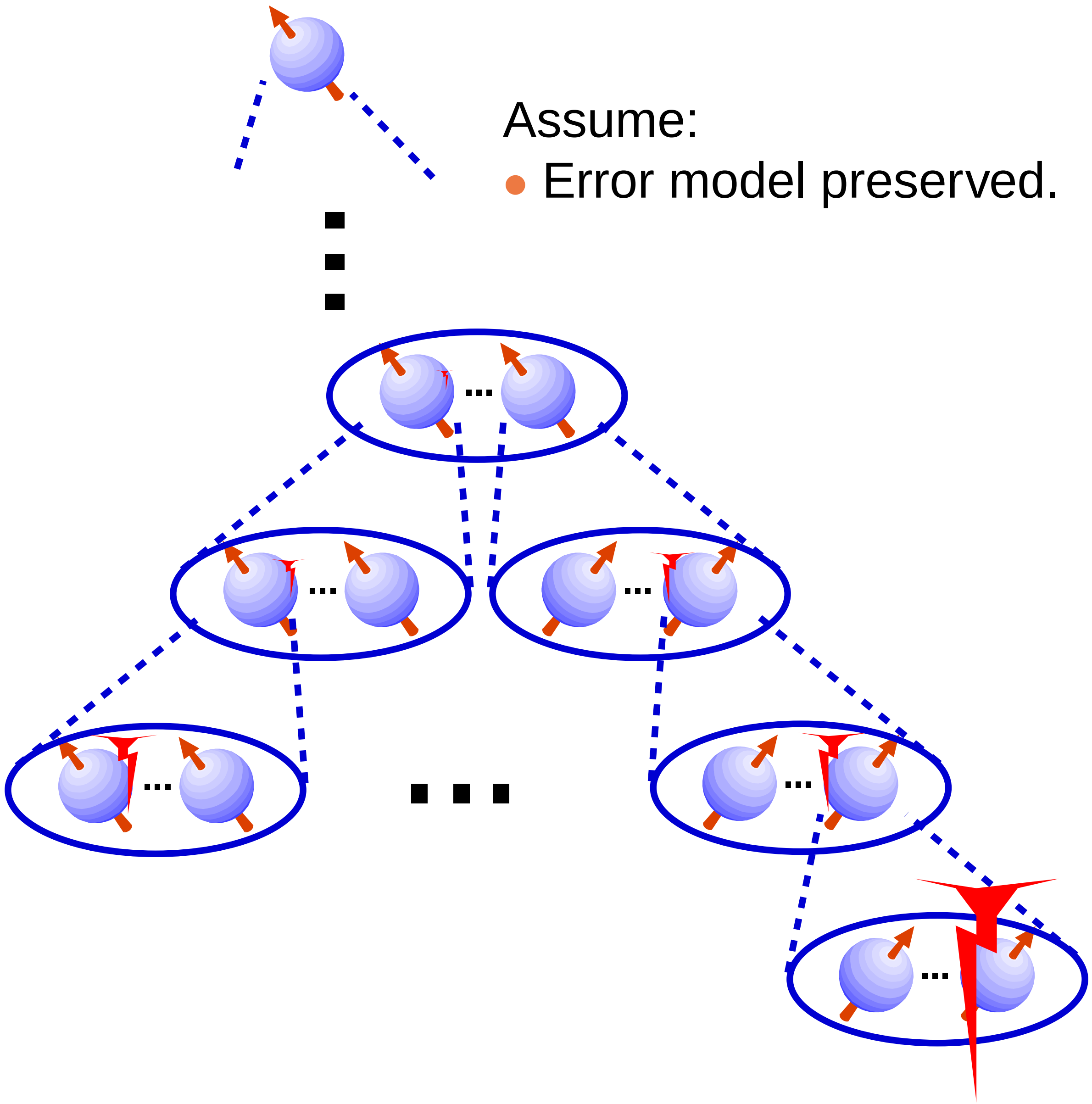
Error-correcting Subsystems



Back to: Methods for Scalability

Concatenation



Assume:

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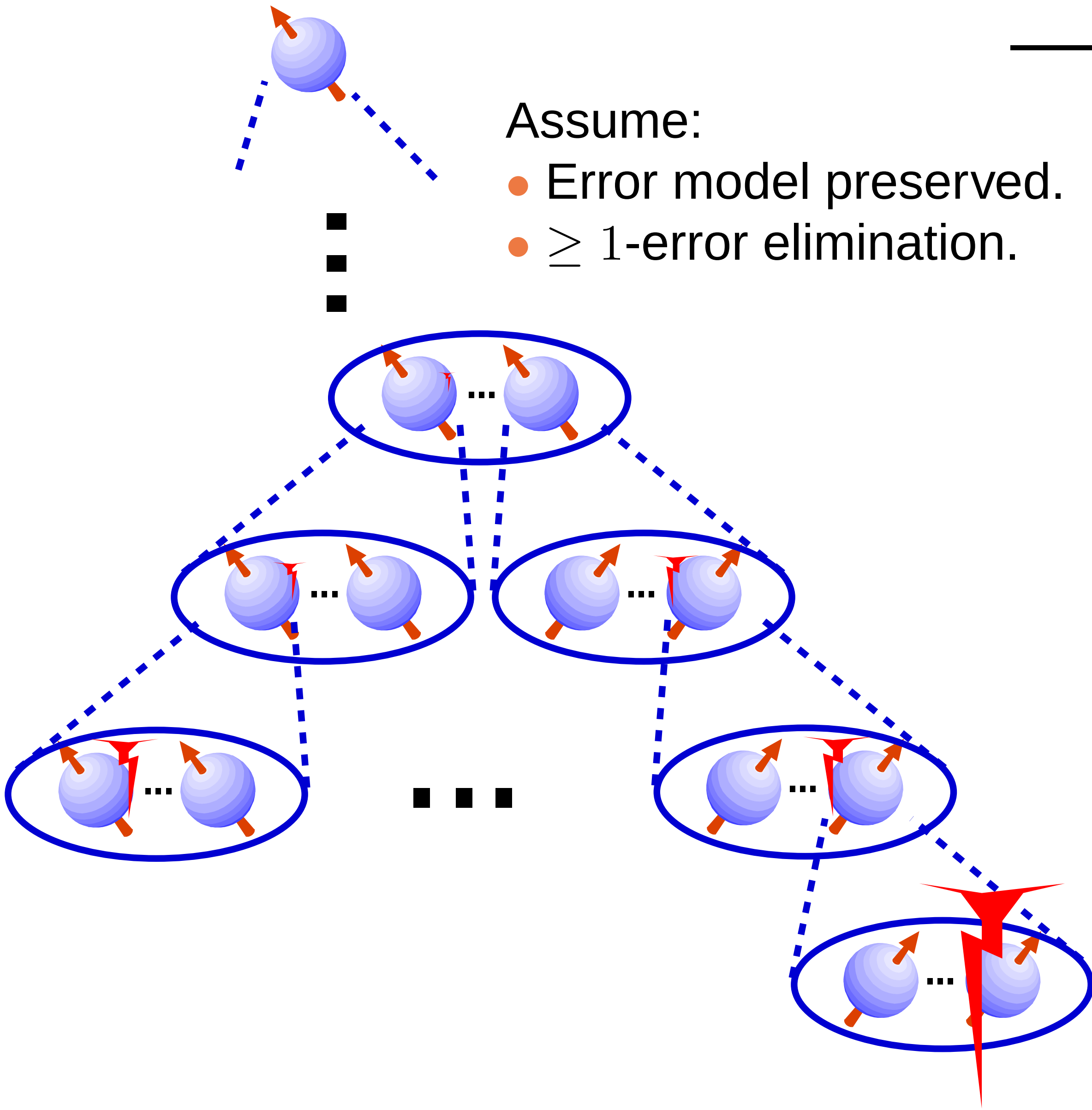

Assume:

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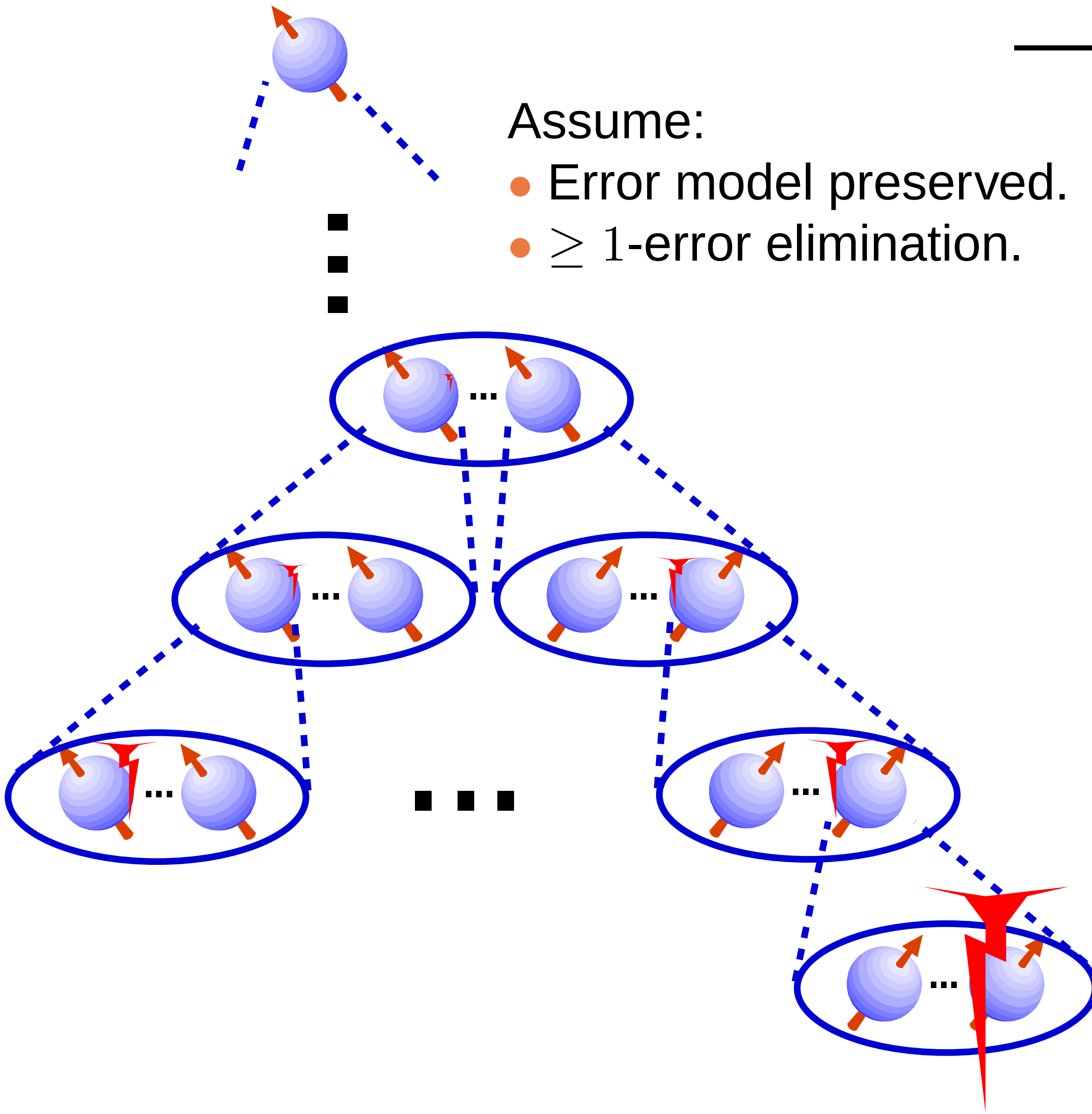
- ≥ 1-error elimination.

