THE FIELDS INSTITUTE
FOR RESEARCH IN MATHEMATICAL SCIENCES

ABSTRACTS 1.2

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Spherical harmonic projectors

The underlying reason for the attractive stability and reliability of the spectral method for modeling geophysical phenomena is the harmonic projection (HP) which is implicit in the numerous harmonic transforms between the physical and spectral spaces. The HP, as currently configured, consists of a forward transform from physical to spectral space (harmonic analysis) immediately followed by an harmonic synthesis back to physical space. Unlike its Fourier counterpart in Cartesian coordinates, the HP does not identically reconstruct the original function on the surface of the sphere but rather replaces it with a weighted least squares approximation. The importance of the HP is that it uniformly resolves waves on the surface of the sphere and therefore eliminates high frequencies that are artificially induced (for example) by the clustering of grid points in the neighborhood of the poles. The HP also maintains spectral accuracy when combined with the double Fourier method. Originally the HP required O(N^3) storage where N is the number of latitudinal points. However, this was recently reduced to O(N^2) using an algorithm that also provided a savings of up to 50 percent in compute time. The HP was also generalized to an arbitrary latitudinal distribution of points. However, the composite components of the HP; namely, the harmonic analysis can be subject to considerable error depending on the point distribution. Here we define a variant of the traditional HP that is well conditioned, with condition number 1, for any point distribution. In addition, storage requirements are further reduced because the projections corresponding to all longitudinal wave numbers m are expressed in terms of a single orthogonal matrix.