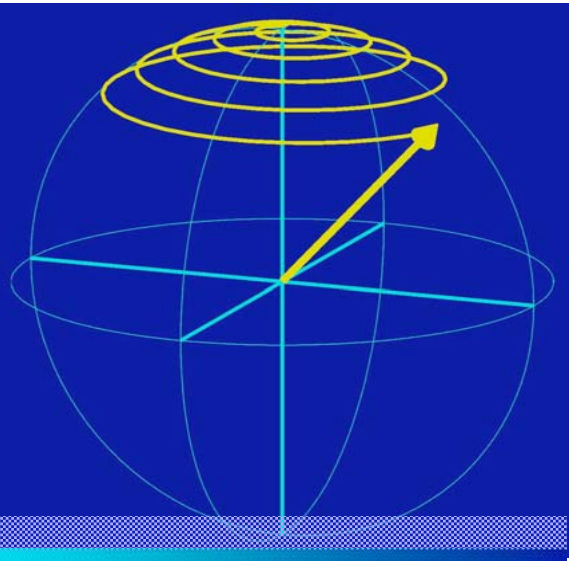
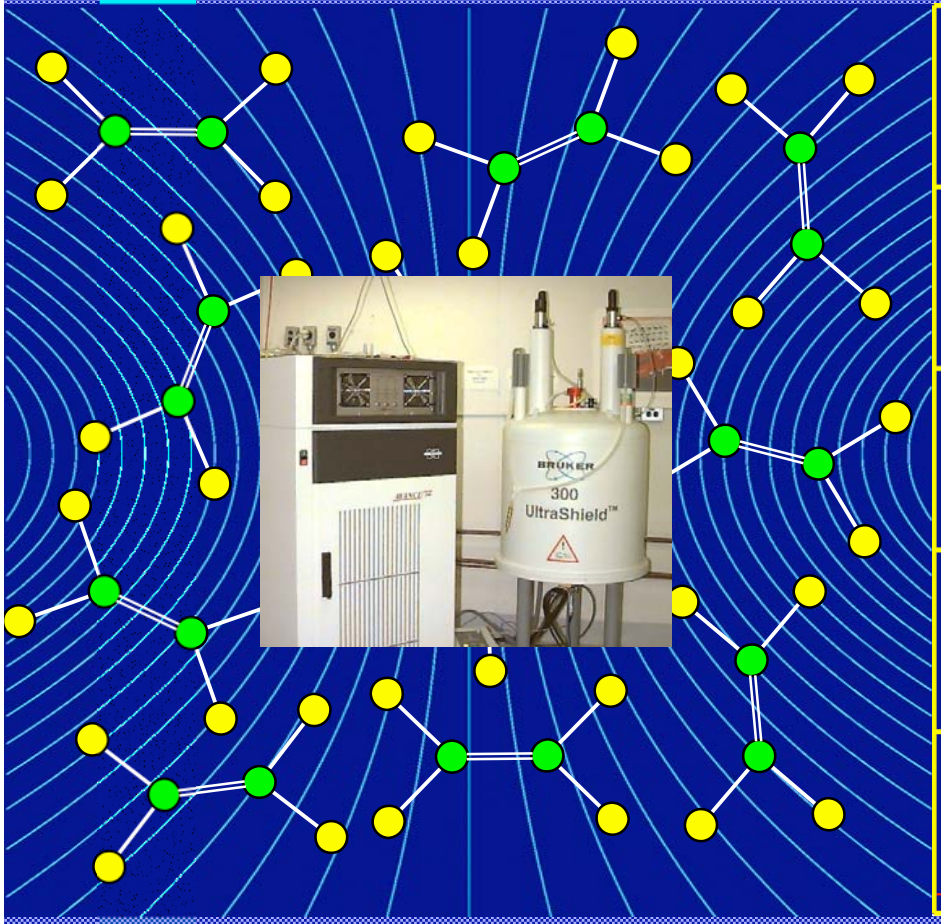
Tim Havel and David Cory
Dept. of Nuclear Engineering
Massachusetts Institute of Technology

