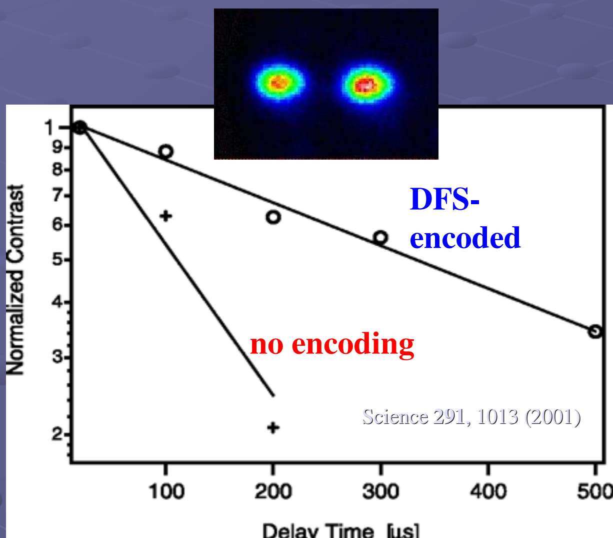
Hybrid quantum error prevention, reduction, and correction methods

Quantum Information & Quantum Control Conference
Toronto, July 23, 2004



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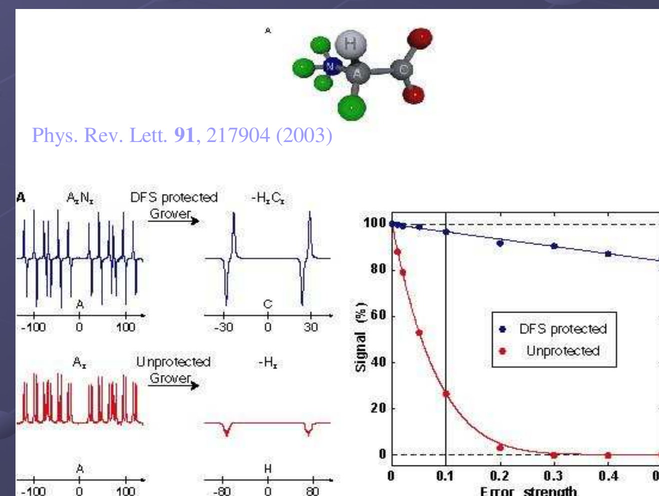

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D-Wave Systems

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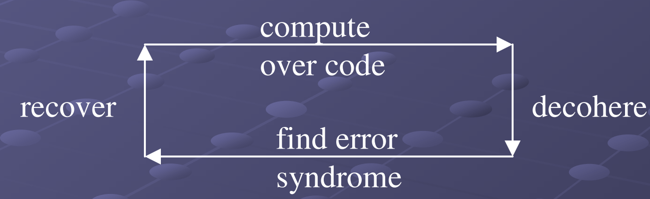
Decoherence-Reduction Methods (Partial List)

Quantum error correcting codes:

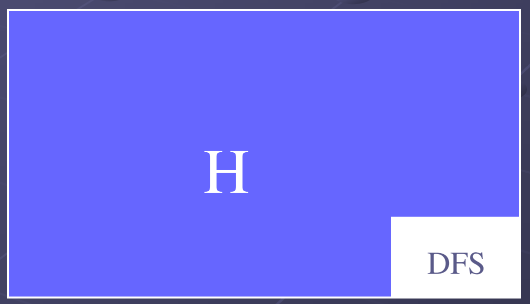
Encoding overhead; works best for errors uncorrelated in space and time (Markovian).



Decoherence-free subspaces/(noiseless) subsystems:

Encoding overhead; assumes symmetry in H_{SB} (strongly correlated errors).

Symmetry Ł conserved quantity = quantum info.



“Bang-Bang” decoupling:

Very rapid, strong pulses, no qubit overhead.
Needs non-Markovian environment.

Control options are primary; they are the experimentally available knobs

"Accept" environment: Inconvenient implication

Decoherence model

enforce!

Naturally available control options

Control options

Collective decoherence

Independent bit flip errors

Collective dephasing

Heisenberg exchange (quantum dots)

XY/XXZ exchange

Sorensen-Mölmer gates (trapped ions)

$|0_L\rangle = |000\rangle, \;\; |1_L\rangle = |111\rangle$

logical $X = \sigma_x \otimes \sigma_x \otimes \sigma_x$

logical $Z = \sigma_z \otimes \sigma_z \otimes \sigma_z$

measure $\sigma_z \otimes \sigma_z \otimes I, \; I \otimes \sigma_z \otimes \sigma_z$

Universal QC and Decoherence Elimination from the Controls up

1. Identify "naturally available" interactions (e.g., Heisenberg exchange in q. dots)
2. Enforce decoherence model by "bang-bang" decoupling pulses generated from naturally available interactions/controls
3. Offer decoherence protection by encoding into decoherence-free subspace (against enforced decoherence model)
4. Quantum compute universally over DFS using only the naturally available interactions
5. Combine with
   - Composite pulse method to deal with systematic gate errors
   - FT-QECC to deal with random gate errors

Why don't you just do QECC?

- In non-Markovian regime FT-QECC and BB are subject to same strength/speed conditions; BB more economical

- Much lower encoding overhead (fewer qubits), fewer gates?

  FT-QECC overhead, Steane [[7,1,3]] code:

  level 1: 7 qubits + 144 ancillas, 38 Hadamards, 288 CNOTs, 108 measurements

  level 2: 49 qubits + 320 ancillas, 154 Hadamards, 1307 CNOTs, 174 measurements

- Compatibility with naturally available controls
  while dealing with as general decoherence;
  threshold improvement – work in progress

Decoherence-Free Subspaces

Find a subspace where $H_{\text{int}} = \sum_\alpha S_\alpha \otimes B_\alpha$ act trivially,

i.e.: make $H_{\text{int}} \propto I_S \otimes O_B$

⇒ DFS:=Subspace of full system Hilbert space in which evolution is purely unitary

Condition for DFS

(Zanardi & Rasetti, Mod. Phys. Lett. B**11**, 1085 (1997); Lidar *et al.*, Phys. Rev. Lett. **81**, 2594 (1998), Phys. Rev. Lett. **82**, 4556 (1999); Knill *et al.*, Phys. Rev. Lett. **84**, 2525 (2000))