Level	Error rate
1	p

Concatenation

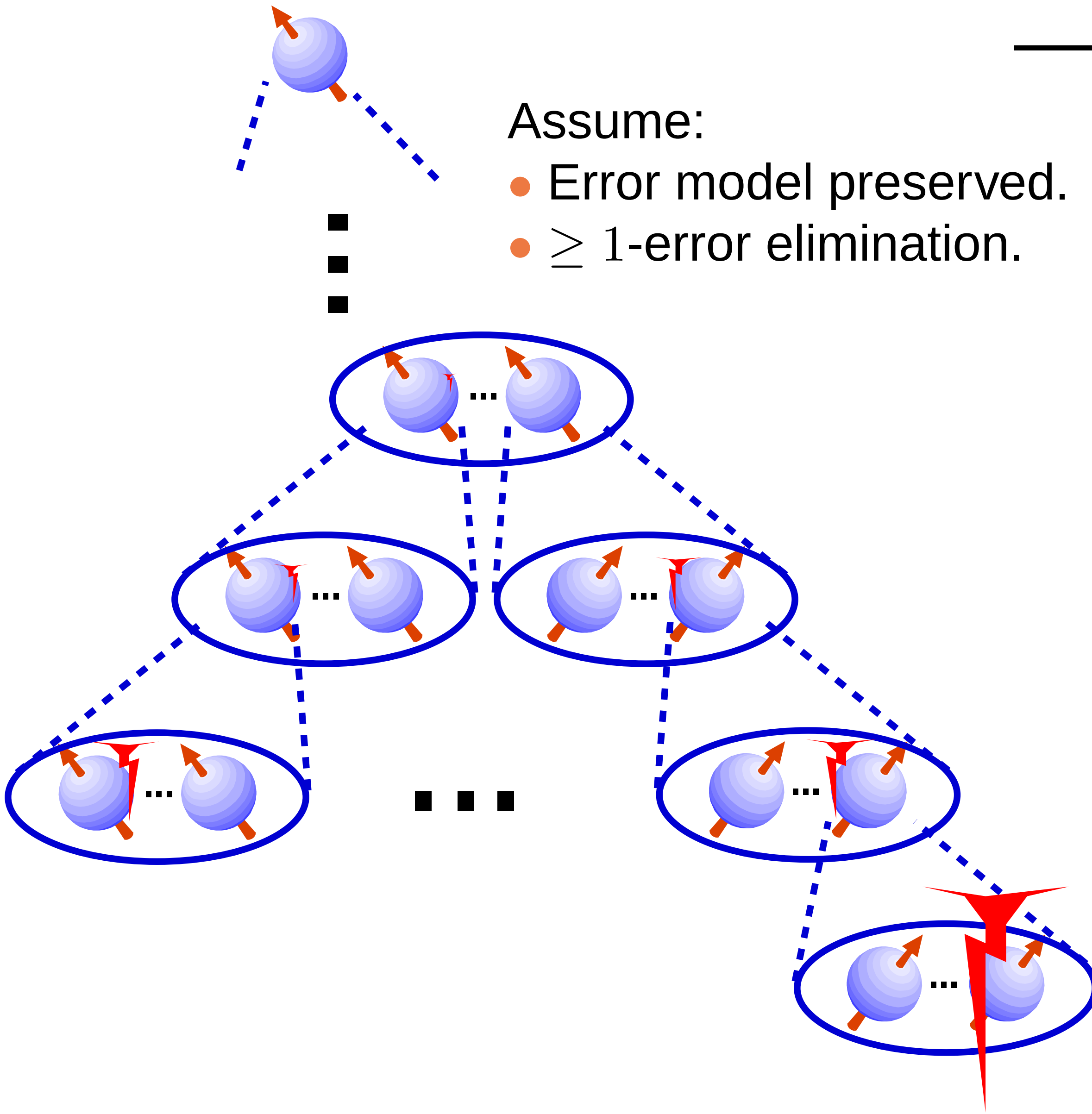


Assume:

- Error model preserved.
- ≥ 1-error elimination.

Level	Error rate
2	$p_2 \leq Cp^2$
1	p

Concatenation



Assume:
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Level	Error rate
$\vdots$	
4	$\begin{cases} p_4 \leq C(C^3 p^{2^2})^2 \\ \quad = C^{2^3-1} p^{2^3} \end{cases}$
3	$\begin{cases} p_3 \leq C(Cp^2)^2 \\ \quad = C^{2^2-1} p^{2^2} \end{cases}$
2	$p_2 \leq Cp^2$
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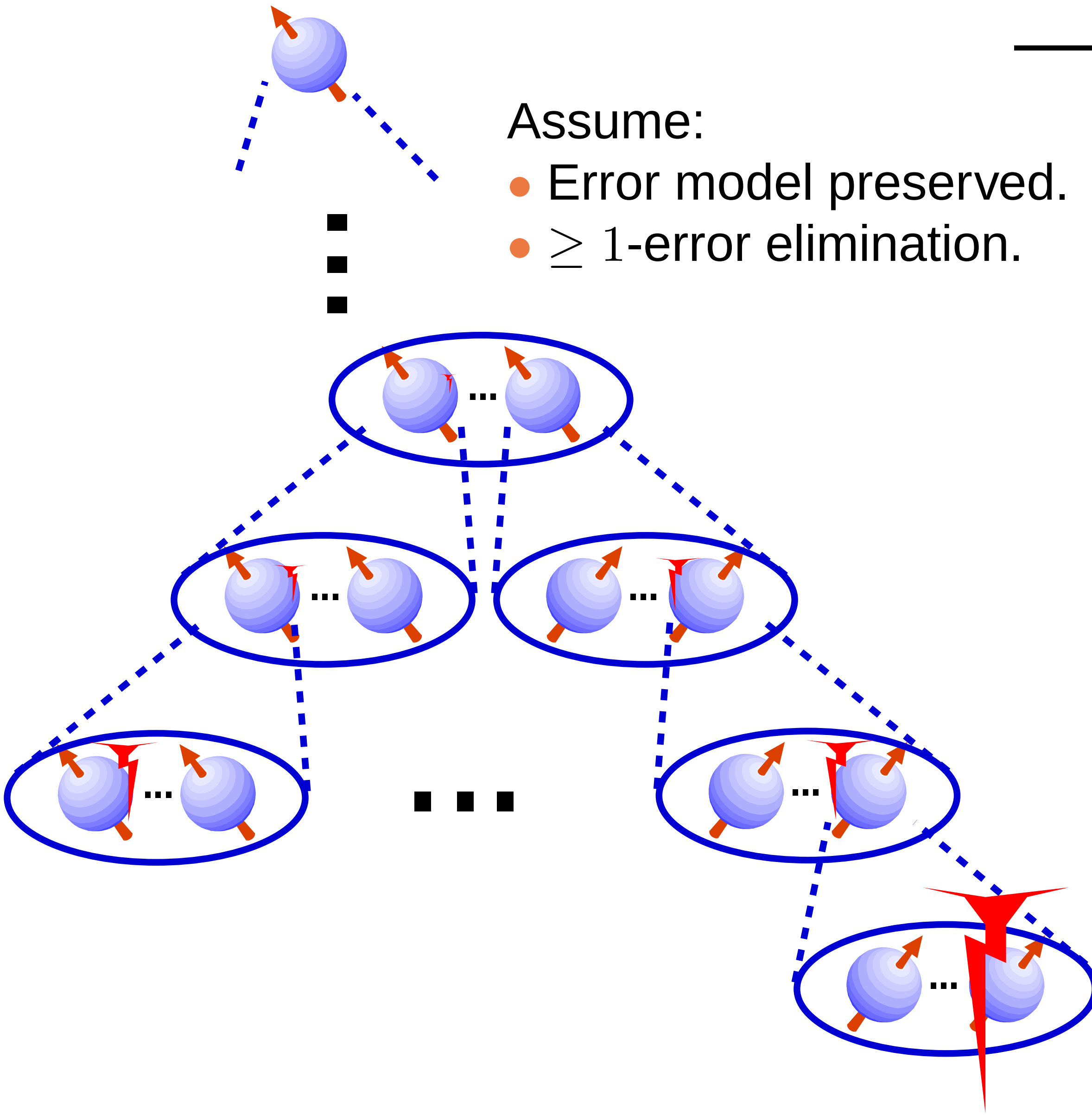
Concatenation



Assume:

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Level	Error rate
k	$\leq C^{2^k-1}p^{2^k}$
⋮	
4	$p_4 \leq C(C^3p^{2^2})^2 = C^{2^3-1}p^{2^3}$
3	$p_3 \leq C(Cp^2)^2 = C^{2^2-1}p^{2^2}$
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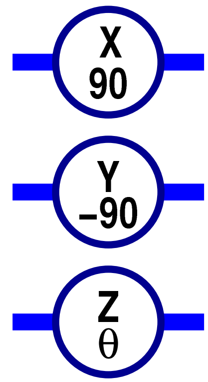
Back to: Methods for Scalability

Pauli Product Operator Rotations

- One qubit rotations and gates:
  - $X_{90^\circ} = e^{-i\sigma_x \pi/4}$
  - $Y_{-90^\circ} = e^{i\sigma_y \pi/4}$
  - $Z_\theta = e^{-i\sigma_z \theta/2}$



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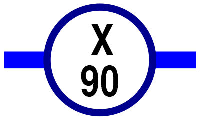

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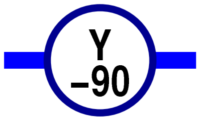
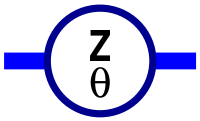
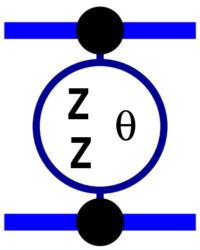


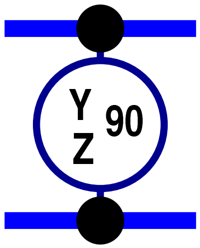
  - $Z_\theta = e^{-i\sigma_z \theta/2}$



- Two qubit rotations and gates:
  - $(Z^{(1)} Z^{(2)})_\theta = e^{-i{\sigma_z}^{(1)}{\sigma_z}^{(2)}\theta/2}$



  - $(Y^{(1)} Z^{(2)})_{90^\circ} = e^{-i{\sigma_y}^{(1)}{\sigma_z}^{(2)}\pi/2}$



Rotation Equivalences

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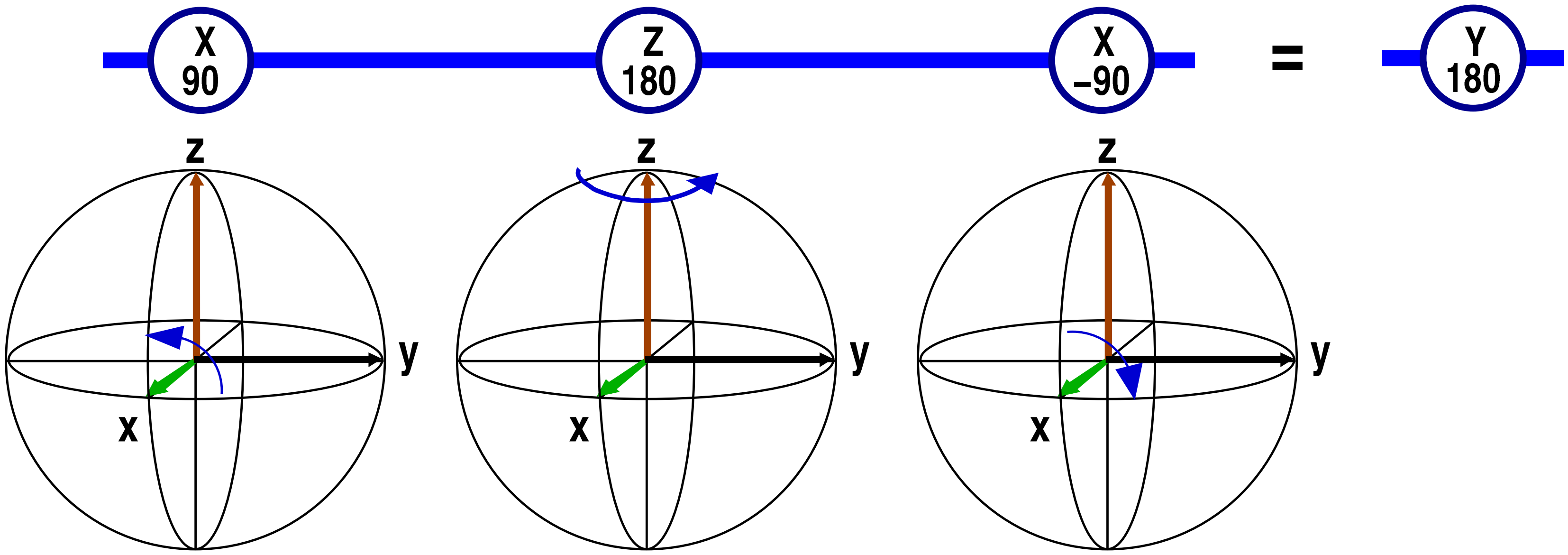