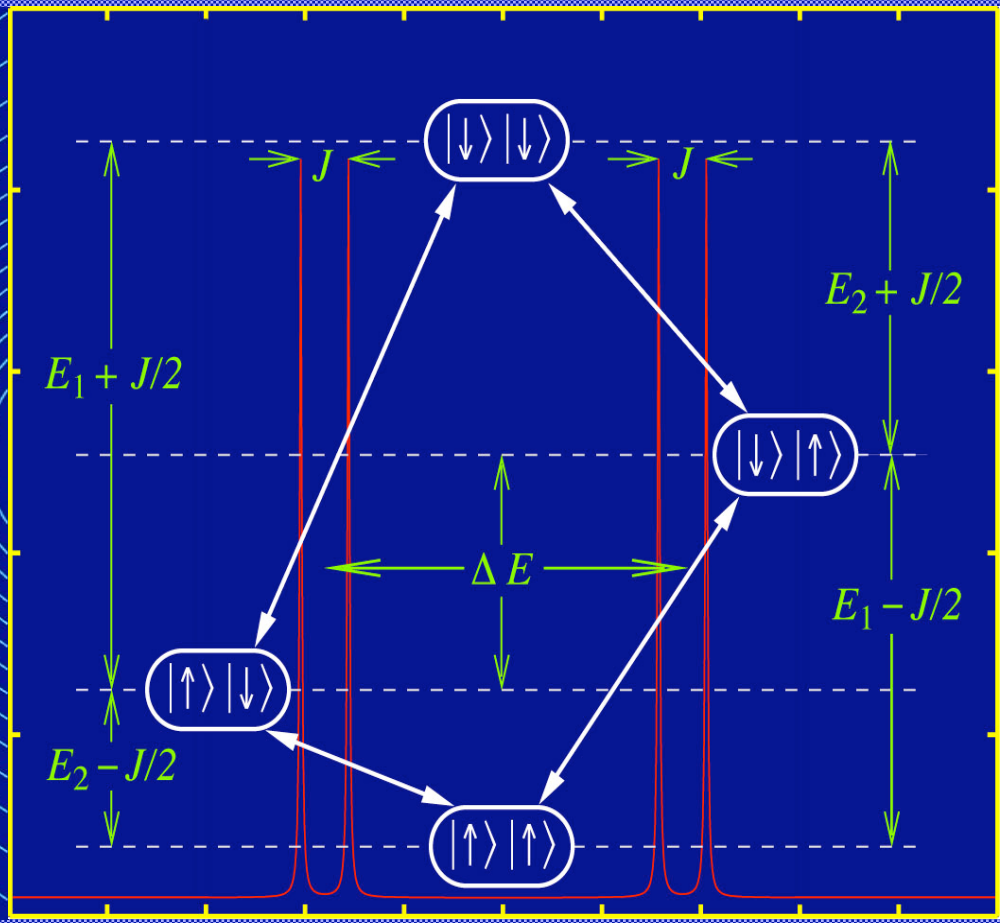


Experimental Methods for Quantum Control in Nuclear Spin Systems

Together with: Jonathan Baugh, Hyang Joon Cho, Paola Cappellaro, Nicolas Boulant, Joseph Emerson, Jonathan Hodges, Raymond Laflamme, Tirthahalli Mahesh, Chandrasekhar Ramanathan, Suddhasattwa Sinha, and Yaakov Weinstein

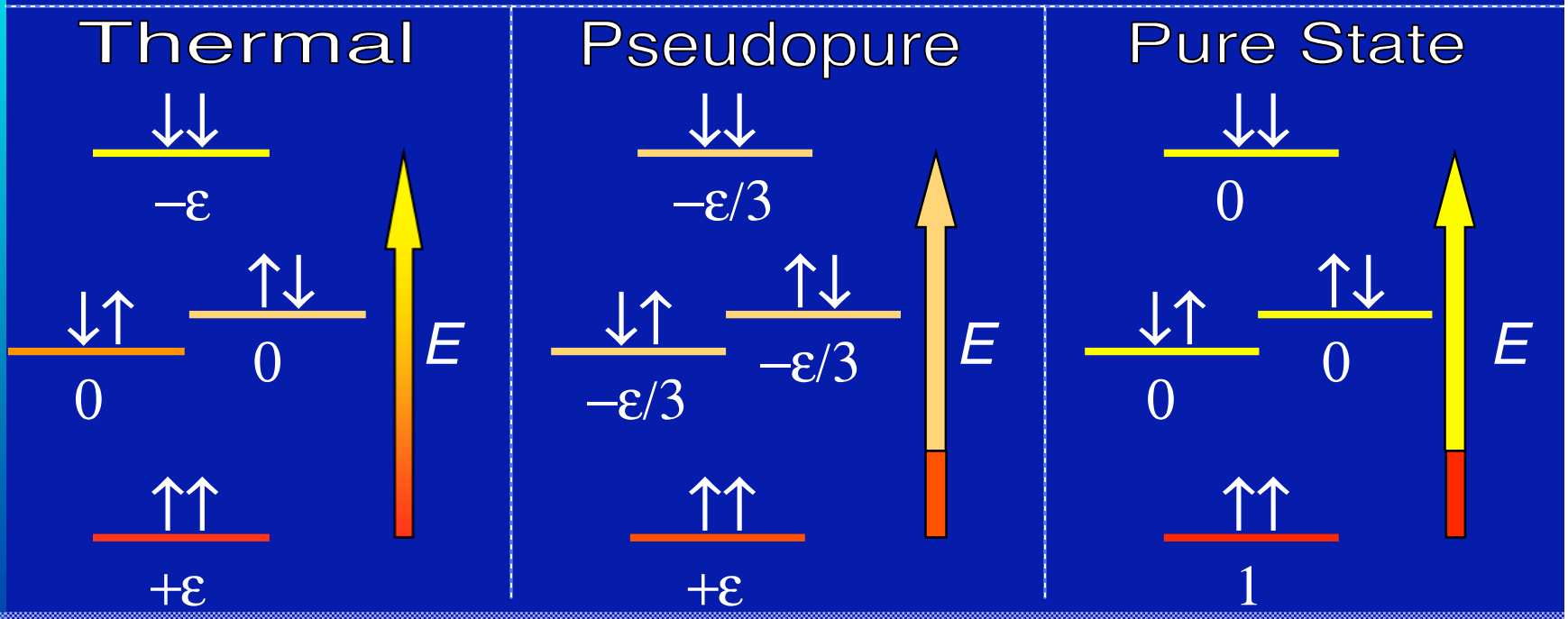
NMR QIP: It really works !