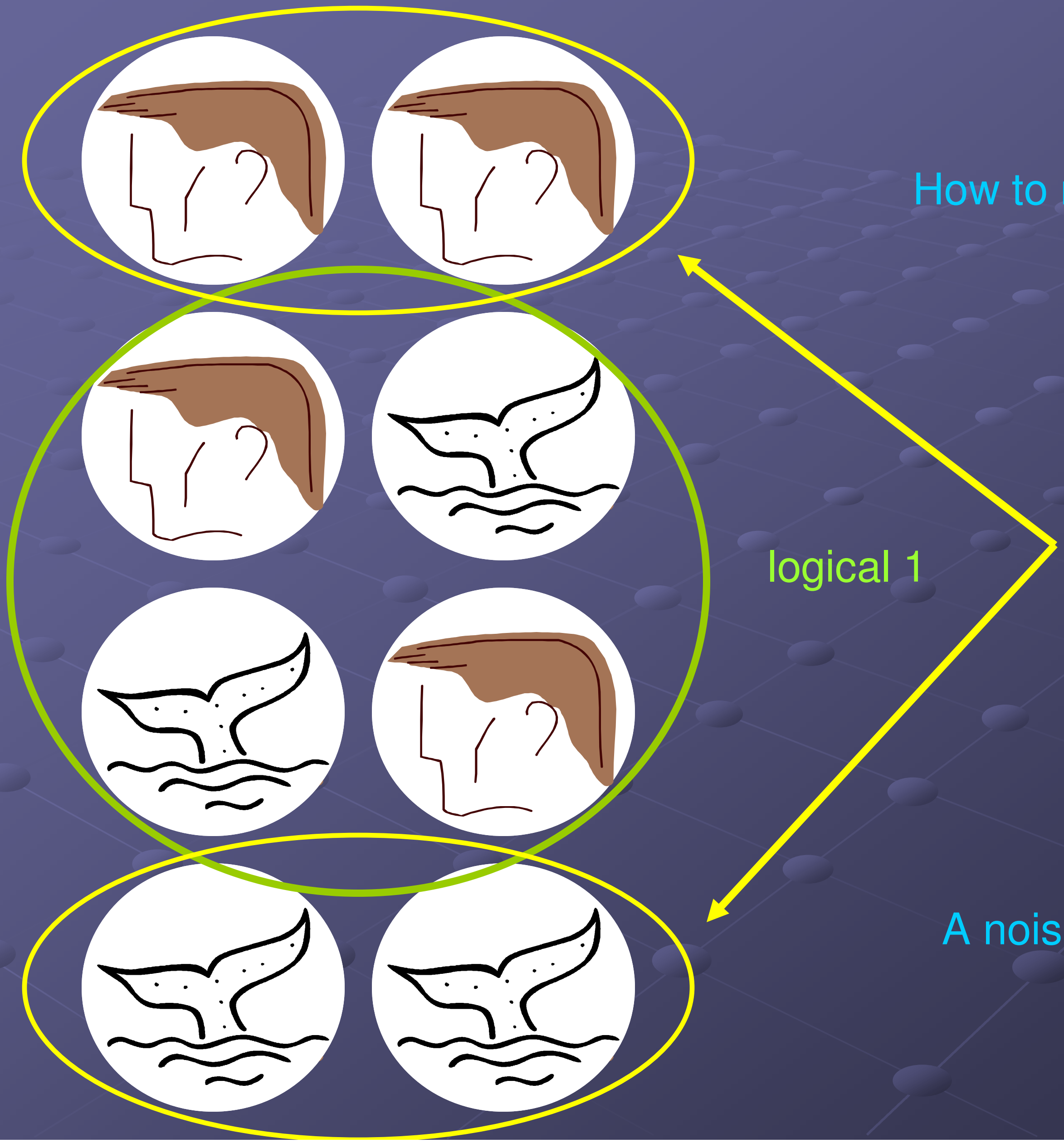
Lie algebra of S_α must have degenerate irreducible representations

DFS = states transforming according to these irreps

Translation: look for **degenerate** states with **fixed** (pseudo-) angular momentum (total, or a component): SYMMETRY

Symmetric coin flipping noise

How to reliably store a single bit?



A noiseless subspace.

Formal Condition for DFS, Computation

Knill, Laflamme & Viola, PRL **84**, 2525 (2000)

System-bath Hamiltonian

$$H_{SB} = \sum_\alpha S_\alpha \otimes B_\alpha$$

Internal + external system Hamiltonian

$$H_S = \sum_\beta S'_\beta \otimes I_\beta$$

A theorem from representation theory:

Error generators span associative algebra $A = \text{polynomials}\{I, S_\alpha, S_\alpha^\dagger\}$

Matrix representation over $\mathbb{C}^{2^N}$:

$$A \cong \bigoplus_J I_{n_J} \otimes M_{d_J}(\mathbb{C})$$

irreducible representations (J), multiplicity (n_J), dimension (d_J)

Commutant = operators commuting with A

$$A' \cong \bigoplus_J M'_{n_J}(\mathbb{C}) \otimes I_{d_J}$$

The control operations that preserve code subspace

Hilbert space decomposition:

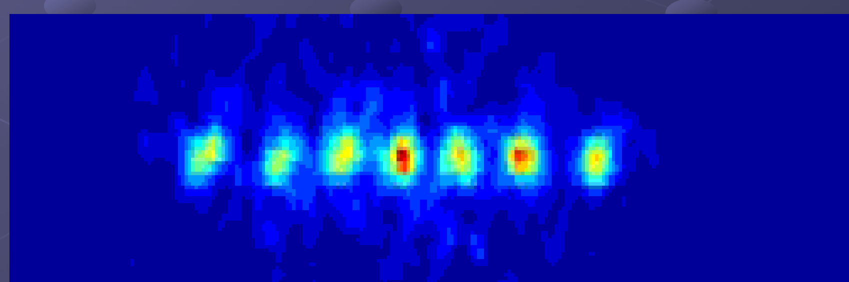
$$\mathbb{C}^{2^N} \cong \bigoplus_J \mathbb{C}^{n_J} \otimes \mathbb{C}^{d_J}$$

DFS ($\mathbb{C}^{n_J}$)

Illustrate with trapped ions, quantum dots.

$n_J > 1$ iff $\exists$ **symmetry** in system-env. interaction

Trapped Ions



few μm

Trapped Ions

- Naturally Available Interactions: E.g., Sorensen-Molmer gates (work with hot ions)

$\propto$ Rabi freq.

Laser phase on ions 1,2

$$U_{12}(\theta,\phi_1,\phi_2)=\exp[i\theta(\sigma_x\cos\phi_1+\sigma_y\sin\phi_1)\otimes(\sigma_x\cos\phi_2+\sigma_y\sin\phi_2)]$$

Naturally compatible decoherence model is “collective dephasing”