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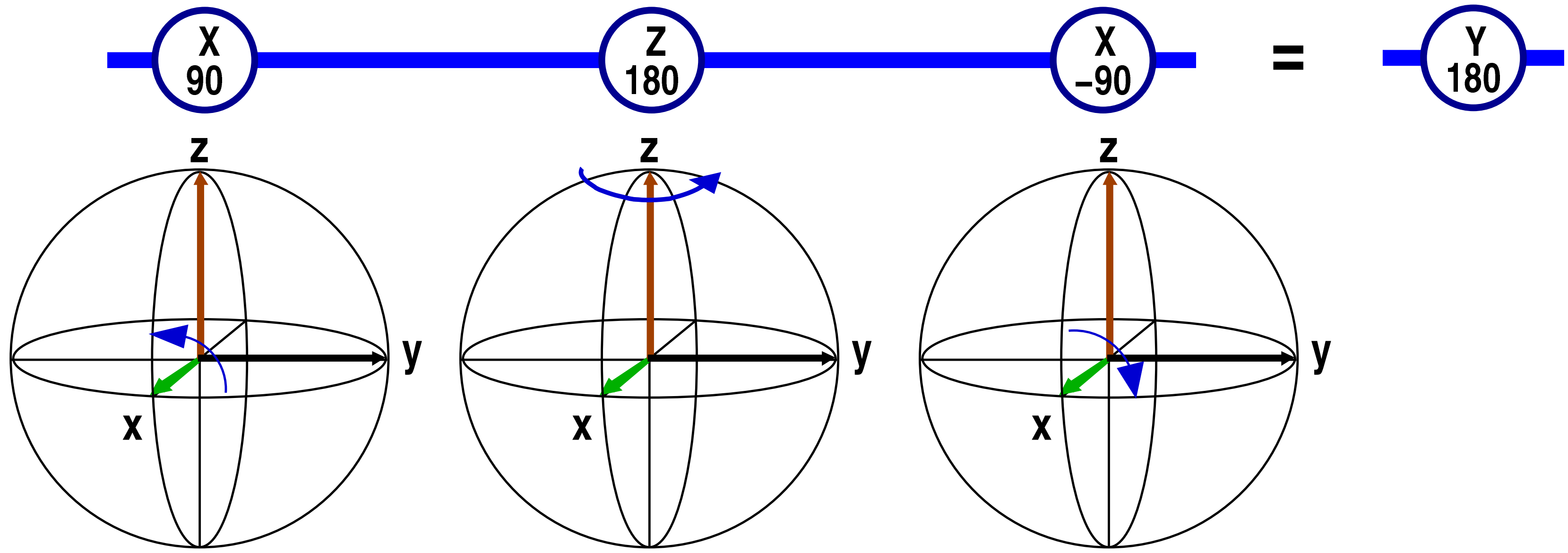
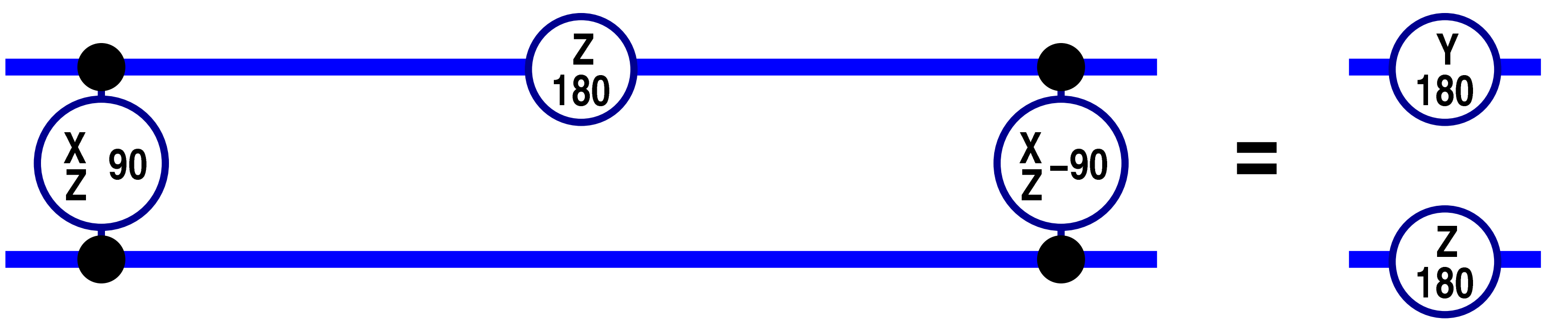
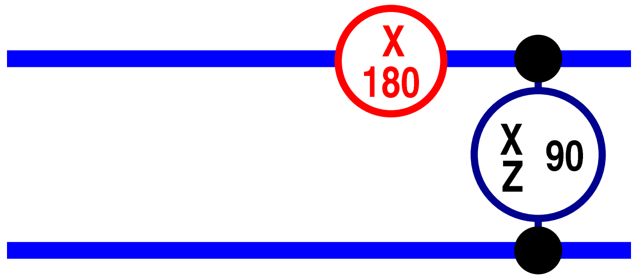




Error Propagation



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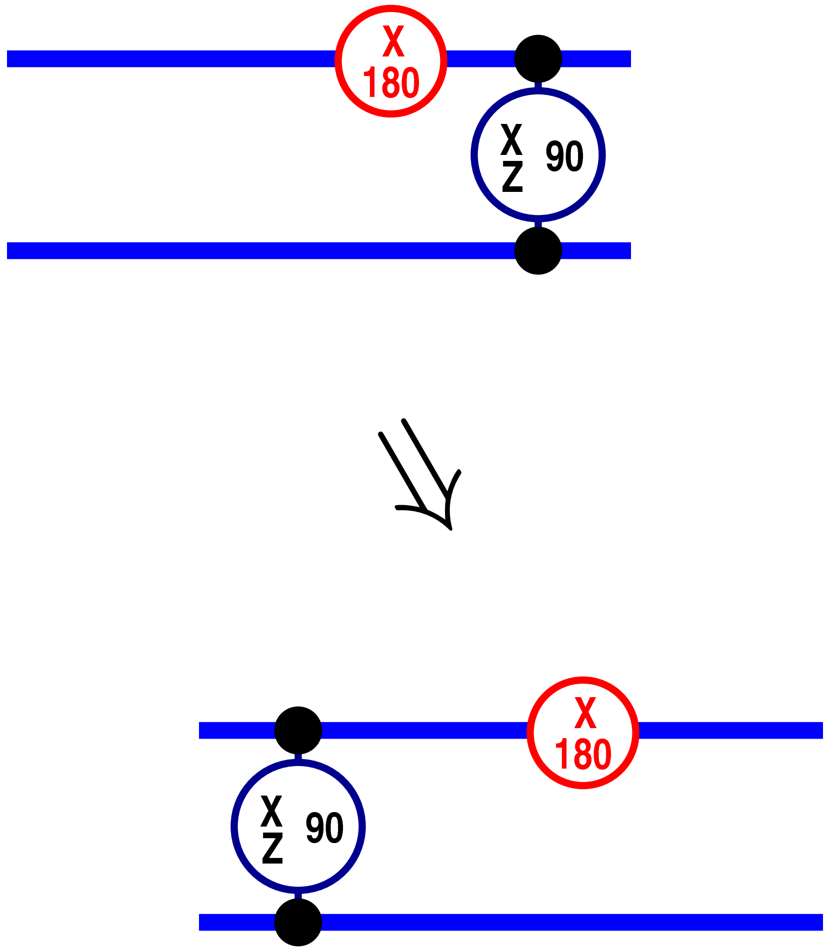

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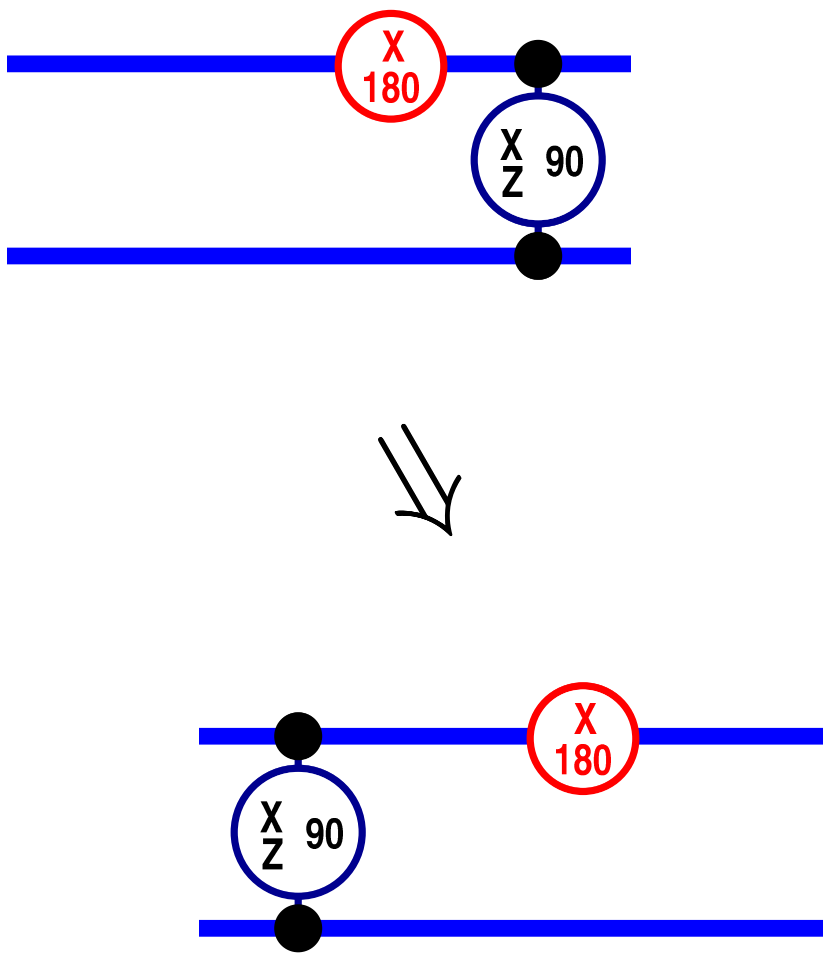

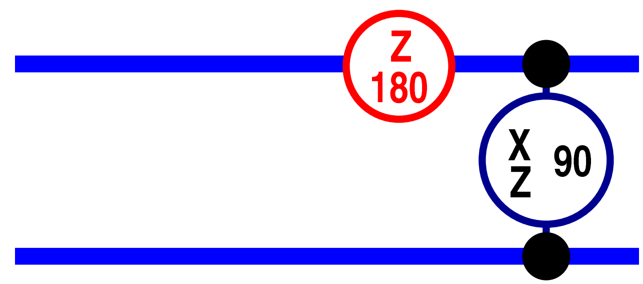

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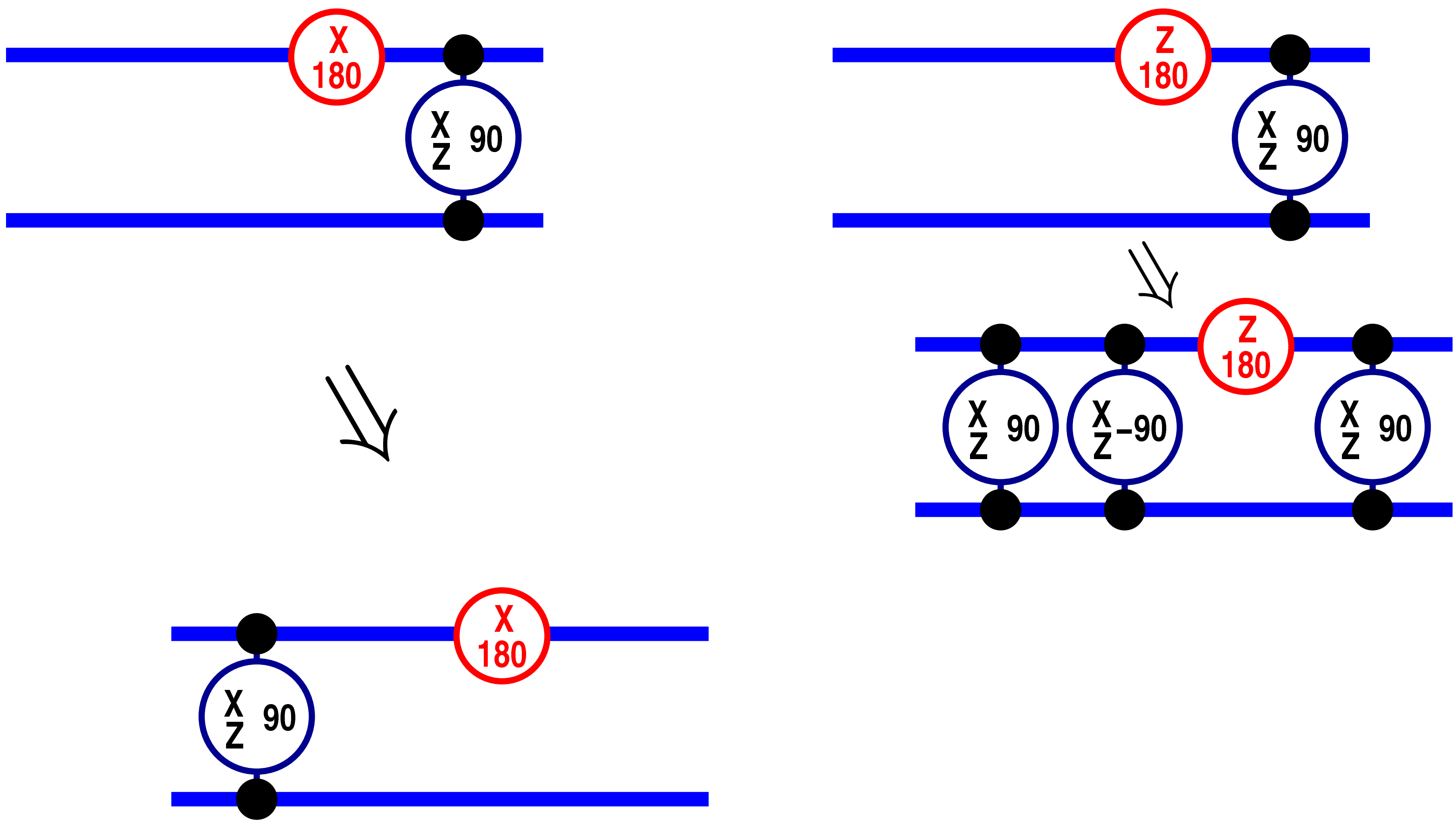