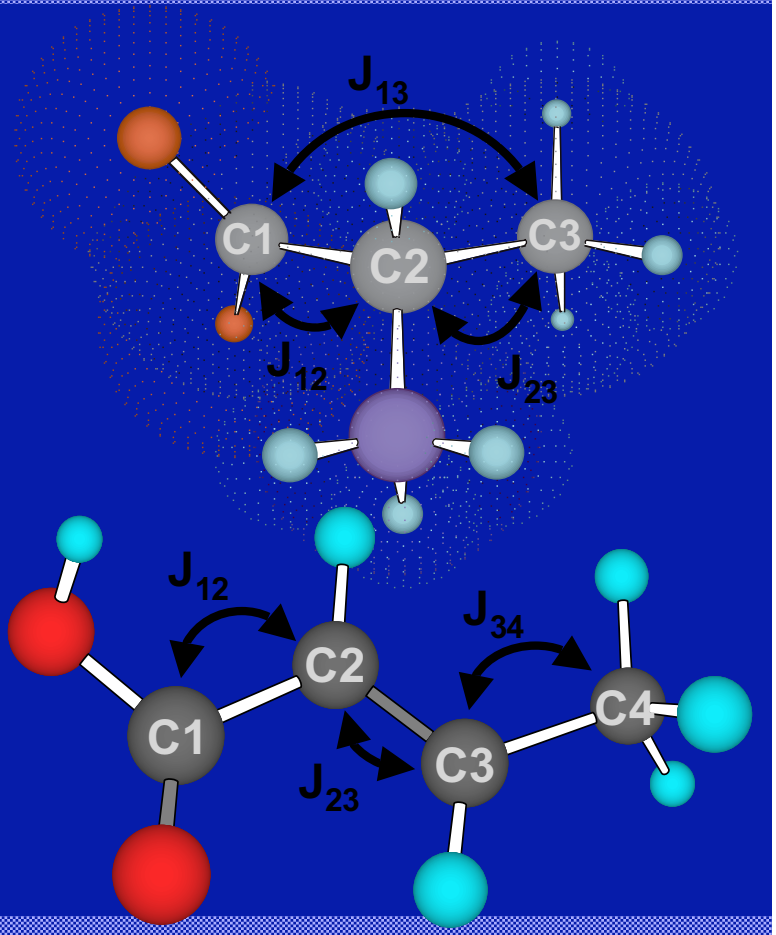
In Principle or In Reality ?

- **Evolution:** $U(1 + \varepsilon |\uparrow\rangle\langle\uparrow|)U^\dagger = 1 + \varepsilon(U|\uparrow\rangle)(\langle\uparrow|U)^\dagger$
- **Measurement:** $tr\left((1 + \varepsilon |\uparrow\rangle\langle\uparrow|)M\right) = \varepsilon\langle\uparrow|M|\uparrow\rangle$



Where Are We Now?