XY Hamiltonian generating SM gates provides commutant structure

$$A' \cong \bigoplus_J M'_{n_J} \otimes I_{d_J}$$

$$\Rightarrow$$

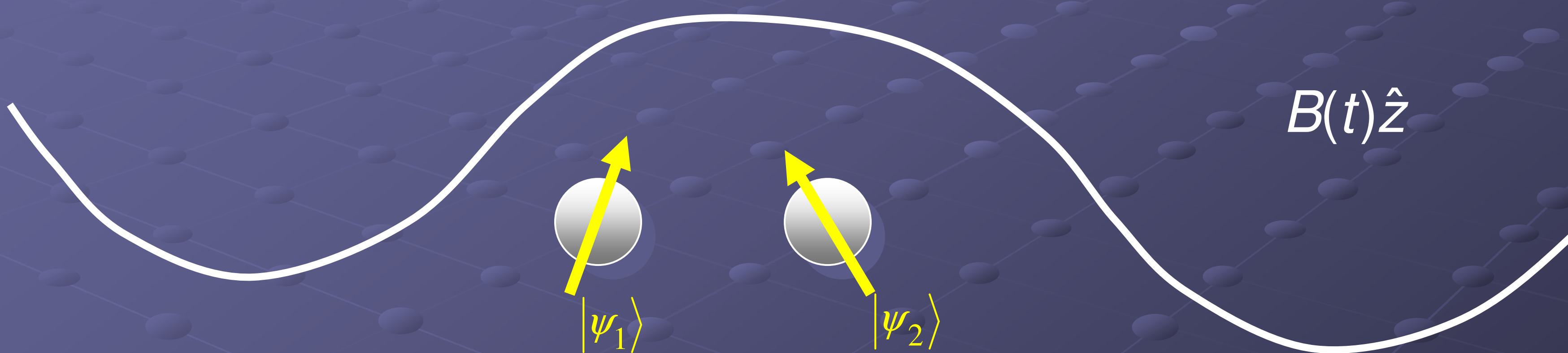
$$A \cong \bigoplus_J I_{n_J} \otimes M_{d_J}$$

The "collective dephasing" algebra

$\Rightarrow$ Have option to encode into collective dephasing DFS

Collective Dephasing

Often (e.g., spin boson model at low temperatures) errors on different qubits are ***correlated***



Long-wavelength magnetic field *B* (environment) couples to spins

Effect: Random "**Collective Dephasing**":

$$\left|\psi_j\right\rangle = a_j\left|0\right\rangle_j + b_j\left|1\right\rangle_j \mapsto a_j\left|0\right\rangle_j + e^{i\theta} b_j\left|1\right\rangle_j$$

random *j*-independent phase
(continuously distributed)

DFS encoding

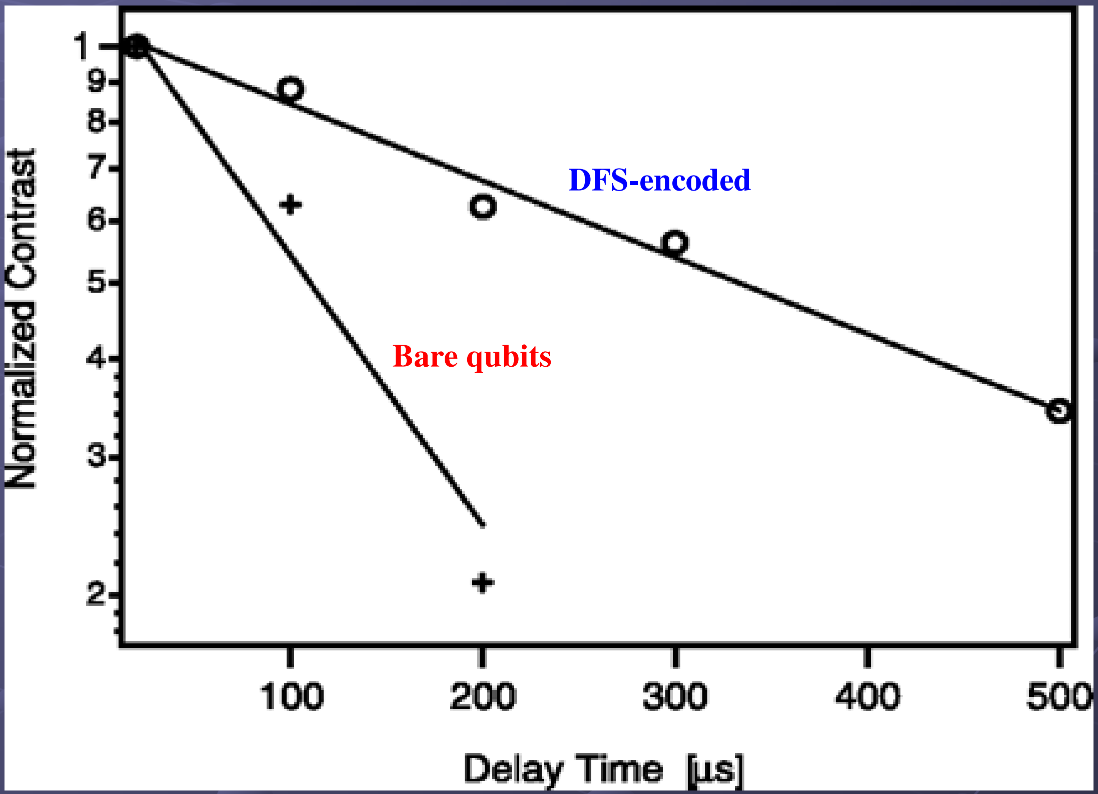
$$\left|0\right\rangle_L = \left|0\right\rangle_1 \otimes \left|1\right\rangle_2$$
$$\left|1\right\rangle_L = \left|1\right\rangle_1 \otimes \left|0\right\rangle_2$$

“A Decoherence-Free Quantum Memory Using Trapped Ions”
D. Kielpinski et al., Science **291**, 1013 (2001)

Bare qubit:
two hyperfine states of trapped $^{9}Be^{+}$ ion

Chief decoherence sources: **(i) fluctuating long-wavelength ambient magnetic fields;** (ii) heating of ion CM motion during computation

DFS encoding: $|0\rangle_L = |0\rangle_1 \otimes |1\rangle_2$
into pair of ions $|1\rangle_L = |1\rangle_1 \otimes |0\rangle_2$



Other sources of decoherence necessarily appear…
Can we *enforce* the symmetry?

Beyond collective dephasing

Classification of *all* decoherence processes on two qubits:

$$H_{SB} = H_{DFS} + H_{Leak} + H_{Logical}$$

$$|0\rangle_L = |0\rangle_1 \otimes |1\rangle_2$$
$$|1\rangle_L = |1\rangle_1 \otimes |0\rangle_2$$

immune

collective dephasing

σ_z

storage

$$H_{DFS} = \{\frac{ZI+IZ}{2}, \frac{XY+YX}{2}, \frac{XX-YY}{2}, ZZ, II\} \otimes B$$

differential dephasing

$$H_{Logical} = \{\bar{X} = \frac{XX+YY}{2}, \bar{Y} = \frac{YX-XY}{2}, \bar{Z} = \frac{ZI-IZ}{2}\} \otimes B$$

motional decoherence

computation

$$H_{Leak} = \{XI, IX, YI, IY, XZ, ZX, YZ, ZY\} \otimes B$$

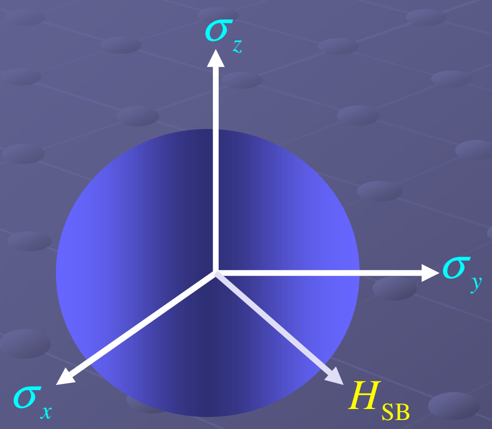
Enforce DFS conditions by “bang-bang” pulses

