Optical Quantum Fingerprinting

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Outline

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A. Motivation

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Fingerprints

- A fingerprint identifies the object/person using relatively little information.
- Especially useful in cases when storage or transmission of data is limited or costly.

Simultaneous Message Passing

- A. C. Yao introduced the simultaneous message passing model in 1979.
- Two parties send bit strings m_A and m_B to a referee who calculates a function $f(m_A, m_B)$.
  - Important for software protection and piracy.
- Fingerprinting in the simultaneous message passing model: f=Eq:

$$\mathrm{Eq}(m_A, m_B) = \begin{cases} 0 & \text{if} m_A \neq m_B, \\ 1 & \text{if} m_A = m_B \end{cases}$$

Quantum Communication Complexity

- Simultaneous message passing is one example of communication complexity, which considers the information transmission required for a specific task.
- Quantum teleportation, remote state preparation and superdense coding are examples of basic q. communication protocols.
- Quantum fingerprinting demonstrates a savings in communication complexity by exploiting quantum vs classical channels.

B. Digital Fingerprinting

Fingerprinting Scheme

Alice

Bob

messages

10001110101….

encoding

{101011 …1000101}

fingerprints

101001…

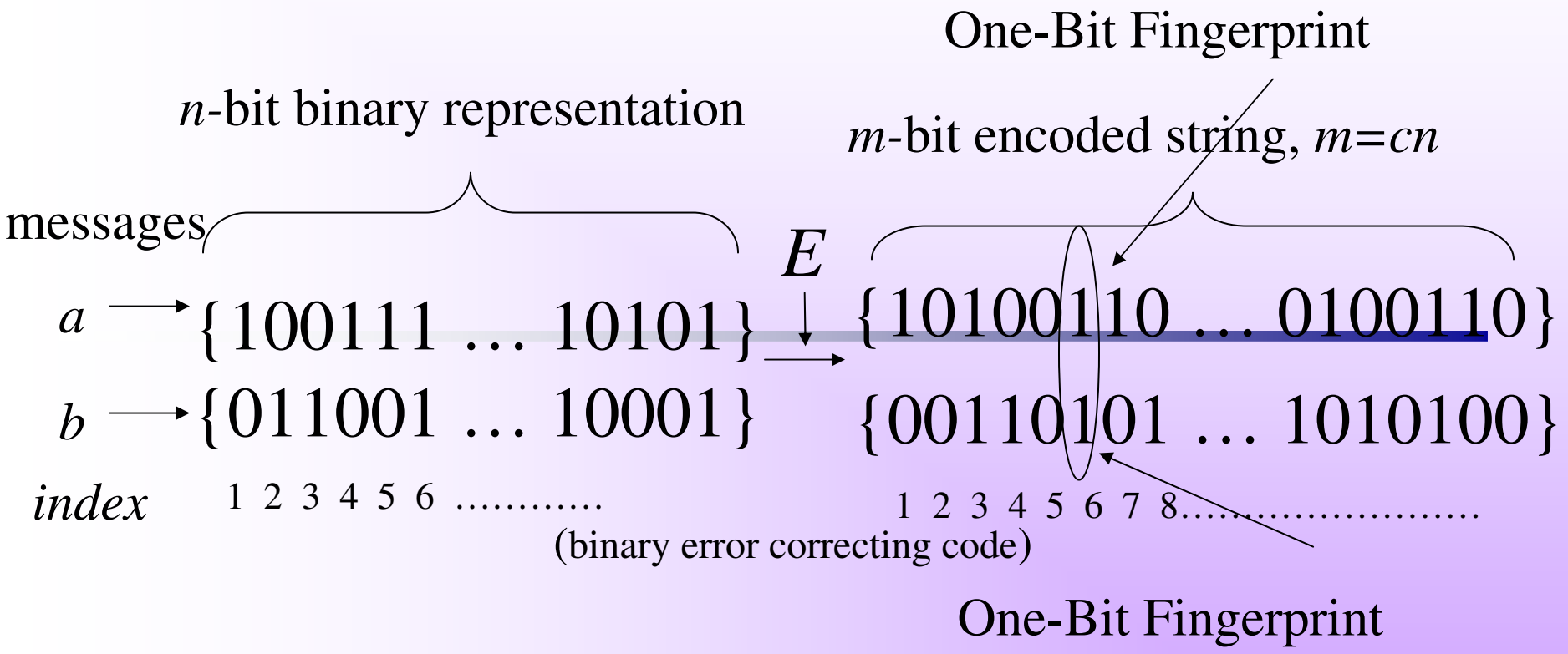
001101…

Roger

Fingerprinting with & w/o shared randomness

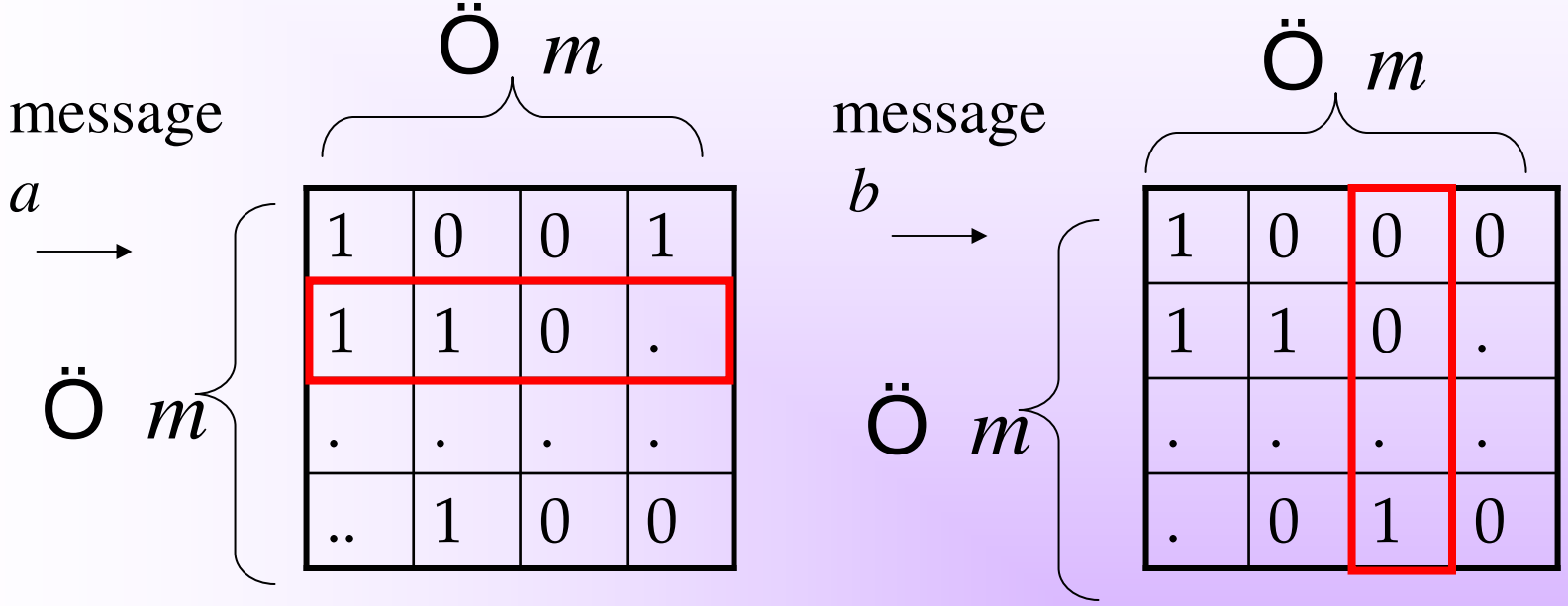
- Without shared randomness: Alice and Bob each independently produce a fingerprint from the message according to some predetermined (but not necessarily deterministic - may have private randomness) process.
- With shared randomness: Alice and Bob use a shared resource to construct the fingerprint.
- For (encoded) message length m, the cost of fingerprinting scales as $\ddot{O}\ m$ (w/o shared randomness) and as $\log m$ (w/o shared randomness).

One-Bit-At-A-Time Fingerprinting



n-bit message to *m*-bit codes to sequence of one-bit fingerprints with index of single bit generated randomly (shared randomness).

Ö m Scaling (no shared randomness)