- Fully programmable systems on 3 - 6 qubits
  - Molecules are ^{13}C-labeled Alanine & Crotonic Acid
  - Efficient, modular implementations of logic gates
  - Achieved via **strongly** modulated RF pulses
  - Designed by simulation & numerical optimization …
  - … to correct incoherent errors



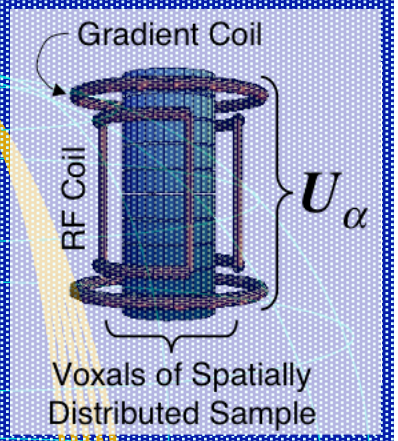
Control of Incoherent Errors

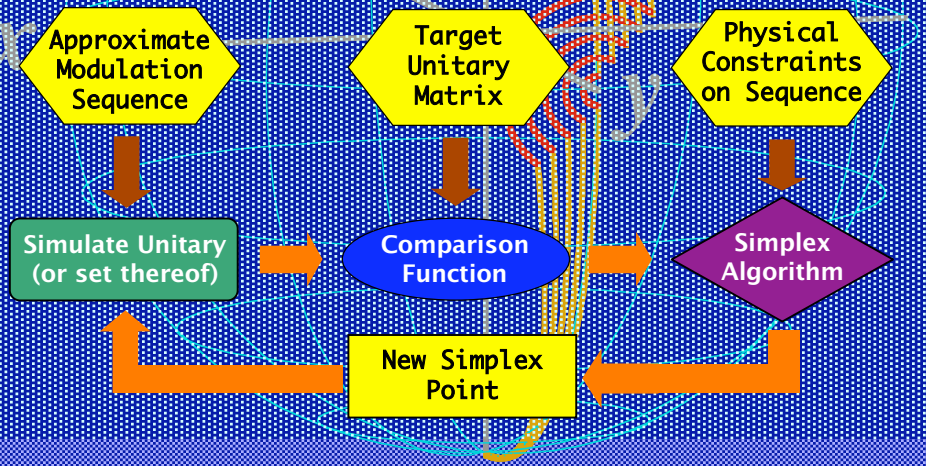
- Errors in quantum control may be classified as:
  - Coherent (inaccurate)
  - Incoherent (imprecise)
  - Decoherent (micro-scopically random)
- Incoherent are correctable by refocusing, as shown by composite pulses
- But strong modulation can get the entire unitary right

$\rho_{out} = \ldots$

$\ldots \int_V d\alpha \, p_\alpha U_\alpha \rho_{in} U_\alpha^\dagger$

distribution p_α can be measured & incorporated into pulse design

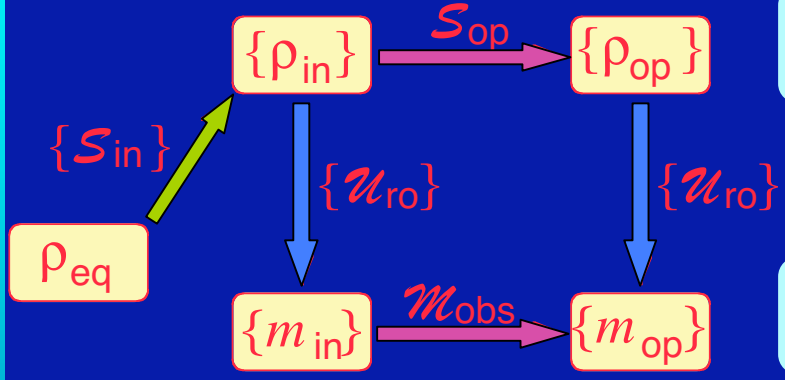




Achievements of NMR QIP

- The first successful demonstrations of:
  - Quantum logic gates on superpositions
  - Application of entangling unitary operations
  - Simple quantum algorithms (QFT, Grover, &c)
  - Quantum error correcting codes (on 3 qubits)
  - Decoherence-free subspaces and systems
  - Quantum simulation as proposed by Feynman
  - Semi-classical/quantum simulations of decoherence
  - Quantum process tomography (on 3 qubits)

Quantum Process Tomography



$$m_{xx} \equiv tr(\mathcal{U}_{ro}(\rho_{xx})\sigma_{-})$$

Due to errors, particularly incoherent, measured superoperator may NOT be completely positive

$$\mathcal{U}_{xx}(\rho) \equiv \int dq\, U_{xx}^{(q)} \rho U_{xx}^{(q)\dagger}$$

Calculating a superoperator from experimental data:

$$\begin{bmatrix} \cdot & & \cdot \\ \rho_1^{in} & \rho_2^{in} \cdots \cdots & \rho_N^{in} \\ \cdots & & \cdots \end{bmatrix} \cdot \cdots = \begin{bmatrix} \cdot & & \cdot \\ \rho_1^{out} & \rho_2^{out} \cdots \cdots & \rho_N^{out} \\ \cdots & & \cdots \end{bmatrix}$$