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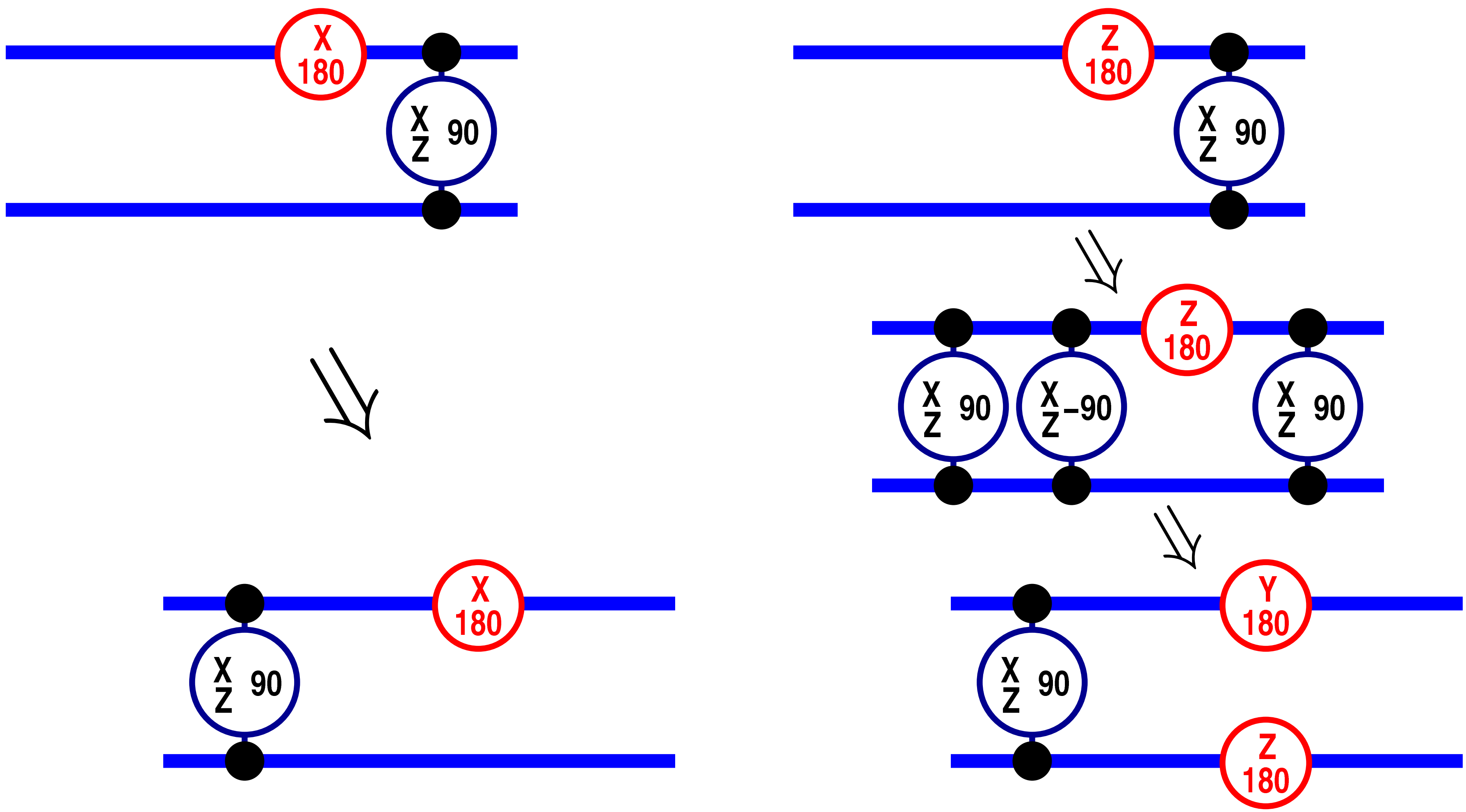
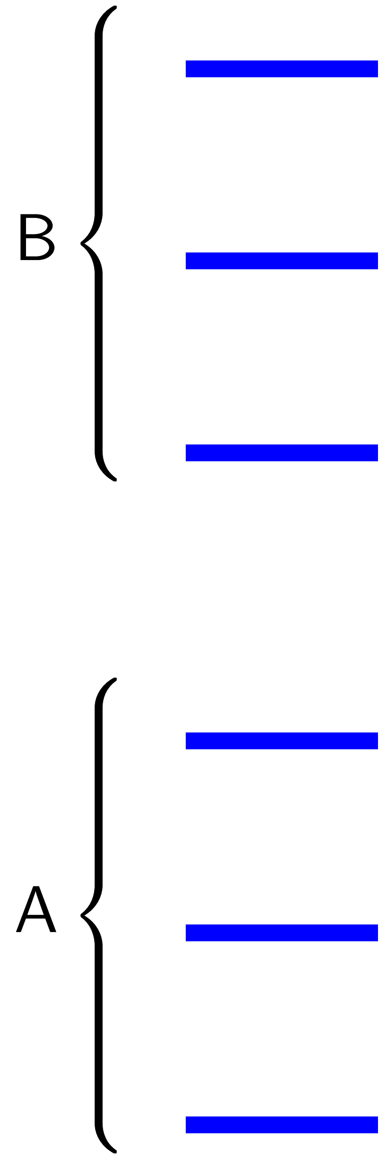


Transversally Encoding Operations

Repetition code for bit flips.

Stabilizer: $\langle \, ZZI, IZZ \, \rangle$

$|0\rangle_L = |000\rangle$, $|1\rangle_L = |111\rangle$



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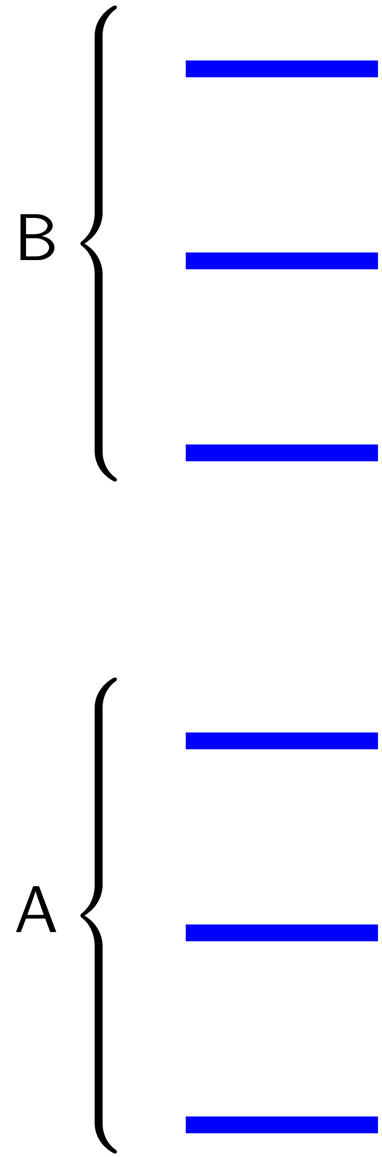

- "Encoded" **cnot** ...

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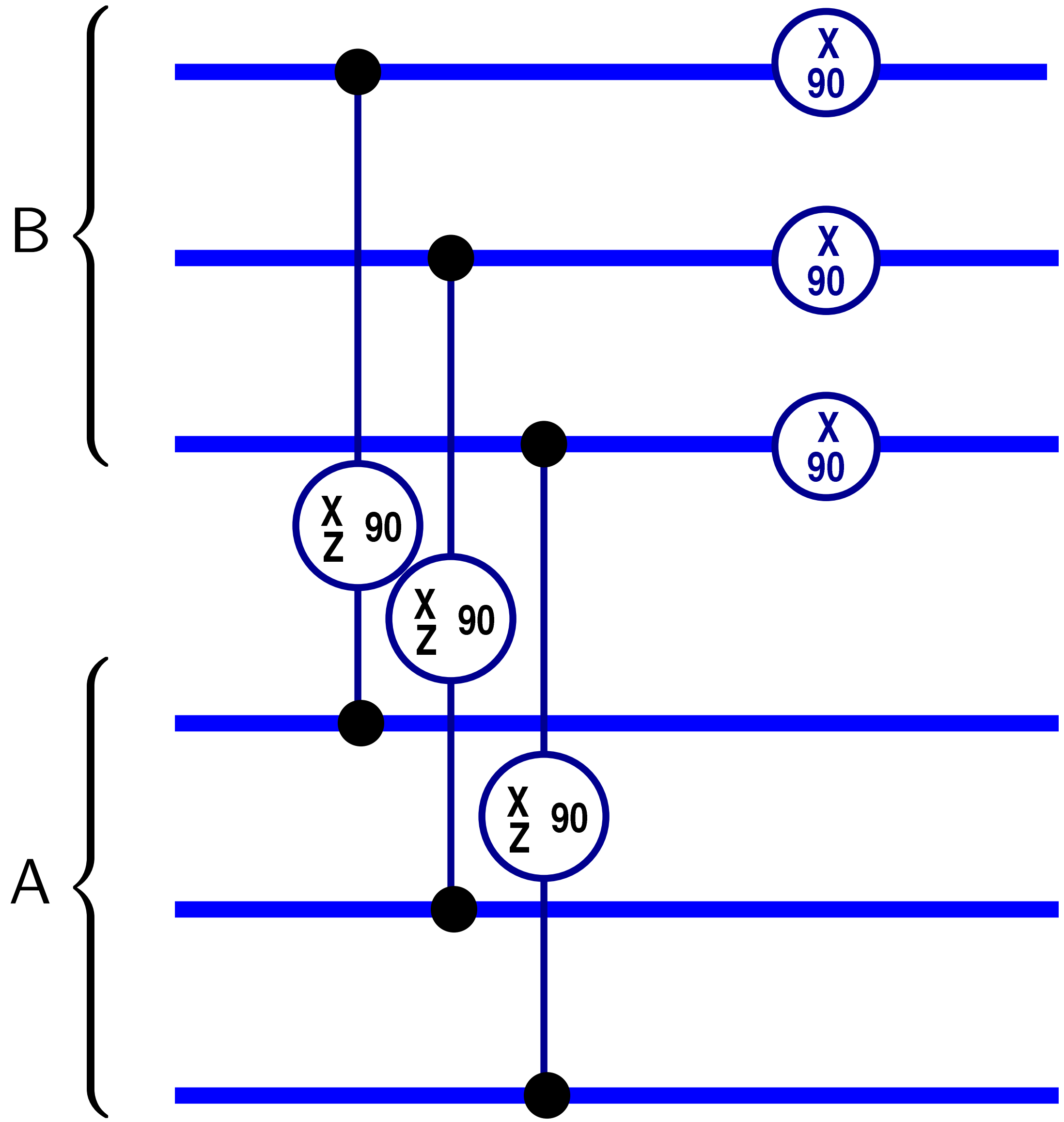