"Bang-Bang" Decoupling

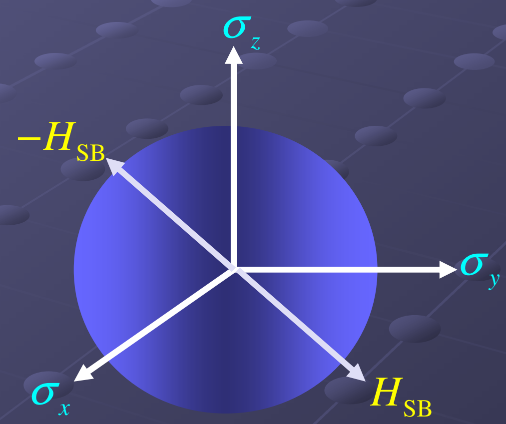
Viola & Lloyd PRA **58**, 2733 (1998), inspired by NMR

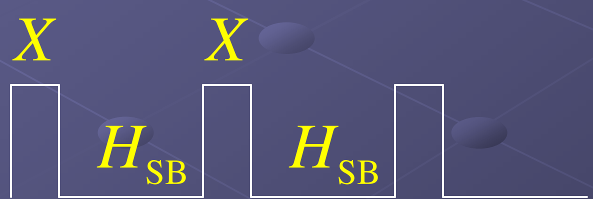
System-bath Hamiltonian: $H_{\text{SB}} = \sum_{\alpha} S_{\alpha} \otimes B_{\alpha}$

system (→ S_{α}) bath (→ B_{α})



Apply rapid pulses flipping sign of S_{α}





$$H_{\text{SB}} = \lambda Z \otimes B$$

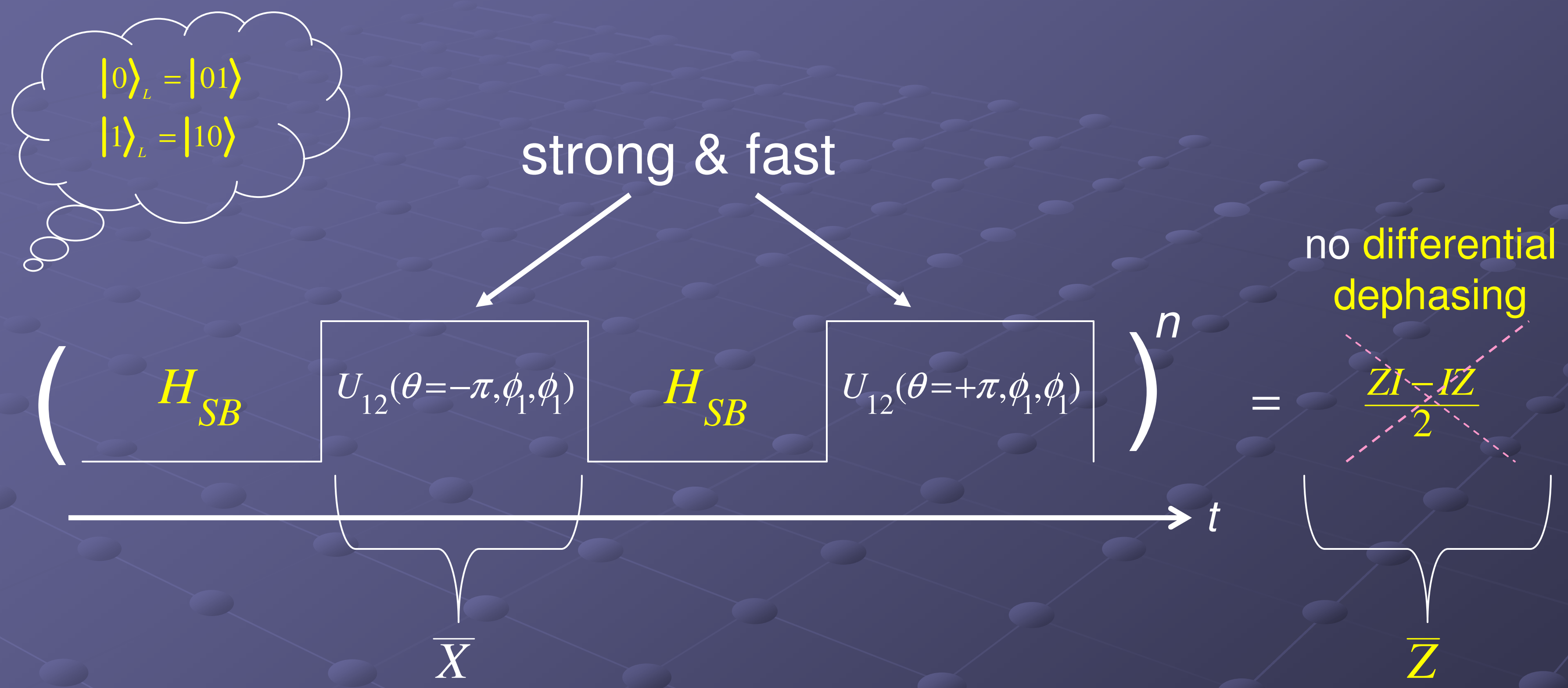
$XZX = -Z \quad \Rightarrow$

"time reversal",

H_{SB} averaged to zero.

Unlike spin-echo, BB relies in essential way on non-Markovian bath; information is retrieved before it's lost to bath.