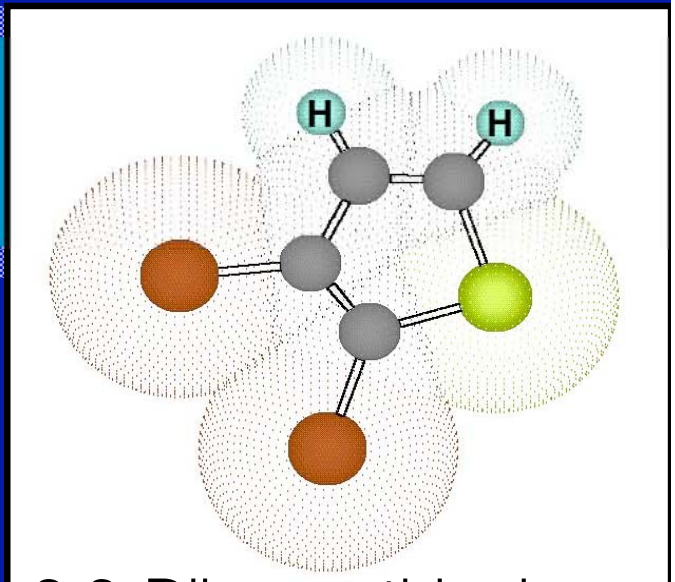
$\mathcal{S}$ ... superoperator matrix vs. induced basis is unknown

columns are stacked input density matrices

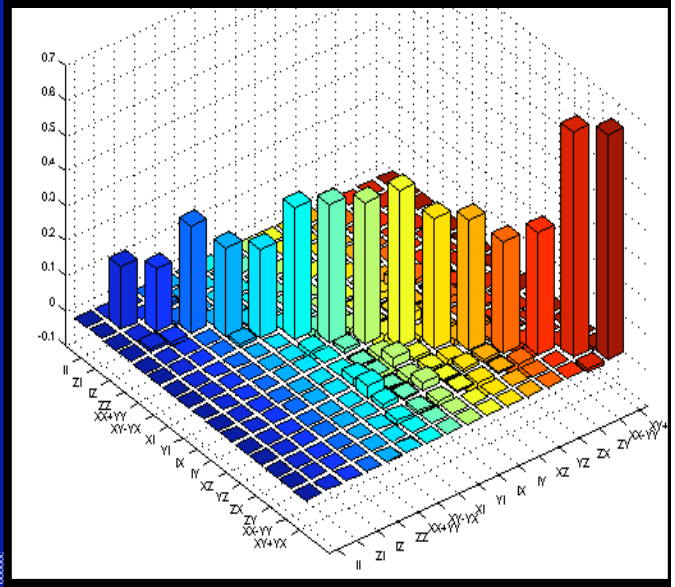
columns are stacked output density matrices

Natural Relaxation

- Our first attempt at QPT sought to determine the natural relaxation superoperator of a 2-spin system, 2,3-dibromothiophene
  - This means to finding all of its relaxation rates, i.e. or Lindblad operators
  - Done by first determining the superpropagator at 4 time points, & fitting the superoperator to these data
  - Fit was ill-conditioned until complete positivity constraint was imposed