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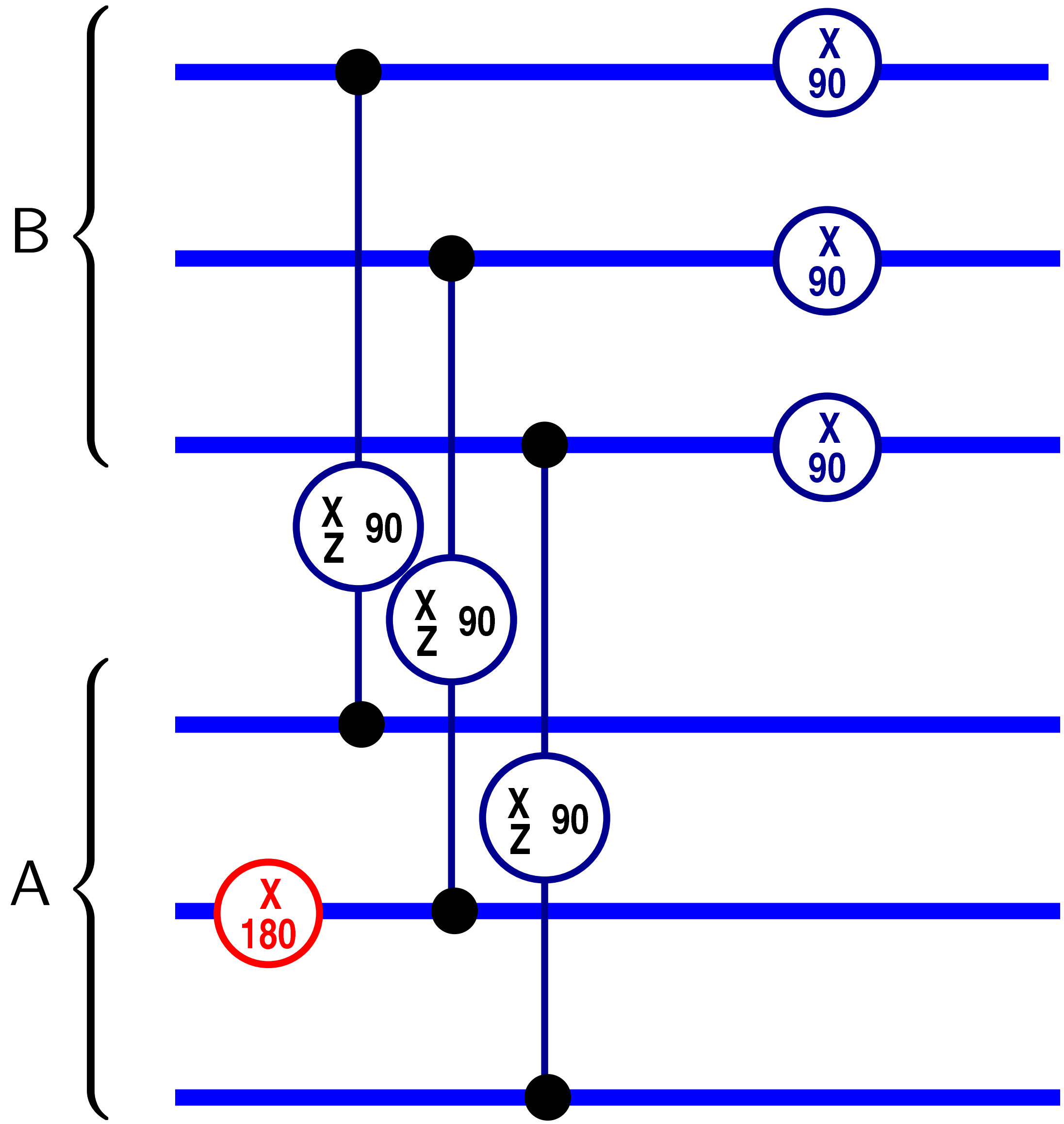

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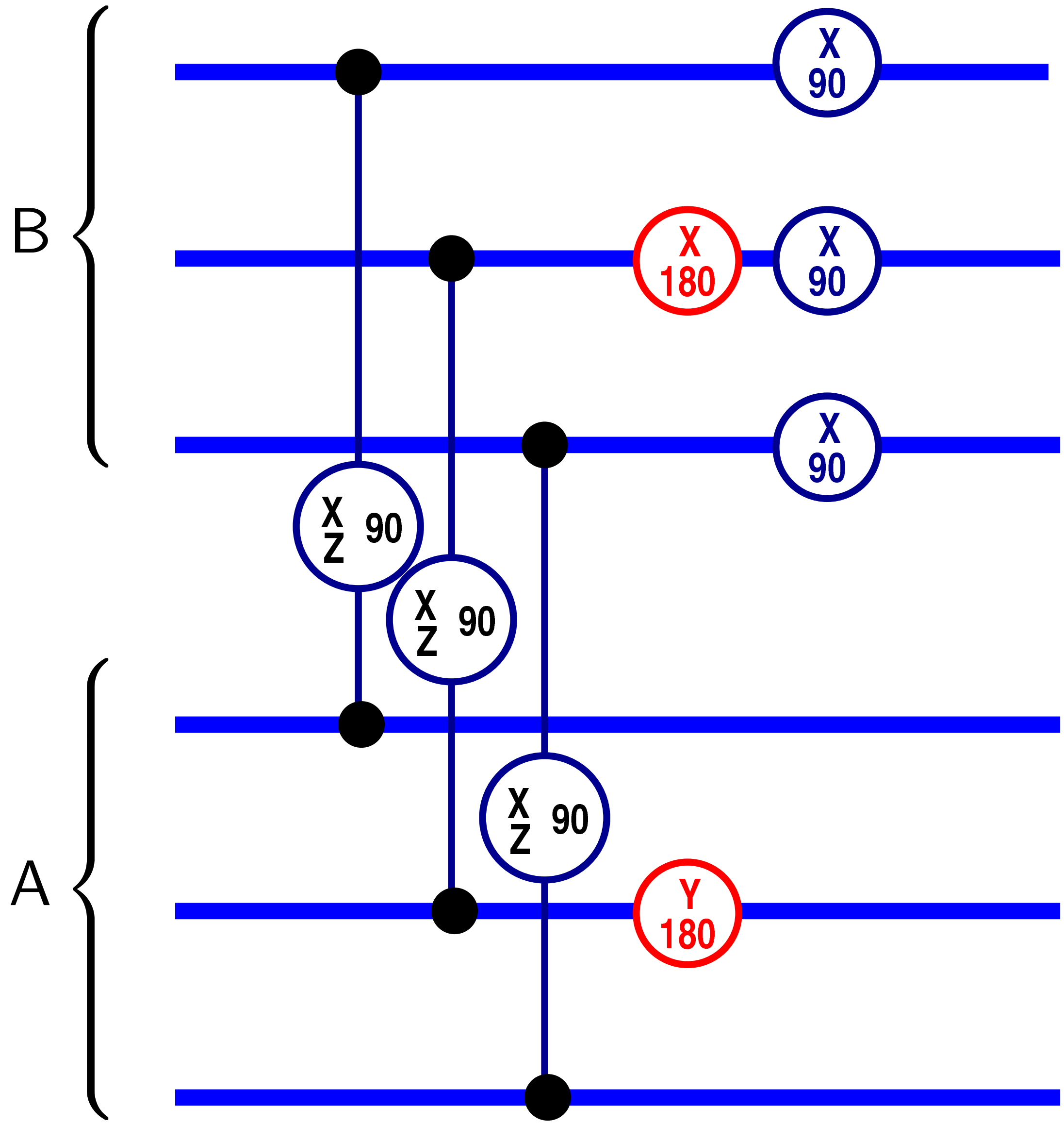

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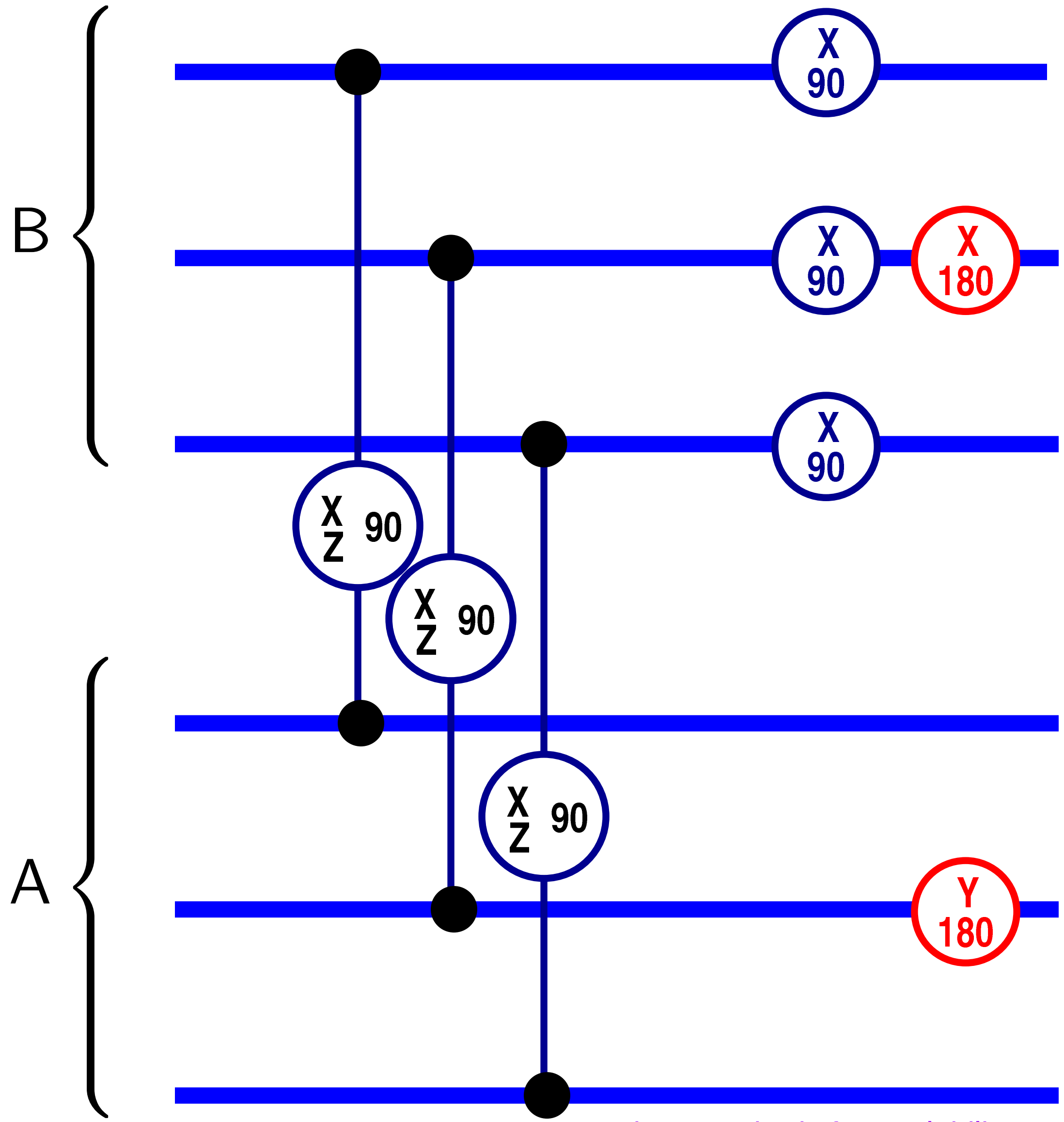
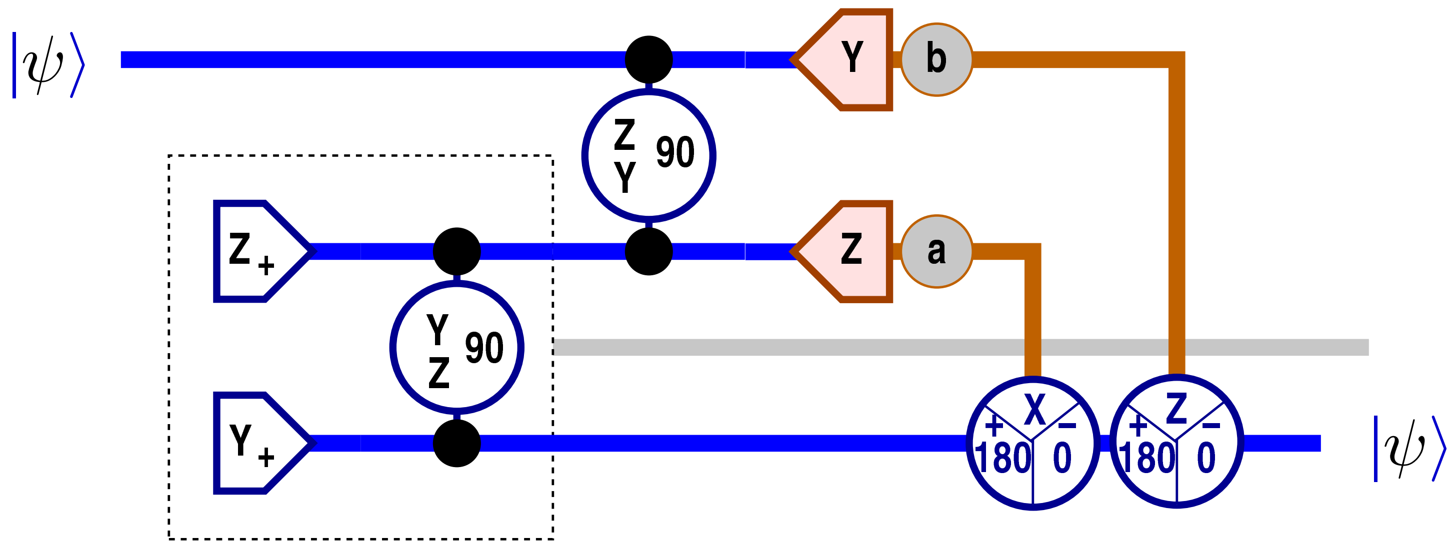