Eliminating Logical Errors Using "Bang-Bang" SM Gate

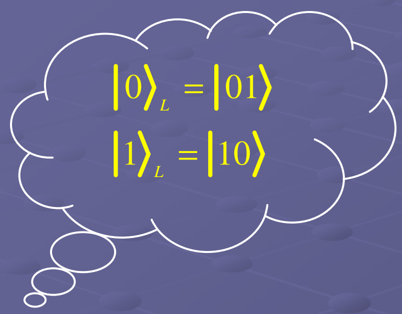


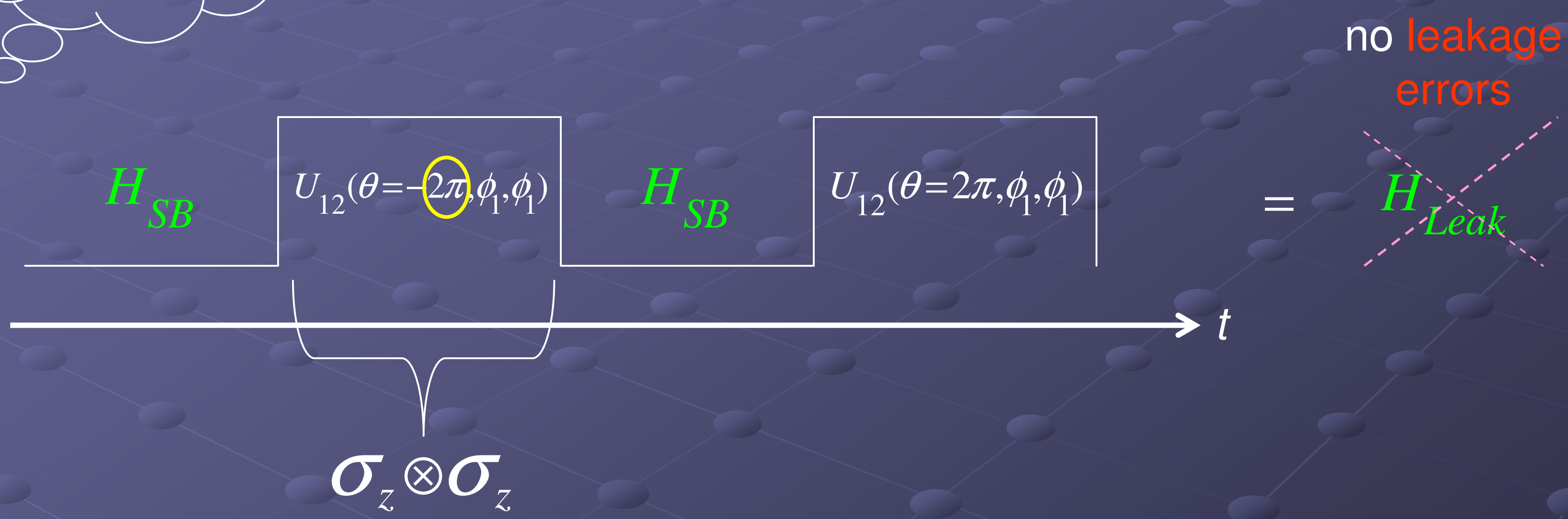
$$\overline{X}\,\overline{Z}\,\overline{X} = -\overline{Z}$$

Also holds for $\overline{Y}$: $\overline{X}\,\overline{Y}\,\overline{X} = -\overline{Y}$

$\therefore \quad \overline{Y} = \frac{YX - XY}{2}$ error also eliminated

Eliminating Leakage Errors Using "Bang-Bang" SM Gate





$$\sigma_z \otimes \sigma_z H_{Leak} \sigma_z \otimes \sigma_z = -H_{Leak}$$

$$H_{Leak} = \{XI, IX, YI, IY, XZ, ZX, YZ, ZY\} \otimes B$$

For general "leakage elimination via BB" see Wu, Byrd, D.A.L., *Phys. Rev. Lett.* **89**, 127901 (2002)

Universal Leakage Elimination Using BB Decoupling

L.-A. Wu, M.S. Byrd, D.A.L., *Phys. Rev. Lett.* **89**, 127901 (2002)

Qubit $\{|0\rangle,|1\rangle\}$ (physical or encoded) is part of larger Hilbert space

Arrange states so $\mathrm{H}=\{|0\rangle,|1\rangle,\ldots,|N\rangle\}$

Leakage is *mixing* of qubit states with other states in H

Can be unitary (Tian & Lloyd, PRA **52**, 050301 (2000)) or bath-induced

Classify all system operators as

$$E := \begin{pmatrix} \overbrace{B}^{2} & \overbrace{0}^{N-2} \\ 0 & 0 \end{pmatrix} \qquad E^{\perp} := \begin{pmatrix} \overbrace{0}^{2} & \overbrace{0}^{N-2} \\ 0 & C \end{pmatrix} \qquad L := \begin{pmatrix} \overbrace{0}^{2} & \overbrace{D}^{N-2} \\ F & 0 \end{pmatrix}$$

Logical operations — Ortho. subspace — Leakage

Define "Leakage elimination operator" as $R := \begin{pmatrix} \overbrace{-I}^{2} & \overbrace{0}^{N-2} \\ 0 & I \end{pmatrix}$

Then $\{R,L\}=0$ i.e., R "time-reverses" L

furthermore $[R,E]=[R,E^{\perp}]=0$ so compatible with logic operations

SM Pulses are Universal on |01>,|10> Code

$$|0\rangle_L = |0\rangle_1 \otimes |1\rangle_2$$
$$|1\rangle_L = |1\rangle_1 \otimes |0\rangle_2$$

$$U_{12}(\theta,\phi_1,\phi_2) = \exp[i\theta(\sigma_x \cos\phi_1 + \sigma_y \sin\phi_1)\otimes(\sigma_x \cos\phi_2 + \sigma_y \sin\phi_2)]$$

$$\overset{DFS}{\mapsto} \exp[i\theta(\bar{X}\cos(\phi_1-\phi_2)+\bar{Y}\sin(\phi_1-\phi_2))]$$

$\therefore$ Can generate a universal set of logic gates by controlling ***relative*** laser phase
- all single DFS-qubit operations
- controlled-phase gate between two DFS qubits

[Also: D. Kielpinski *et al*. Nature **417**, 709 (2002), K. Brown *et al*., PRA **67**, 012309 (2003)]

Similar conclusions apply to XY & XXZ models of solid-state physics (e.g., q. dots in cavities, electrons on He): D.A.L., L.-A. Wu, *Phys. Rev. Lett*. **88**, 017905 (2002)

$$H_S = \sum_i \varepsilon_i \sigma_i^z + \sum_{i<j} \frac{J_{ij}^x}{2}\left(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y\right) + J_{ij}^z \sigma_i^z\sigma_j^z$$

