THE FIELDS INSTITUTE
FOR RESEARCH IN MATHEMATICAL SCIENCES

ABSTRACTS 1.2

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Traces, Ideals and Arithmetic Means

This article (PNAS-US to appear) grew out of recent work of Dykema, Figiel, Weiss and Wodzicki (Commutator structure of operator ideals) which inter alia characterizes commutator ideals in terms of arithmetic means.

In this paper we study ideals that are arithmetically mean (am) stable, am-closed, am-open, soft-edged and soft- complemented. We show that many of the ideals in the literature possess such properties. We apply these notions to prove that for all the ideals considered, the linear codimension of their commutator space (the "number of traces on the ideal") is either 0, 1, or infinity. We identify the largest ideal which supports a unique nonsingular trace as the intersection of certain Lorentz ideals. An application to elementary operators is given.

We study properties of arithmetic mean operations on ideals, e.g., we prove that the am-closure of a sum of ideals is the sum of their am-closures. We obtain cancellation properties for arithmetic means: for principal ideals, a necessary and sufficient condition for first order cancellations is the regularity of the generator; for second order cancellations, sufficient conditions are that the generator satisfies the exponential delta 2-condition or is regular. We construct an example where second order cancellation fails, thus settling an open question. We also consider cancellation properties for inclusions. And we find and use lattice properties of ideals associated with the existence of "gaps".