Back to: Methods for Scalability

Teleportation



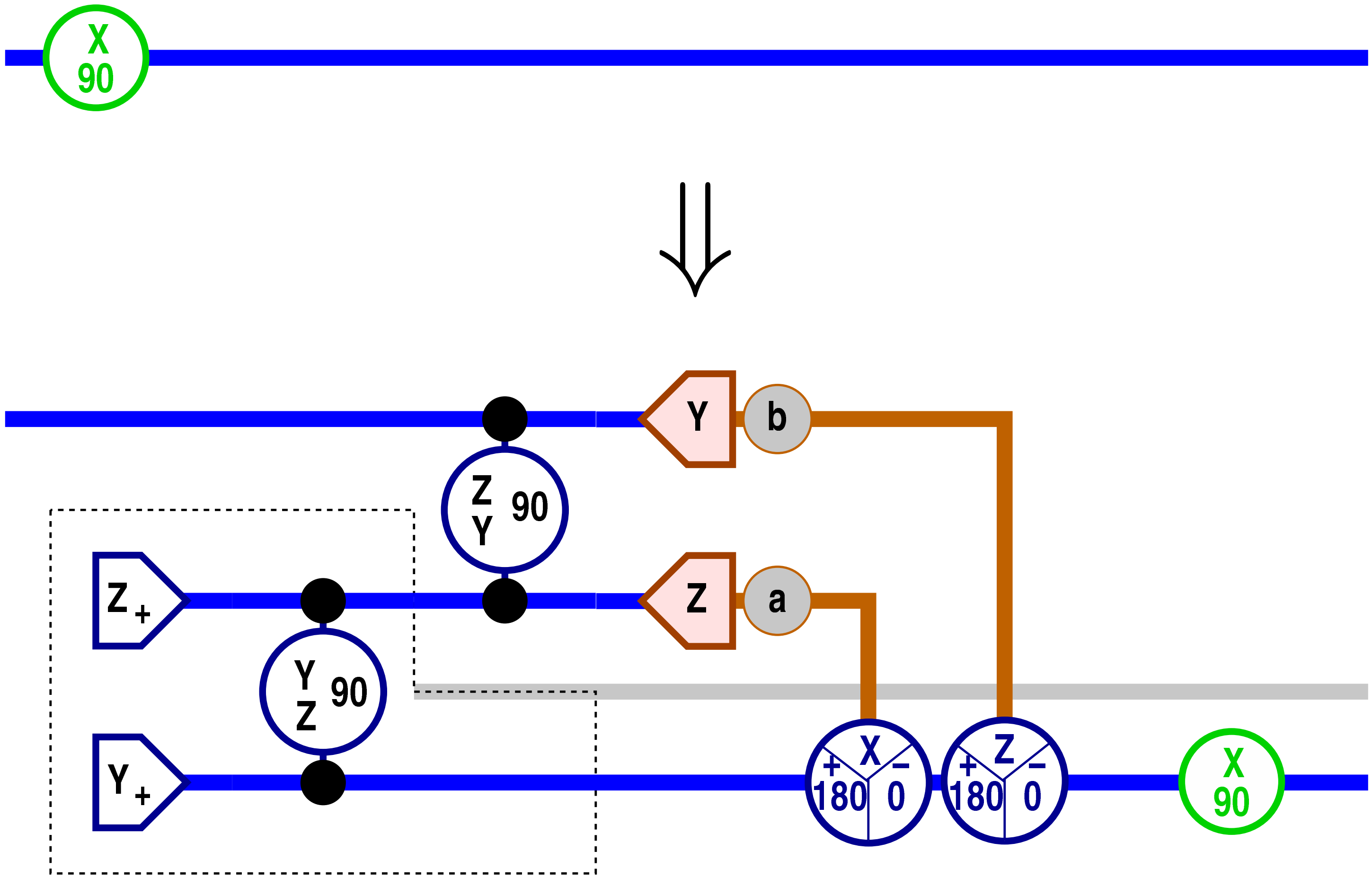
$$\text{a} = +1 \Rightarrow X_{180^\circ} \quad , \quad \text{a} = -1 \Rightarrow \mathbb{I}$$

and

$$\text{b} = +1 \Rightarrow Z_{180^\circ} \quad , \quad \text{b} = -1 \Rightarrow \mathbb{I}$$

Operation $=$ Preparation $+$ Teleportation I

Operation = Preparation + Teleportation I



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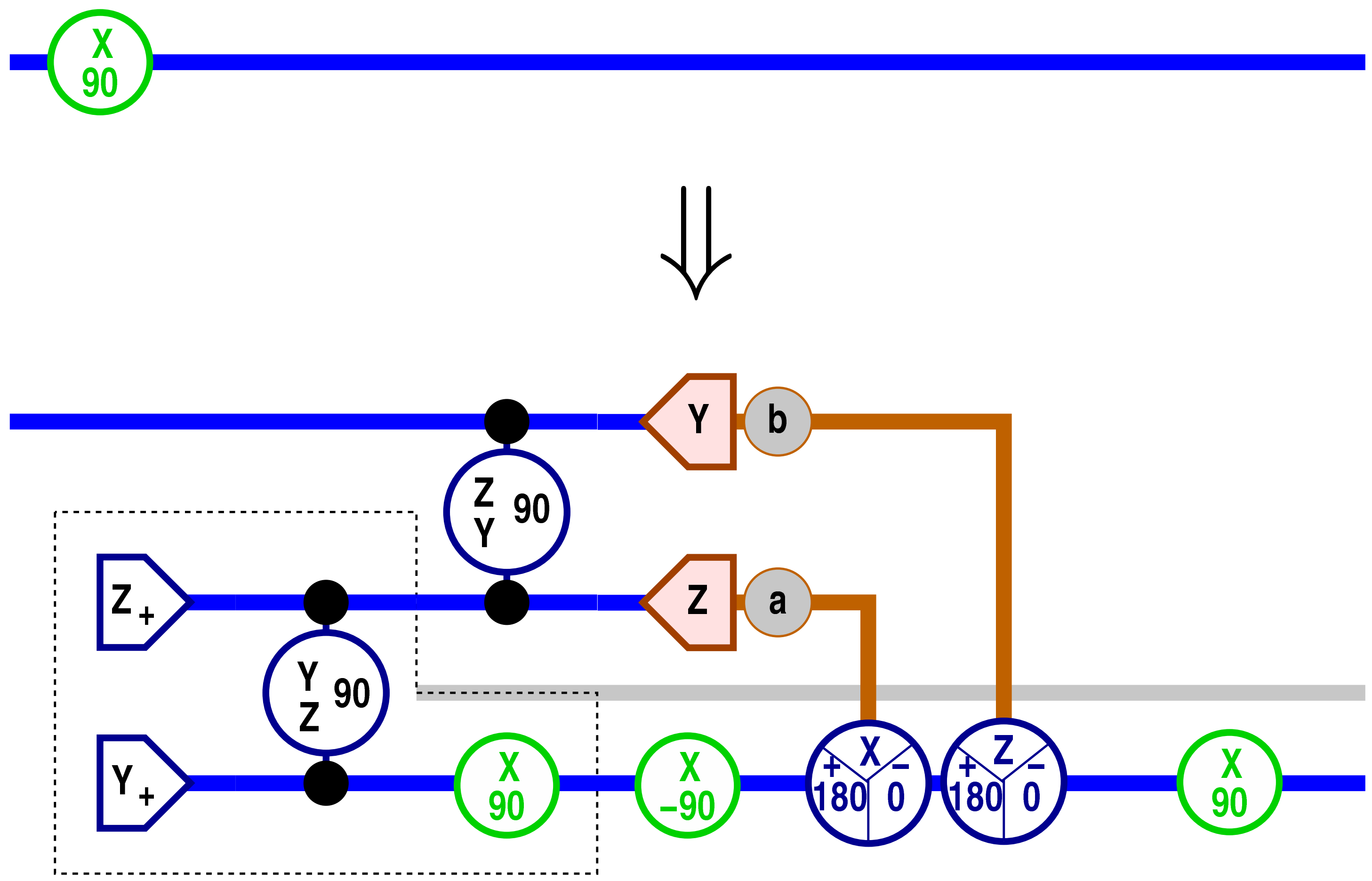

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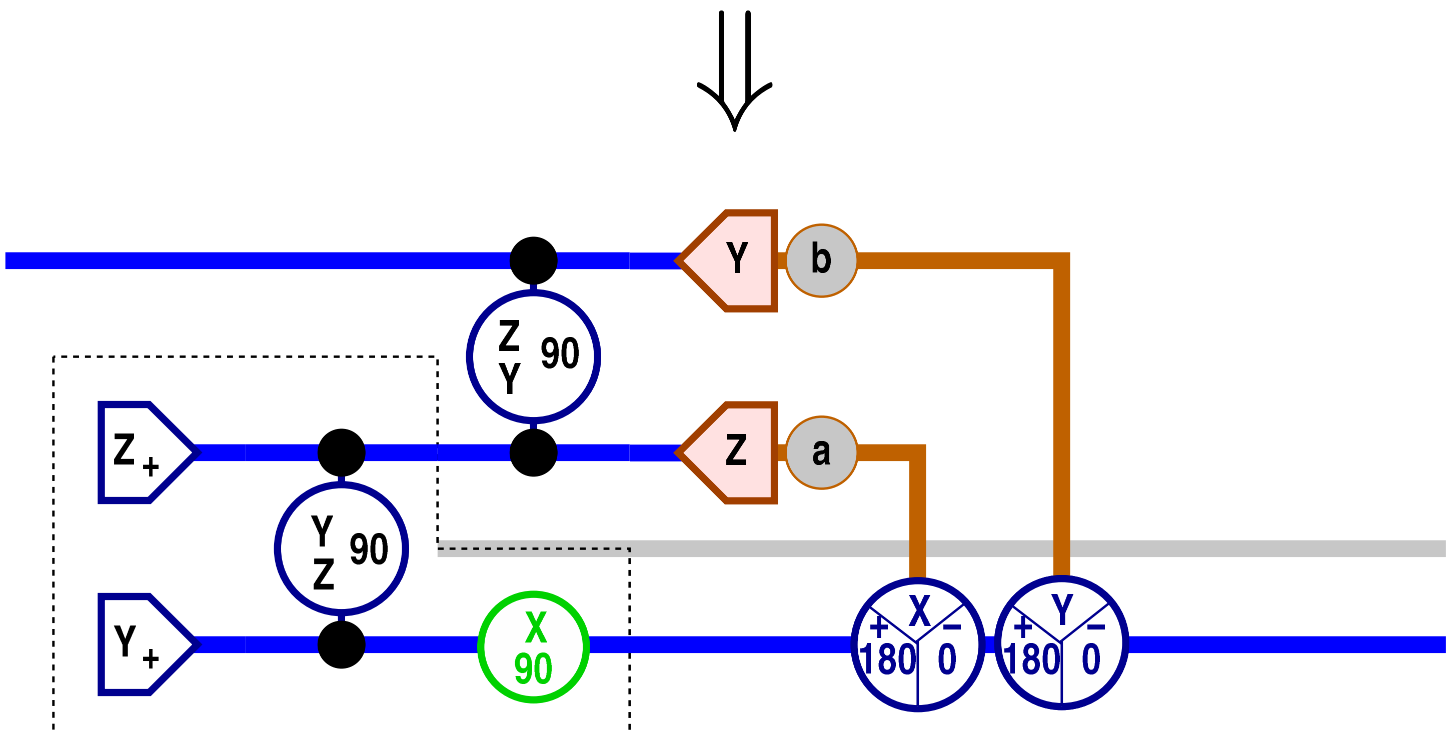
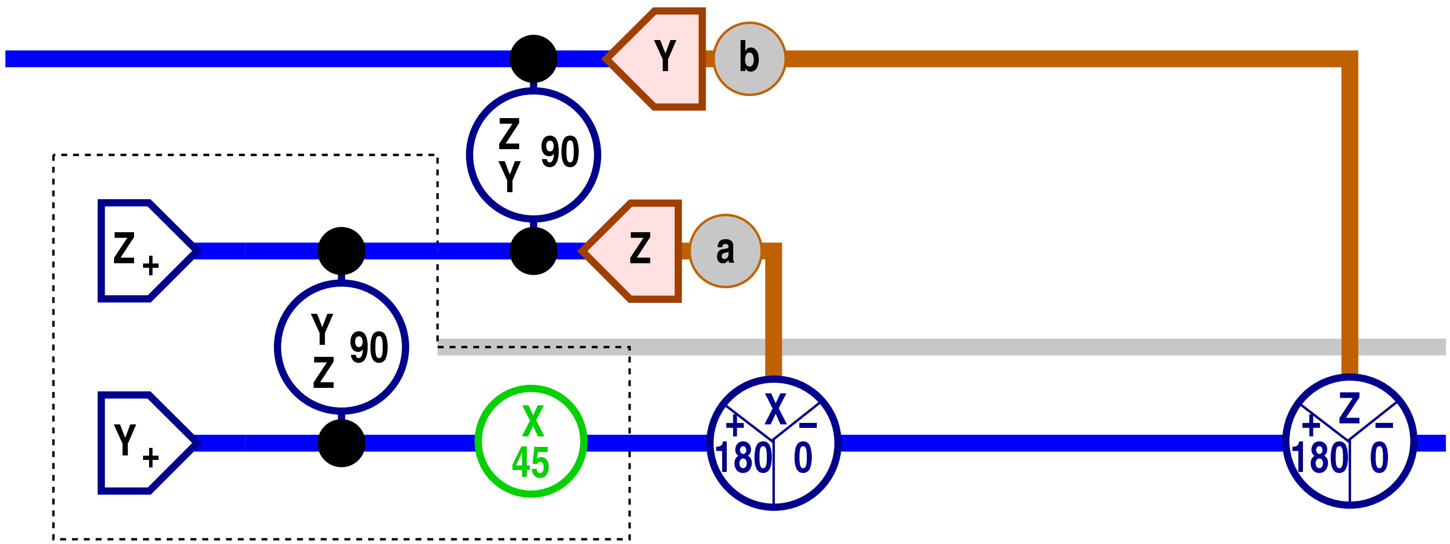