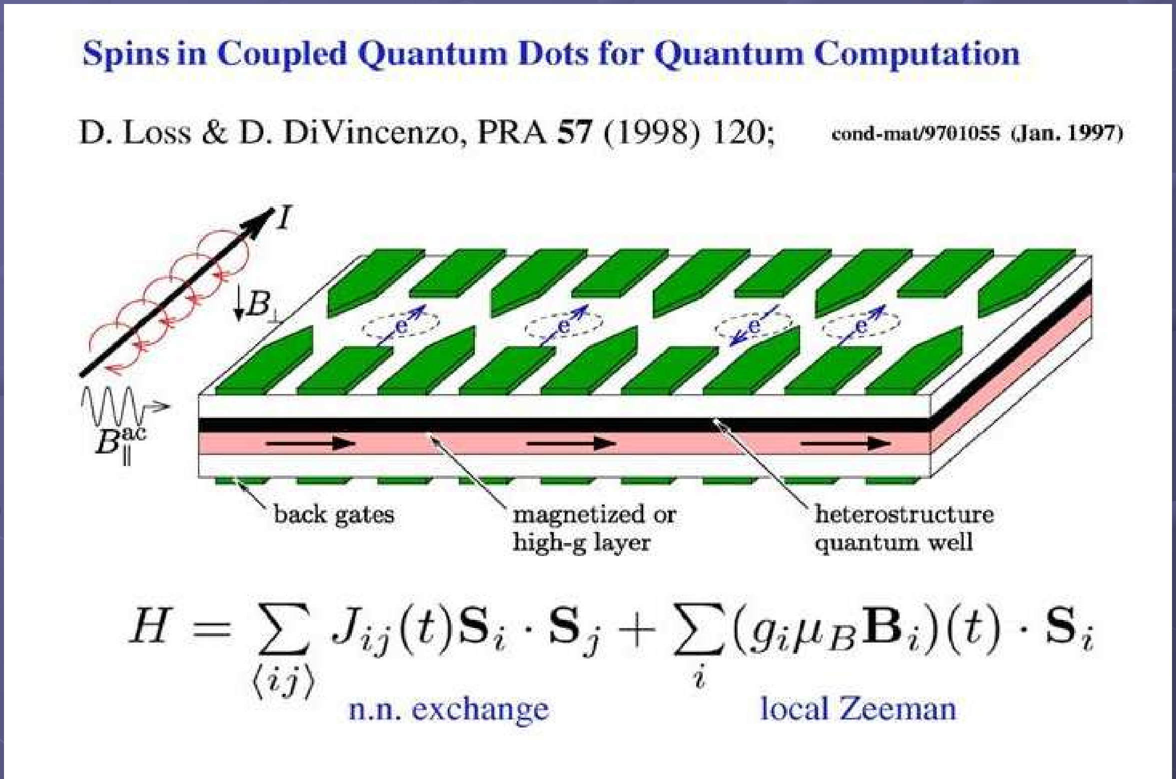
Control assumption for universality over $|01\rangle, |10\rangle$: $\varepsilon_i - \varepsilon_{i+1}, J_{i,i+1}^x$.

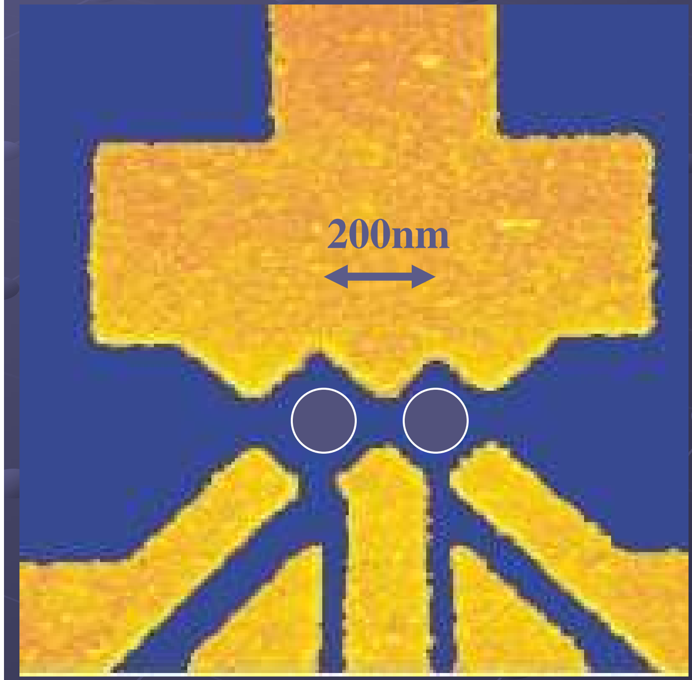
SM and XY/XXZ Pulses are "Super-Universal"

- For trapped ions can eliminate all dominant errors (differential dephasing + leakage) in a 4-pulse sequence
- To eliminate ALL two-qubit errors (including $\bar{X}$) need a 10-pulse sequence.
- Scheme entirely compatible with SM or XY/XXZ-based gates to perform universal QC inside DFS.

For details, see: D.A.L. and L.-A. Wu, *Phys. Rev. A* **67**, 032313 (2003).

Further applications: Quantum Dots





Heisenberg Systems

Same method works, e.g., for *spin-coupled quantum dots* QC:

By BB pulsing of $H_{\text{Heis}} = \frac{J}{2}\left(\sigma_1^x\sigma_2^x + \sigma_1^y\sigma_2^y + \sigma_1^z\sigma_2^z\right)$

collective decoherence conditions can be created:

$$H_{\text{SB}} = \sum_{i=1}^{n} g_i^x \sigma_i^x \otimes B_i^x + g_i^y \sigma_i^y \otimes B_i^y + g_i^z \sigma_i^z \otimes B_i^z$$
$$\longrightarrow S_x \otimes B_x + S_y \otimes B_y + S_z \otimes B_z$$

Requires sequence of 6 $\pi/2$ pulses to create collective decoherence conditions over blocks of 4 qubits. Leakage elimination requires 7 more pulses.

Details: L.-A. Wu, D.A.L., *Phys. Rev. Lett.* **88**, 207902 (2002); L.A. Wu, M.S. Byrd, D.A.L., *Phys. Rev. Lett.* **89**, 127901 (2002).

Earlier DFS work showed universal QC with Heisenberg interaction alone possible [Bacon, Kempe, D.A.L., Whaley, *Phys. Rev. Lett.* **85**, 1758 (2000)]:

Heisenberg interaction is "super-universal"

On to fault-tolerance…

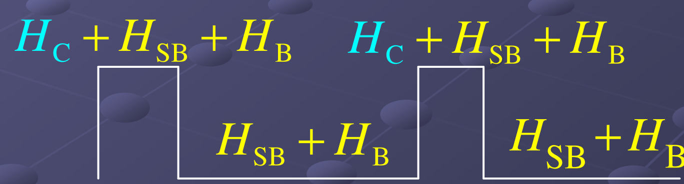
(with Kaveh Khodjasteh)

We have neglected so far:

- Control inaccuracy in BB pulse implementation (systematic + random)

Composite pulses (NMR)   Concatenated QECC

- H_{SB}, H_B on during BB pulse



- Time constraints on BB pulses

Related to transition q. Zeno à inverse q. Zeno effect; form of bath spectral density plays crucial role

K. Shiokawa, D.A.L., *Phys. Rev. A* **69**, 030302(R) (2004); P. Facchi, D.A.L., Pascazio *Phys. Rev. A* **69**, 032314 (2004)

All of these issues are shared by QECC:

