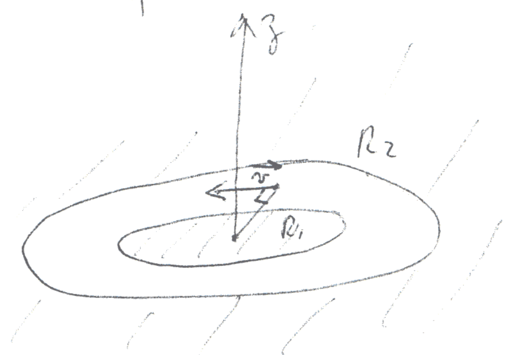
Exercise 1

① (ii) $v = v_\theta u_\theta$

In (x,y) coordinates (the flow is 2-dimensional) we have



$$v = \begin{pmatrix} y \\ -x \end{pmatrix} \cdot \frac{1}{r} \cdot \left(\frac{A}{r} + Br \right)$$

$$\left(\vec{u_\theta} = \frac{1}{r} \begin{pmatrix} y \\ -x \end{pmatrix} , \quad r = (x^2+y^2)^{1/2} \right)$$

So $$v = \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} y \cdot \left[\frac{A}{x^2+y^2} + B \right] \\ -x \left[\frac{A}{x^2+y^2} + B \right] \end{pmatrix}$$

→ The computation gives: $\text{curl} \begin{pmatrix} y/x^2+y^2 \\ -x/x^2+y^2 \end{pmatrix} = 0$

→ $\text{curl } v = \begin{pmatrix} 0 \\ 0 \\ 2B \end{pmatrix}$

(i) For an axisymmetric flow,

$$\frac{\partial v}{\partial t} + v.\nabla v = \text{acceleration} = -\vec{u_r} \frac{|v|^2}{r}$$

$$\rightarrow \frac{\partial v}{\partial t} + v.\nabla v = f(r)\, u_r \Rightarrow f(r) u_r = \nabla \left(\int_{R_1}^{r} f(r')dr' \right)$$

$$\Rightarrow \frac{\partial v}{\partial t} + v.\nabla v = \nabla p \text{ for some } p$$

Moreover in (x,y) coord, we check that

$$\partial_x v_x + \partial_y v_y = 0.$$

$\rightarrow$ v is a solution of Euler, and $\partial_r v = 0$.

(iv) angular velocity $= \Omega = \frac{v_\theta}{r}$

at $r = R_1$, $\Omega = -\frac{R_2^2(\omega_2-\omega_1)}{R_2^2 - R_1^2} + \frac{R_1^2(\omega_1) - R_2^2\omega_2}{R_2^2 - R_1^2}$

$= \omega_1$

same at $r = R_2$.