Gottesman&Chuang 1999 [10]

To: Advantages of Teleportation

Operation = Preparation + Teleportation II



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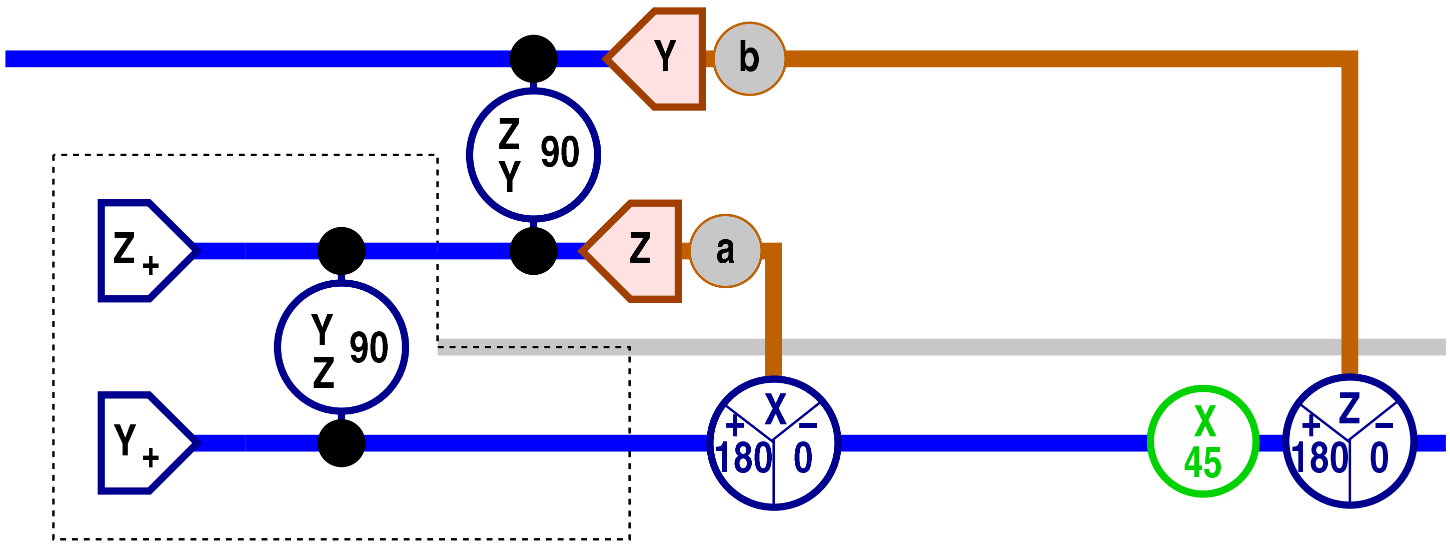

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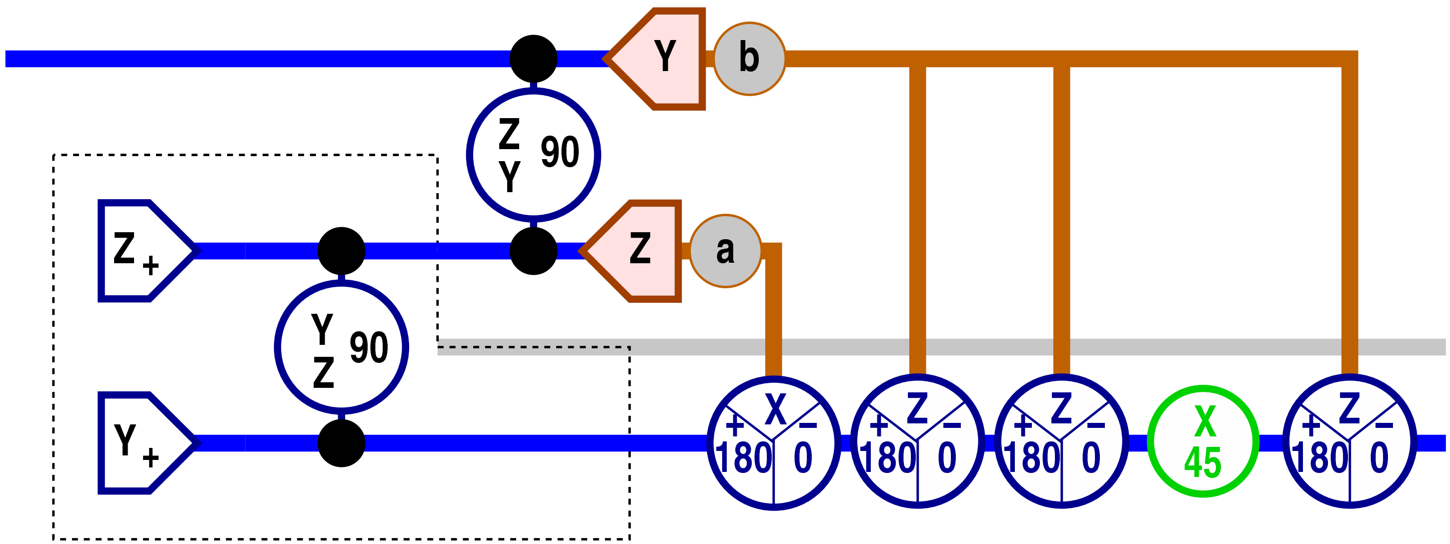

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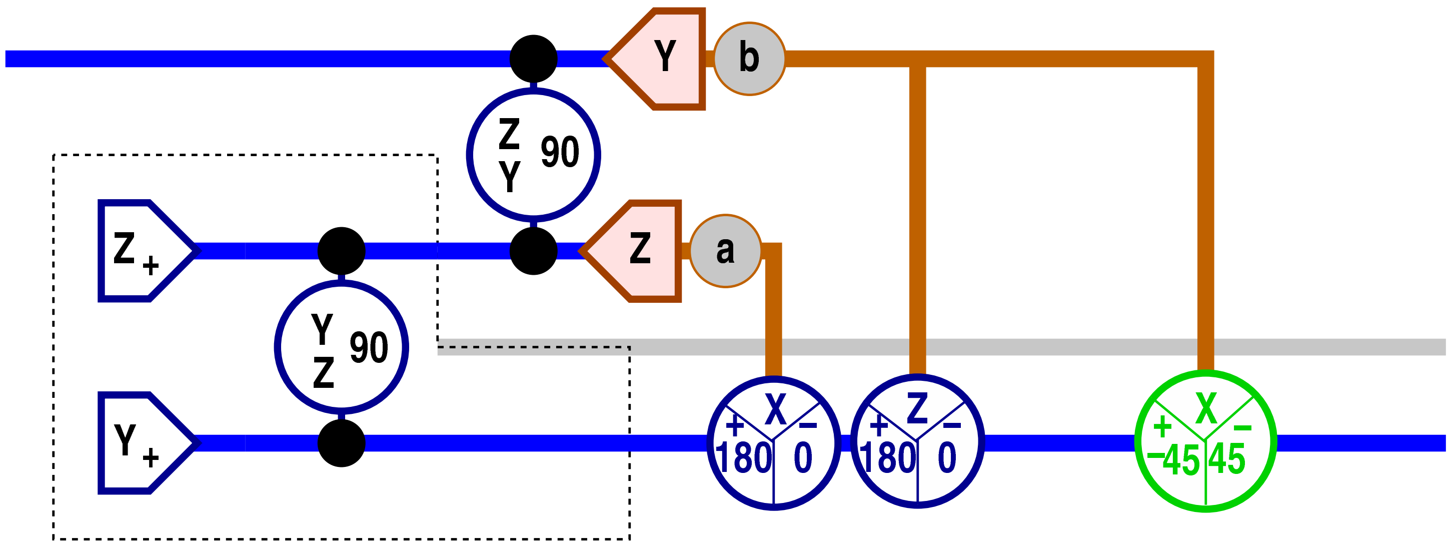
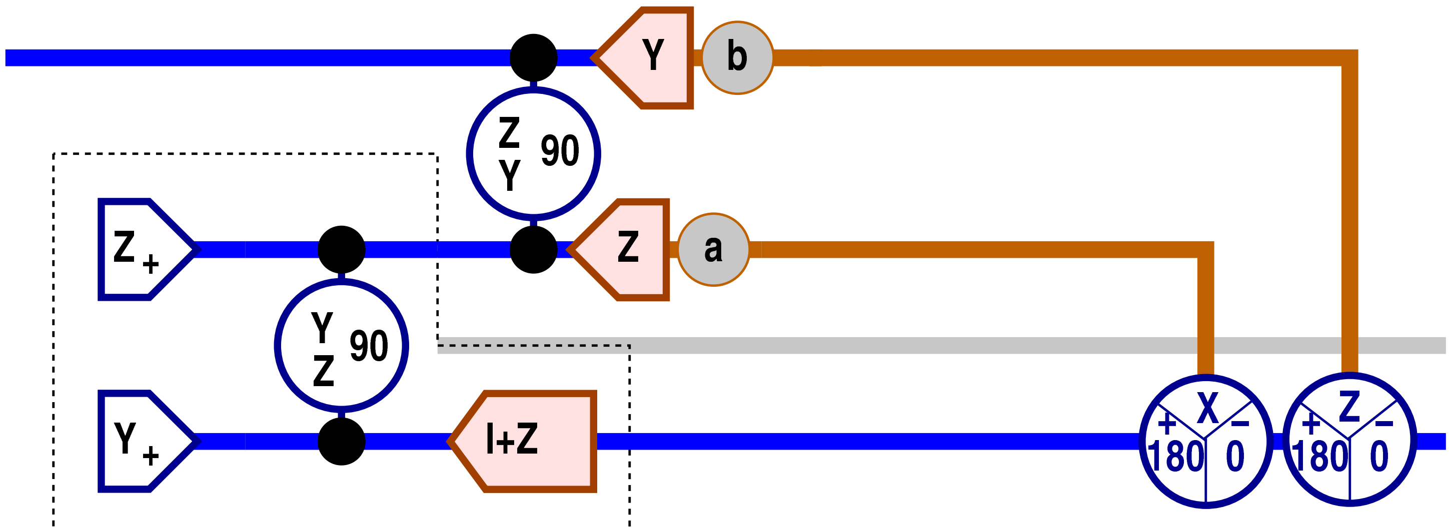


To: Advantages of Teleportation

Measurement = Preparation + Teleportation



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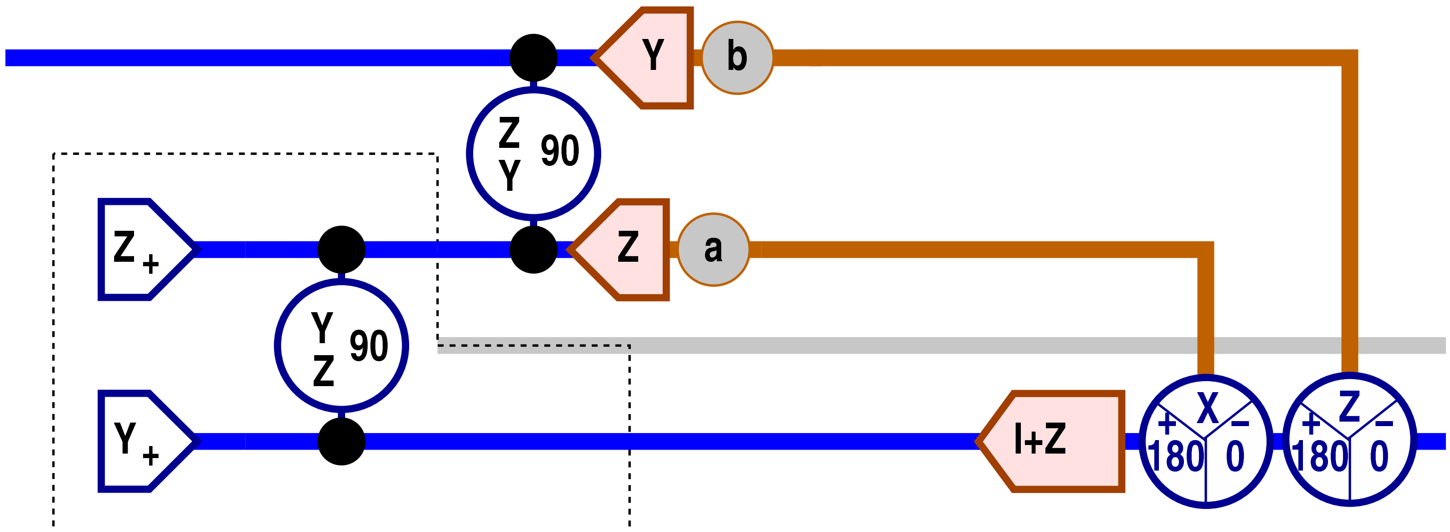

