Abstract: From Spectral Flow to the Odd Local Index Formula

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We generalise the odd local index formula of Connes and Moscovici to the case of unbounded spectral triples $(\mathcal{A}, \mathcal{N}, \mathcal{D})$ for a $*$-subalgebra $\mathcal{A}$ of a general semifinite von Neumann algebra, $\mathcal{N}$ with a fixed faithful, normal, semifinite trace, τ. In this setting it gives a cohomological formula for the pairing of Conne's Chern Character $Ch(u)$ of an element in $K_1(\mathcal{A})$ (a unitary $u \in \mathcal{A}$) with a (b, B) cocycle constructed from the spectral triple.

We start from the spectral flow formula for the index (of the Toeplitz operator PuP) for finitely summable spectral triples developed by Carey-Phillips. This spectral flow formula is given by the integral of a one-form along the straight line path from $\mathcal{D}$ to $u\mathcal{D}u^*$ together with a normalising "constant." We show how the seemingly innocuous normalising constant in the formula actually gives a new approach to the Connes-Moscovici results.

Time permitting, we will indicate how we can prove the even index theorem using similar techniques, but starting from a semifinite version of the McKean-Singer formula.