Fault Tolerant QECC: Assumptions & Requirements

Terhal & Burkard quant-ph/0402104, Alicki quant-ph/0402139:
FT-QECC for *non*-Markovian baths, completely uncorrelated errors

t_0 = time to execute elementary single or two-qubit gate

$\Delta[q_i] =$ (max-min eigenvalues of $H_{\mathrm{SB}}[q_i]$)/2

$\lambda_0 = \max_{i,j}\{\Delta[q_i], \Delta[q_i, q_j]\}$

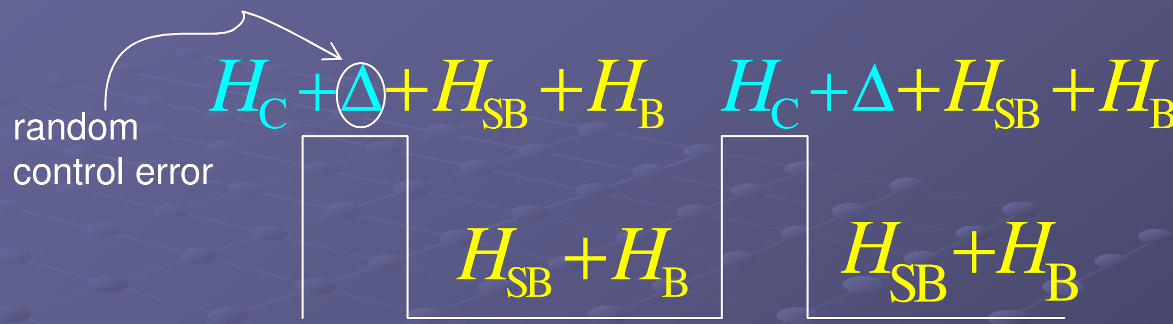
τ_D = non-Markovian decoherence time
[$\tau_D = f$ (fastest bath timescale); Markovian: $\tau_D \sim T_2$]

Threshold condition:
$$\frac{t_0}{\tau_D} \sim (\lambda_0 t_0)^2 \sim 10^{-8} - 10^{-12}$$

∴ Need small t_0: fast gates (time-scale set by bath spectral density/radius)
Need small λ_0: system-bath interaction □ gate amplitude

Not different from BB assumptions!

Dealing with control inaccuracies and “bath on” during BB



Main Effect of BB:

- Renormalize H_{SB}: $\quad H_{SB} = \lambda S \otimes B; \quad \lambda \xrightarrow{\text{BB}} \lambda', \quad \lambda' < \lambda$

Concatenate BB sequences!

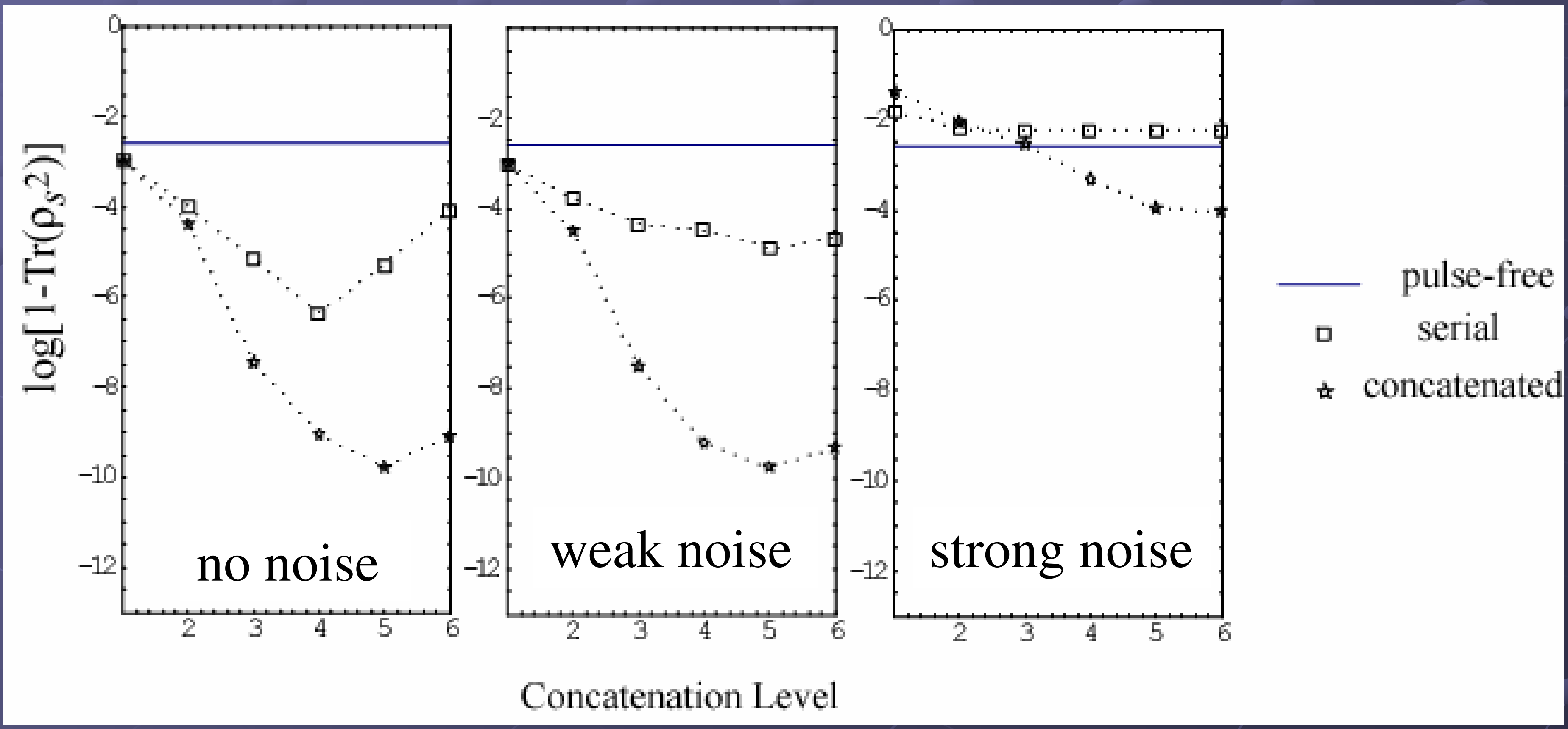
\- Renormalization $\Rightarrow$ effective λ shrinks super-exponentially

total pulse sequence time grows exp.

