Abstract

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Numerics for Bifurcation and Dynamics in Partial Differential Equations

We start with the short demonstration of nonlinear phenomena and the well known Liapunov-Schmidt method.

Application of these methods to PDEs usually is not directly possible but we need discretization methods. The classical discretization methods have to be modified such that we are able to yield convergence results for bifurcation scenarios (a first important result in this direction is due to H.B. Keller and B. Langford 72) and to center manifolds.
We want to get convergence results for nonlinear problems in PDEs. Here we discuss space discretizations.
The goal is to develop a theory for a wide class of operator equations and the actual discretization methods. This theory should be as simple as possible, however as general as necessary to incluce these problems.

The class of operators should cover elliptic problems and include at least Navier Stokes problems. The class of discretization methods should include difference-, finite element-, spectral- and wavelet methods. We certainly will have to consider symmetric discretizations for equivariant problems with finite and infinite symmetry groups as well.

It turned out that this goal could be unfolded to include convergence results directly for all the above operators and nonlinear problems and to extend this to the case of bifurcation numerics. For this approach we need mainly four basic concepts:
We combine monotone operators A (or the corresponding coercive bilinear forms) with compact perturbations C. The approximating spaces, e.g., grid funtions in difference methods, finite elements, wavelets have to appropriately approximate the solution functions. In that case we have essentially the following result
$(A + C)^h$ is stable $\Leftrightarrow A + C$ boundedly invertible.
Just recently we have been able to prove convergence for wavelet methods for all the above operators (B. -Dahlke). For finite elements with variational crimes we additionally need estimates for the consistency errors. This approach allowed an extension to collocation methods and just recently the proof for convergence for one of the special cases of Doedels collocation methods.
Application to bifurcation and center manifolds is possible if we use the standard concept of bordered systems. In fact we have proved

$$\begin{pmatrix} A & \| \\ = & 0 \end{pmatrix}^h \text{ stable } \Leftrightarrow \begin{pmatrix} A & \| \\ = & 0 \end{pmatrix} \text{ invertible} \qquad (1)$$

The well known Jepson Spence paper shows that we exactly need this condition for generalized Liapunov-Schmidt methods. We need the same for the case of center manifolds. That means for this large class of operators and discretization

methods we do have convergence results for both problems.
Fianally, we want to indicate a recent result: It is possible to retransform the full bifurcation scenarios from the reduced normal form back to the variable and parameter space of the original situation.