Estimates of automorphic forms and representation theory.

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This is a joint work with Andre Reznikov.

Abstract.

Let H be the upper half plane with the hyperbolic metric of constant curvature -1. We consider the natural action of the group $G = SL(2, \mathbf{R})$ on H and identify H with G/K by means of this action (here $K = SO(2) \subset G$).

Fix a lattice (discrete subgroup) $\Gamma \subset G$ and consider the Riemann surface $Y = \Gamma \setminus H$.

The Laplace-Beltrami operator Δ acts in the space of functions on Y. When Y is compact it has discrete spectrum; we denote by $\mu_1 \leq \mu_2 \leq$ ... its eigenvalues on Y and by ϕ_i the corresponding eigenfunctions. (We assume that $||\phi_i||_{L^2} = 1$.) These functions ϕ_i are usually called *automorphic functions* or *Maass forms* .

The study of automorphic functions and the corresponding eigenvalues is important in many areas of representation theory, number theory and geometry.

In my talk I will discuss several problems of estimating numbers arising from modular forms.

In particular I will discuss the following problem.

Consider three Maass forms ϕ_i, ϕ_j, ϕ_k and set $c_{ijk} = \int \phi_i \phi_j \phi_k$. These coefficients are called triple products; many problems in number theory can be reduced to estimating of these triple products.

I will explain why these triple products are of interest and how they are related to the theory of Rankin-Selberg L-functions.

The main topic of the talk is a new method which allows us to give estimates of triple products when one of the indexes tends to ∞ in terms of the eigenvalues of Δ.

This method allows to see how to separate the exponentially decaying factor in the triple product and give a polynomial, tight on the average, estimate of the remaining term.

The main idea of the method which I will try to explain is to associate to every automorphic function ϕ an automorphic representation of the group $G = SL(2, \mathbf{R})$ and study the properties of this representation. The main new tool which we use is some uniqueness result in representation theory.