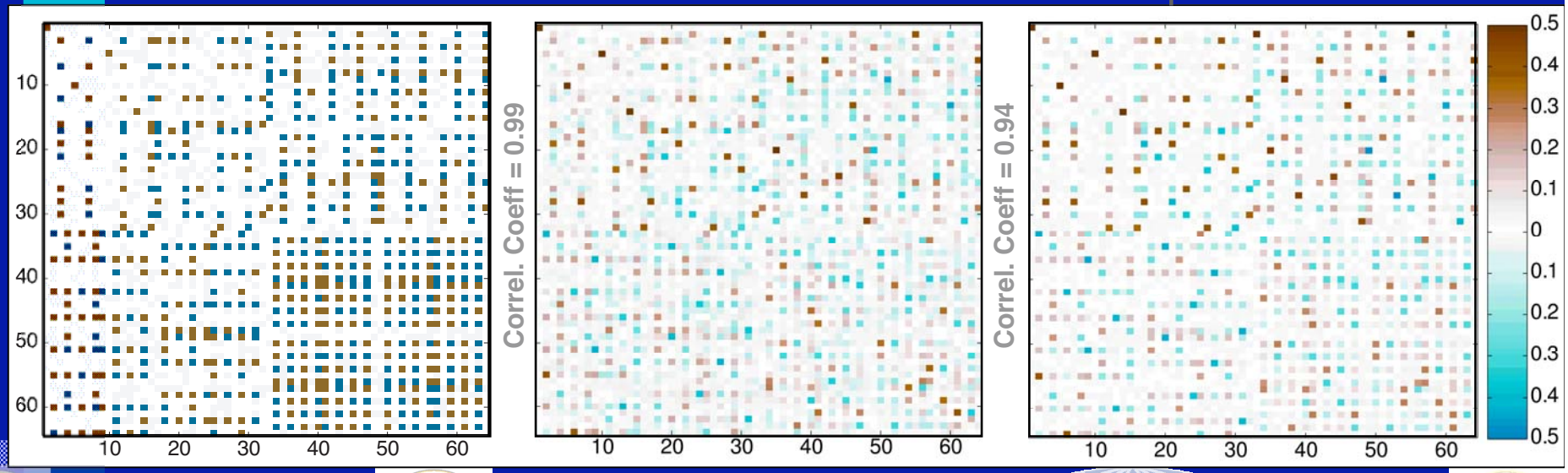


2,3-Dibromothiophene

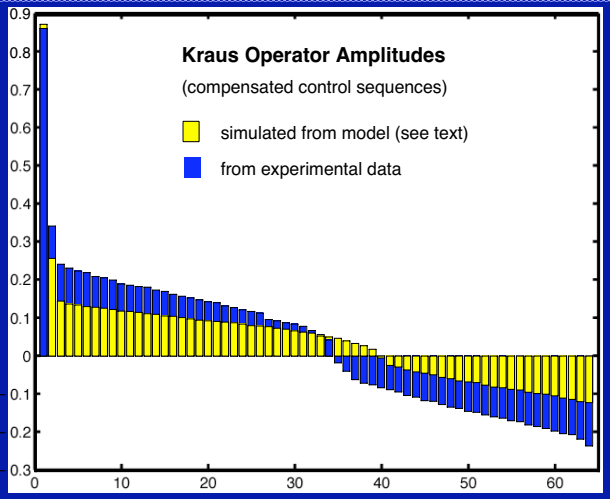


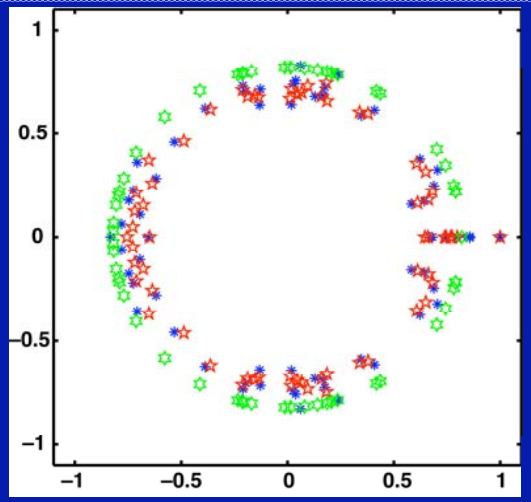
3-qubit QFT with our SM-pulses

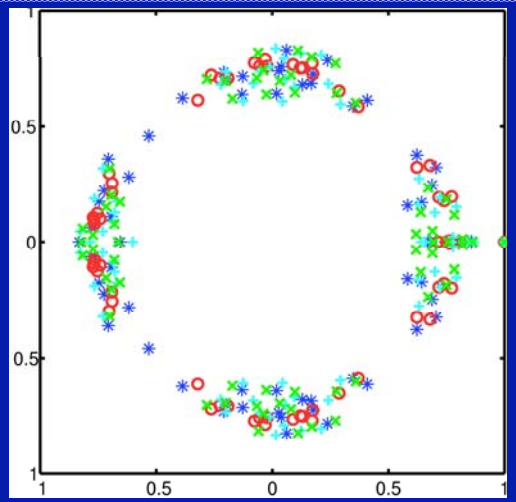
- The plots below show the theoretical, simulated and experimental superoperator on the 3-qubit QFT as implemented by strongly modulating pulses in alanine, all versus the product operator ($\sigma_\alpha \otimes \cdots \otimes \sigma_\zeta$) basis