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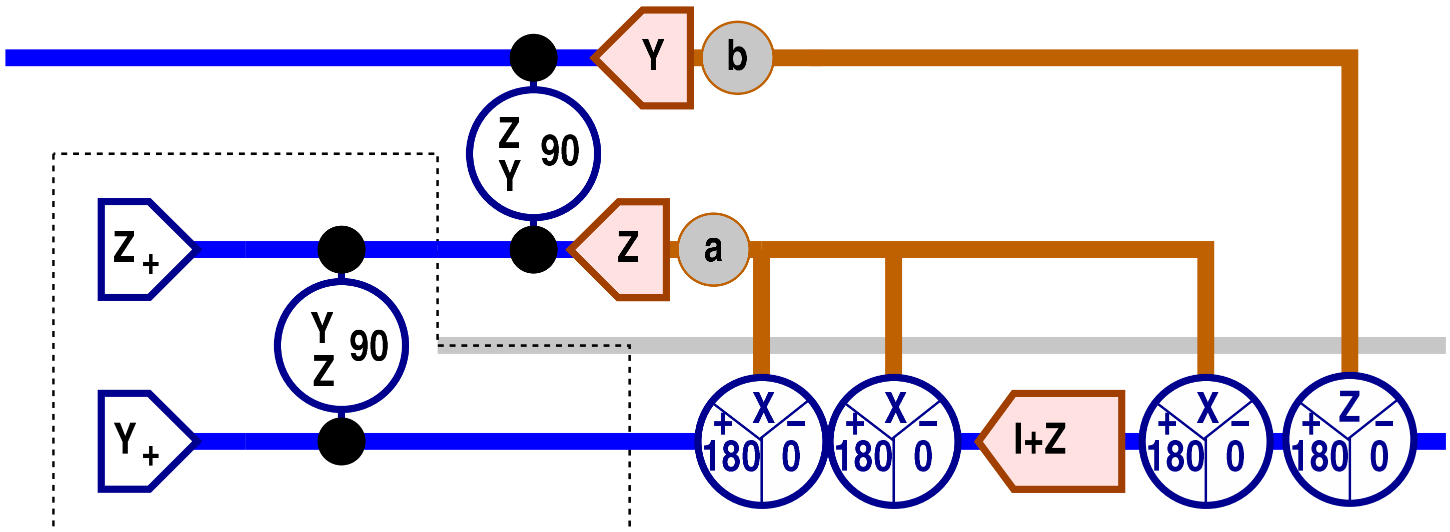

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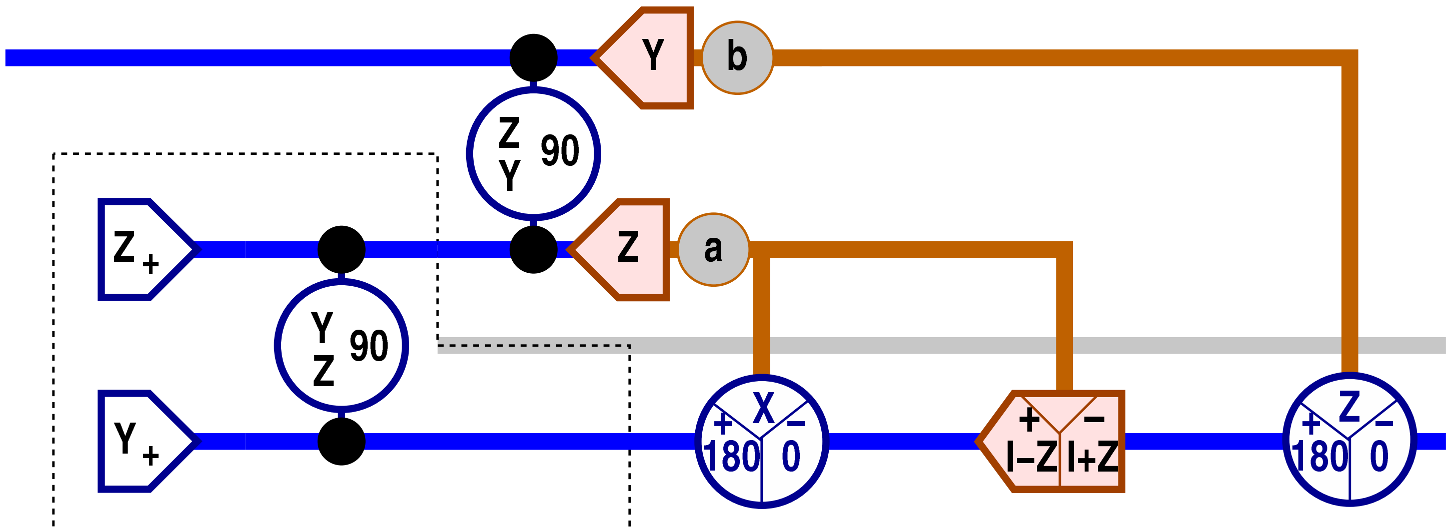

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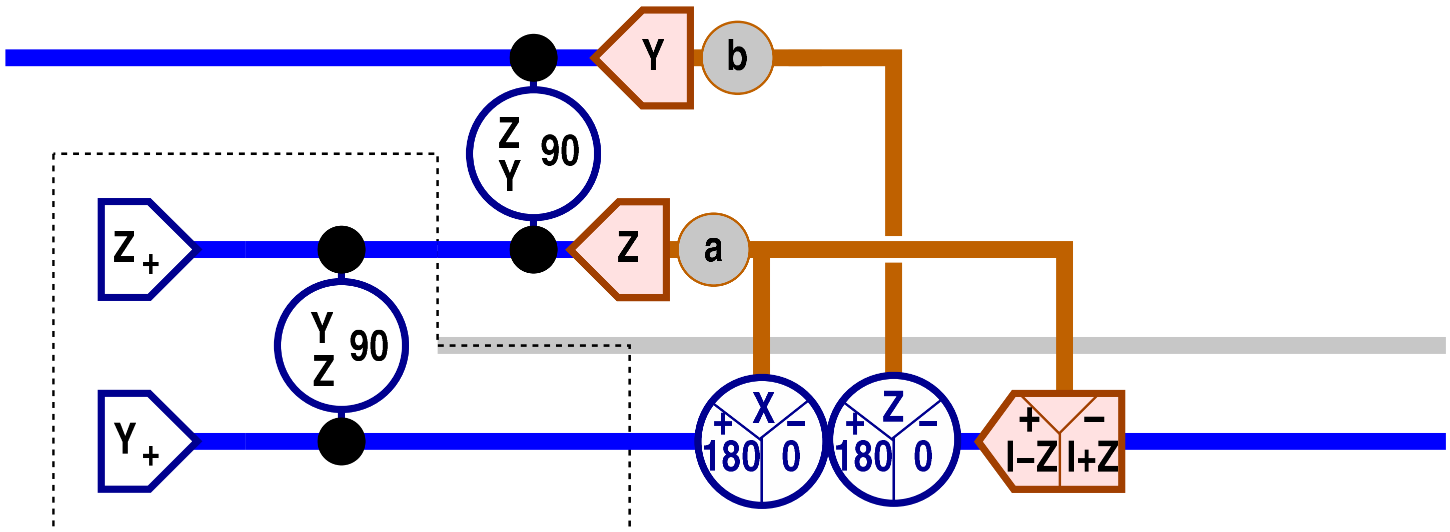

Advantages of Teleportation

- Transversal after successful state preparation.
  - Fault tolerant universality.
  - Robust syndrome detection for recovery from error.

Advantages of Teleportation

- Transversal after successful state preparation.
  - Fault tolerant universality.
  - Robust syndrome detection for recovery from error.
- Good error detection suffices.
  - Reject attempted state preparations if errors are detected.

Back to: Methods for Scalability

Conclusion

- Accuracy threshold questions:
  - Bit flip error model?
  - Erasure error model?
  - Depolarizing error model?

Conclusion

- Accuracy threshold questions:
  - Bit flip error model?
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- Worst-case dimension of a maximum size error-detecting quantum code in an n-dimensional space subject to an e-dimensional error model?

  (Best bound known: $\Omega(n/e^2)$.)

References

[1] D. Aharonov and M. Ben-Or. Fault-tolerant quantum computation with constant error. In *Proceedings of the 29th Annual ACM Symposium on the Theory of Computation (STOC)*, pages 176–188, New York, New York, 1996. ACM Press.

[2] D. Aharonov and M. Ben-Or. Fault-tolerant quantum computation with constant error. quant-ph/9906129, 1999.

[3] A. Barenco, A. Berthiaume, D. Deutsch, A. Ekert, R. Jozsa, and C. Macchiavello. Stabilization of quantum computations by symmetrization. *SIAM J. Comput.*, 26:1541–1557, 1997.

[4] C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, and W. K. Wootters. Mixed state entanglement and quantum error-correcting codes. *Phys. Rev. A*, 54:3824–3851, 1996.

[5] A.R. Calderbank, E.M. Rains, P.W. Shor, and N.J.A. Sloane. Quantum error correction and orthogonal geometry. *Phys. Rev. A*, 78:405–408, 1997.

[6] A.R. Calderbank and P.W. Shor. Good quantum error-correcting codes exist. *Phys. Rev. A*, 54:1098–1105, 1996.

[7] W. Dür, H.-J. Briegel, J. I. Cirac, and P. Zoller. Quantum repeaters based on entanglement purification. *Phys. Rev. A*, 59:169–181, 1999.

[8] D. Gottesman. A class of quantum error-correcting codes saturating the quantum hamming bound. *Phys. Rev. A*, 54:1862–1868, 1996.

[9] D. Gottesman. A theory of fault-tolerant quantum computation. *Phys. Rev. A*, 57:127–137, 1998.

[10] D. Gottesman and I. L. Chuang. Demonstrating the viability of universal quantum computation using teleportation and single-qubit operations. *Nature*, 402:390–393, 1999.

[11] A. Yu. Kitaev. Quantum computations: algorithms and error correction. *Russian Math. Surveys*, 52:1191–1249, 1997.

[12] E. Knill and R. Laflamme. A theory of quantum error correcting codes. *Phys. Rev. A*, 55:900–911, 1997.

[13] E. Knill, R. Laflamme, and G. Milburn. A scheme for efficient linear optics quantum computation. *Nature*, 409:46–52, 2001.

[14] E. Knill, R. Laflamme, and L. Viola. Theory of quantum error correction for general noise. *Phys. Rev. Lett.*, 84:2525–2528, 2000.

[15] E. Knill, R. Laflamme, and W. Zurek. Resilient quantum computation: Error models and thresholds. *Proc. R. Soc. Lond. A*, 454:365–384, 1998.

[16] E. Knill, R. Laflamme, and W. H. Zurek. Resilient quantum computation. *Science*, 279:342–345, 1998.

[17] D. A. Lidar, I. L. Chuang, and K. B. Whaley. Decoherence-free subspaces for quantum computation. *Phys. Rev. Lett.*, 81:2594–2597, 1998.

[18] B. Misra and E. C. G. Sudarshan. Zenos paradox in quantum theory. *J. Math. Phys.*, 18:756–763, 1977.

[19] M. A. Nielsen and I. L. Chuang. *Quantum Computation and Quantum Information*. Cmabridge University Press, 2001.

[20] J. Preskill. Reliable quantum computers. *Proc. R. Soc. Lond. A*, 454:385–410, 1998.

[21] P. W. Shor. Scheme for reducing decoherence in quantum computer memory. *Phys. Rev. A*, 52:2493–2496, 1995.

[22] P. W. Shor. Fault-tolerant quantum computation. In *Proceedings of the 37th Symposium on the Foundations of Computer Science (FOCS)*, pages 56–65, Los Alamitos, California, 1996. IEEE press.

[23] A. Steane. Multiple particle interference and quantum error correction. *Proc. R. Soc. Lond. A*, 452:2551–2577, 1996.

[24] L. Viola, E. Knill, and S. Lloyd. Dynamical decoupling of open quantum systems. *Phys. Rev. Lett.*, 82:2417–2421, 1999.

[25] L. Viola and S. Lloyd. Dynamical suppression of decoherence in two-state quantum systems. *Phys. Rev. A*, 58:2733–2744, 1998.

[26] L. Viola, S. Lloyd, and E. Knill. Universal control of decoupled quantum systems. *Phys. Rev. Lett.*, 83:4888–4891, 1999.

[27] P. Zanardi and M. Rasetti. Noiseless quantum codes. *Phys. Rev. Lett.*, 79:3306–3309, 1997.