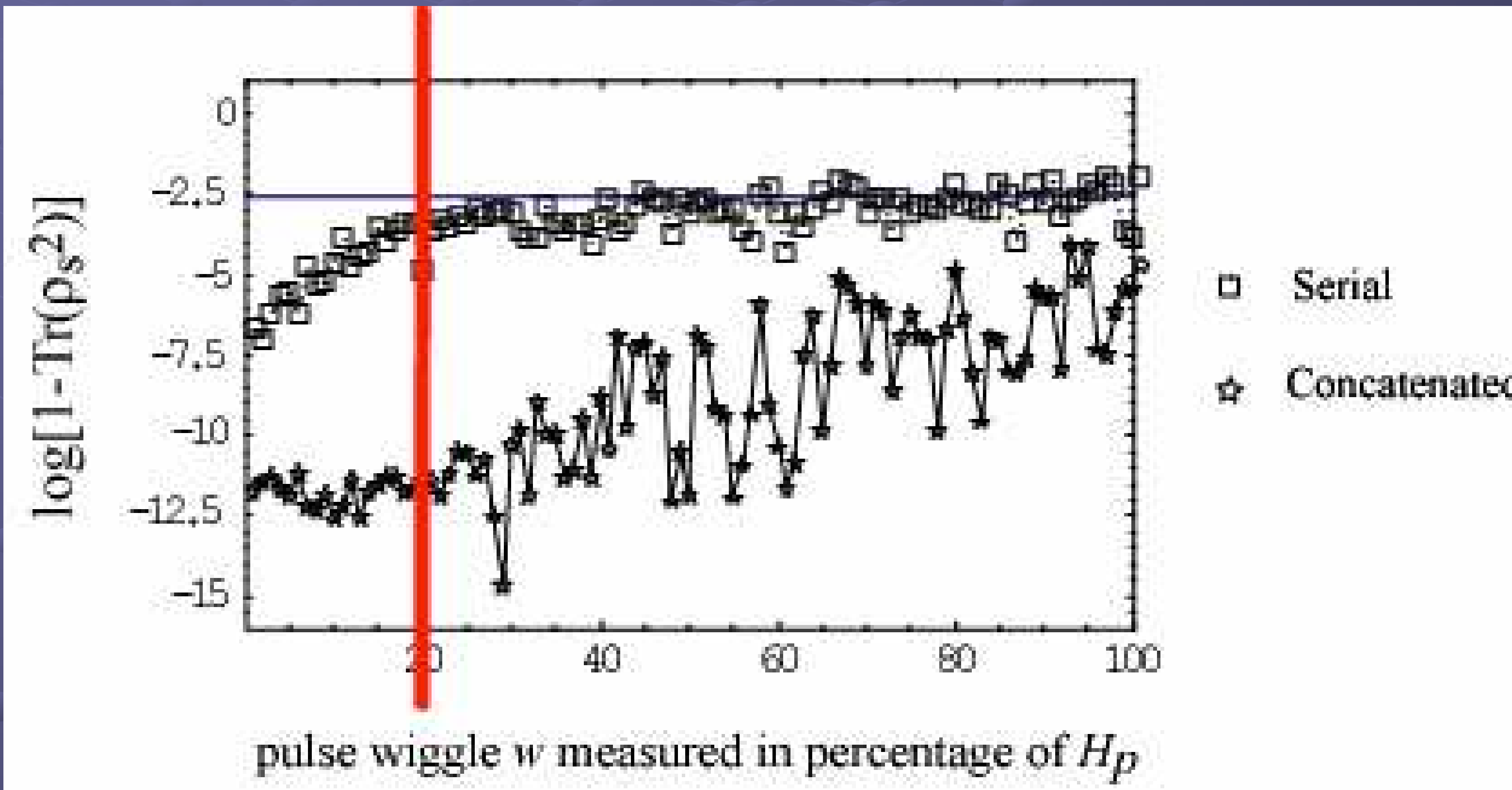
Concatenated BB – Numerical Results

$$H = \omega_s Z_1 + \omega_b \sum_{i=2}^{6} Z_i + \sum_{i>j}^{i,j<6} j_{i,j} H_{ij}$$

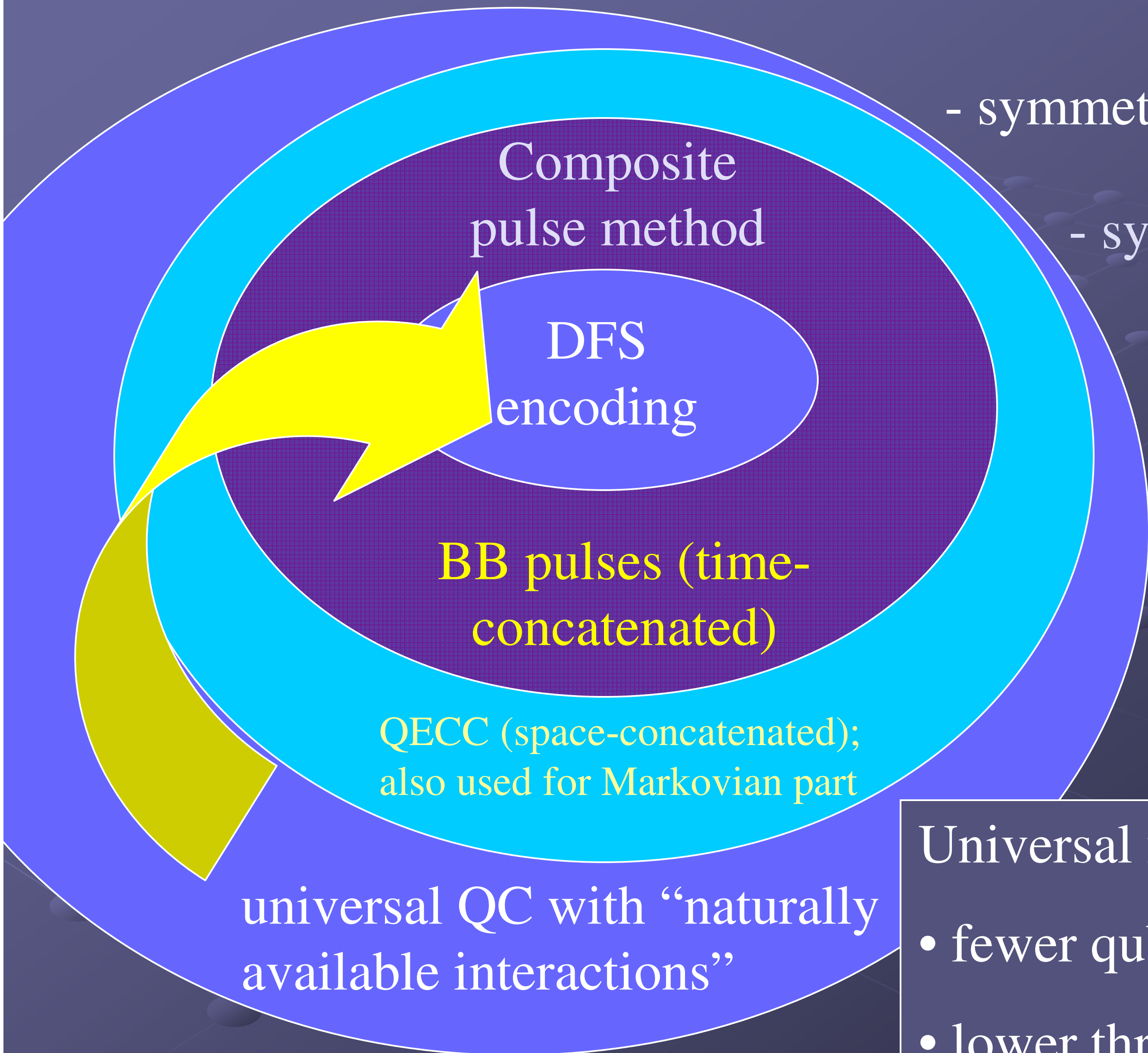
where $H_{ij} = X_i X_j + Y_i Y_j + Z_i Z_j$ is the Heisenberg interaction, $j_{i,j}$ is exponentially decaying coupling.



A phase transition?



Hybrid QECC: The Big Picture



- symmetry not for free…

- systematic (unknown) gate errors

- random gate errors

Universal fault tolerant QC with

- fewer qubits, fewer gates
- lower threshold