Identifying Errors via the Model



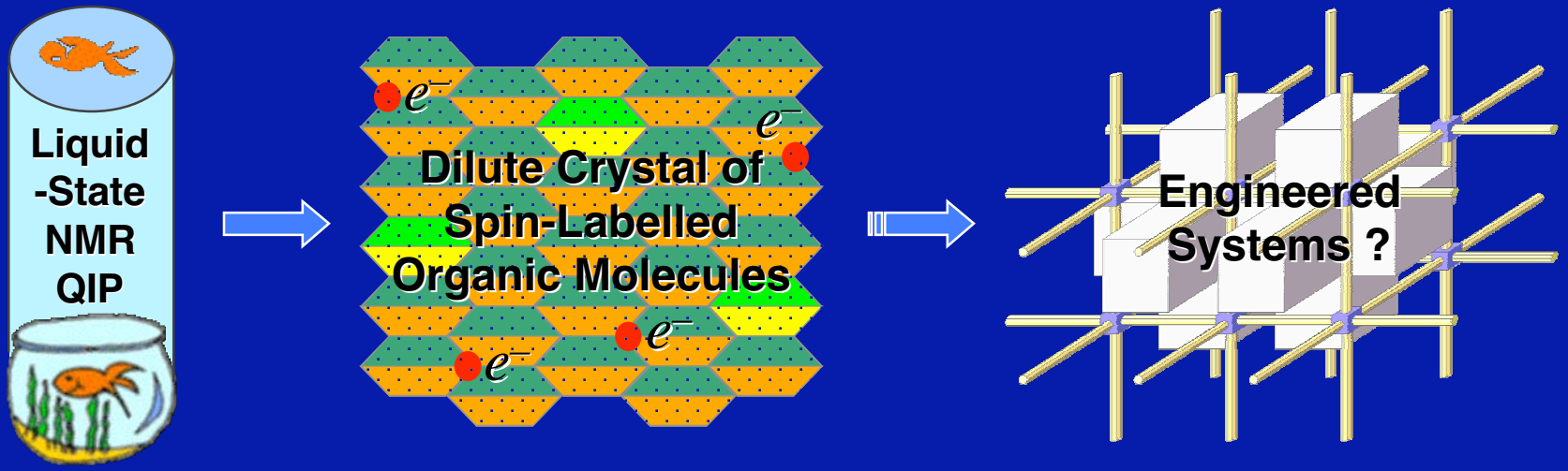




- The lack of complete positivity, though significant, could be fixed without large changes in the superoperator's eigenvalues, implying that the incoherent errors were not large (as we had hoped!)
- The clustering of the eigenvalues at [0, i, –1, $-i$] was much improved by finding the product of qubit rotations that maximized the correlation with the simulated, implying that the main coherent errors were the cumulative result of many small single qubit errors

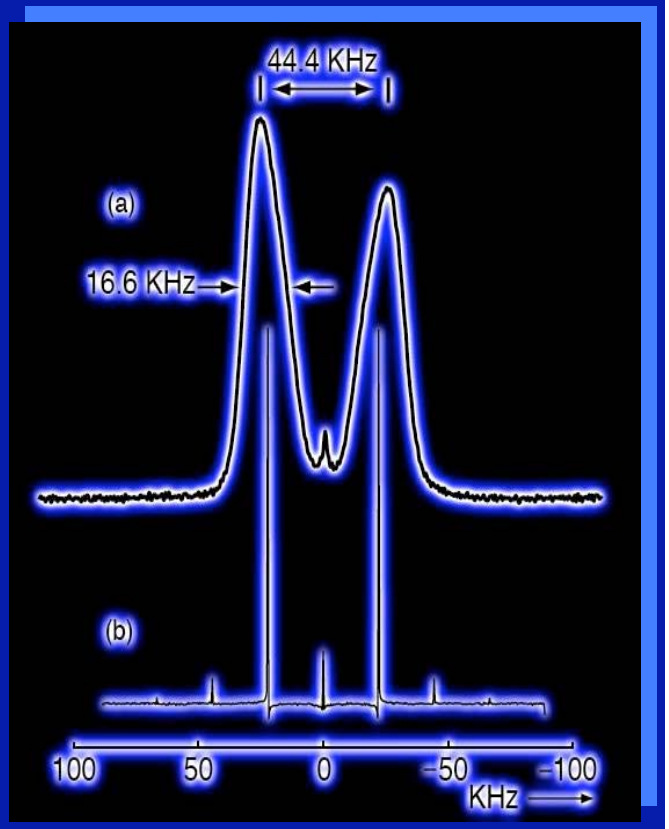
Where Are We Going with NMR?

- Solid-state NMR offers several practical benefits:
  - Longer decoherence times and faster gates
  - Adjustable chemical shifts to address more qubits
  - The ability to perform Dynamic Nuclear Polarization



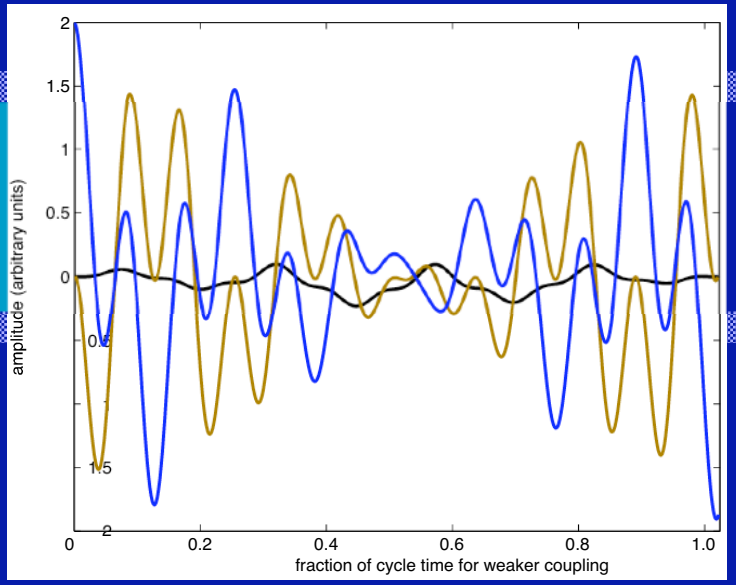
Restricting Dipole Couplings in Crystal Lattices to Nearest Neighbors

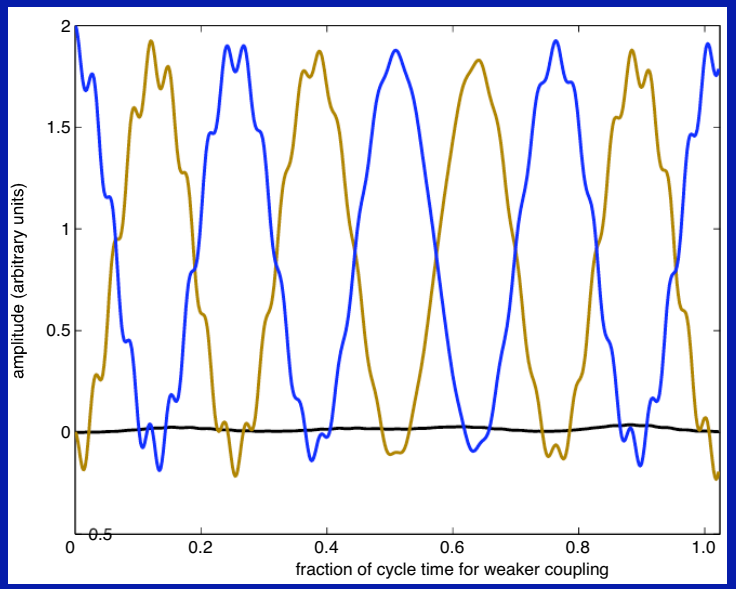
- In dilute spin crystals, this would improve isolation of spins in different molecules, and greatly reduce the complexity of the intramolecular Hamiltonian
- In cubic lattices of spins (e.g. CaF_2), it would enable simulation of massive 3D quantum Ising models
- The figure shows this effect in the case of Gypsum, a crystal containing strongly-coupled pairs of protons with many weak interpair couplings



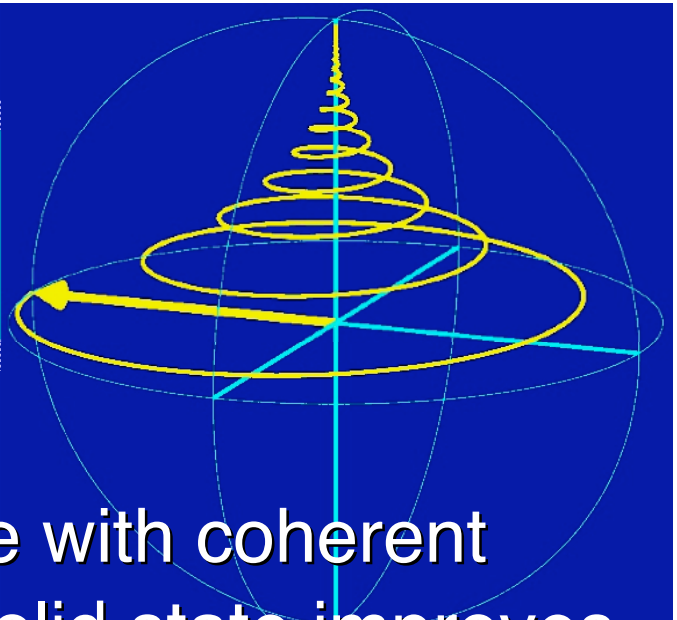
How Do We Do It ?

- The dipolar Hamiltonian of a pair of spins has eigen-values $[-1,0,1,2]\ D$
- Thus on-resonance RF cosine modulated at $3D/2$ tracks the spins' natural evolution
- An RF power of $D/2$ is enough to average any weaker couplings to zero





Conclusions and Acknowledgements



- We hope that as our experience with coherent control of nuclear spins in the solid state improves, a route to a truly scalable architecture, based on spatial addressing, will open up – though we are not ready to promise scalability today
- Those interested in further details should drop by either my poster or Jonathan Baugh's
- This work supported largely by DARPA & ARDA.

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- Free sample issues available at poster sessions (one per customer, please)!
- A quadruple issue on experimental aspects of QIP is now in press, with Our Patron Saint, Henry Everitt, as the guest editor (also to be published as an independent book)!