Row index 1

Fingerprint for b = {01 110…}

Column index 2

Fingerprint for a = {10 00…1}

Errors and Guarantees

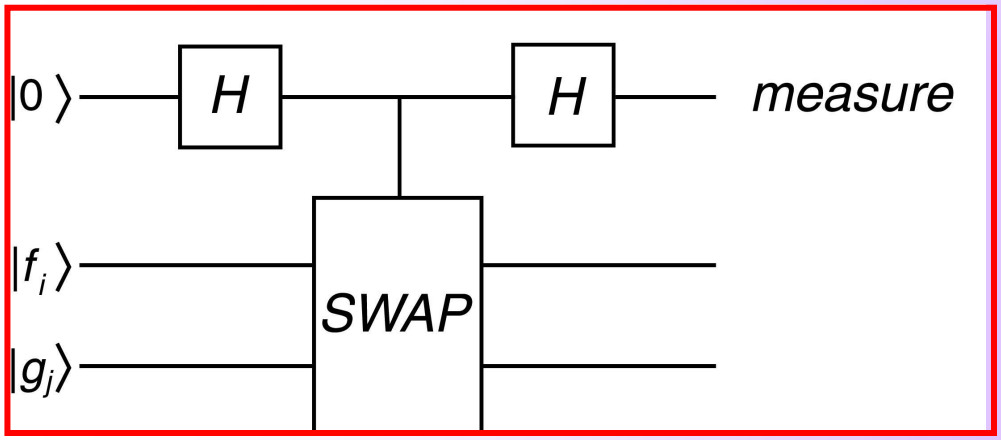
- Fingerprinting can have errors - we concentrate on one-sided errors such that inferring different fingerprints is always correct but inferring same is prone to error.
- One-sided errors can be valuable; for example holding a suspect whilst checking fingerprints: don't want to release a felon but holding an innocent person longer for further checking is acceptable.
- Guarantee is based on Worst-Case Scenario (WCS): malicious supplier produces messages to maximize p_{err}.
- For one-sided error scheme, Roger employs a deterministic protocol (no random numbers)

C. Quantum Fingerprinting

Quantum Fingerprinting

- Buhrman (2001) showed that q. fingerprinting reduces the cost of fingerprinting w/o shared randomness from $\ddot{O}\ m$ to $\log_2 m$.
- Use error correcting codes that maximize Hamming distance between code words $|E_i(m)\rangle$, then transmit state

$$|f(m_A)\rangle = \frac{1}{\sqrt{m}} \sum_{i=1} |i\rangle |E_i(m_A)\rangle$$



- Compare states using ancilla & cSWAP
  - § Referee measures $r=1$ with probability

$$(1 - |\text{Æ}\, f / \ g\text{æ}\,|^2)/2$$

$$(H \otimes I \otimes I)(\text{cSWAP})(H \otimes I \otimes I)|0\rangle_{\text{anc}} |f(m_A)\rangle |f(m_B)\rangle$$

Measurement outcome and inference

- Roger obtains $r=0,1$; infers $\mathrm{Eq}(m_A, m_B)=1-r$.
- One-sided error scheme: same messages yield $r=0$ and different messages yield either $r=0$ (with prob p_{err}) or $r=1$ (with prob p_{success}).

$$p_{\mathrm{err}} = 1 - p_{\mathrm{success}} = (1+\delta)/2 \text{ for } \delta \equiv |\langle f|g\rangle|^2.$$

- In the WCS, supplier always sends different messages so $r=0$ results are errors; thus p_{err} is the one-sided error for the WCS.

D. Optical Quantum Fingerprinting

Optical Q. Fingerprinting

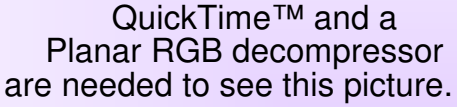
- Qubits can be realized as polarization-encoded single photons.
- The 'quantum advantage' in fingerprinting is not just asymptotic: scales all the way to single-qubit level (de Beaudrap, PRA, 2004).
- cSWAP is not achievable in linear quantum optics, but an optical realization is possible for single-qubit fingerprinting of two-bit messages; we propose such an experiment.

Polarization States

- A single photon can be encoded in a superposition of the two polarization states $|0\rangle$ (eg horizontal) and $|1\rangle$ (eg vertical).
- The single qubit state is parametrized by polar and azimuthal angles on the 'Bloch sphere':

$$|\theta, \phi\rangle = |(\cos\theta, \mathrm{e}^{i\phi}\sin\theta)^{\mathrm{T}}\rangle = \cos\theta|0\rangle + \mathrm{e}^{i\phi}\sin\theta|1\rangle$$

- Best states are widely separated on Bloch sphere; for $M=4$, use 'tetrahedral states'.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Planar RGB decompressor
are needed to see this picture.

‘Tetrahedral States’

$$\text{Introduce } \Omega \equiv (\theta, \phi).$$

The four maximally separated fingerprint states are thus

$$F = \{|\Omega_k\rangle; \Omega_0 = (0,0) \text{ or } \Omega_k = (2\cos^{-1}(1/\sqrt{3}), \frac{2\pi}{3}ik) \text{for } k = 1,2,3\}$$

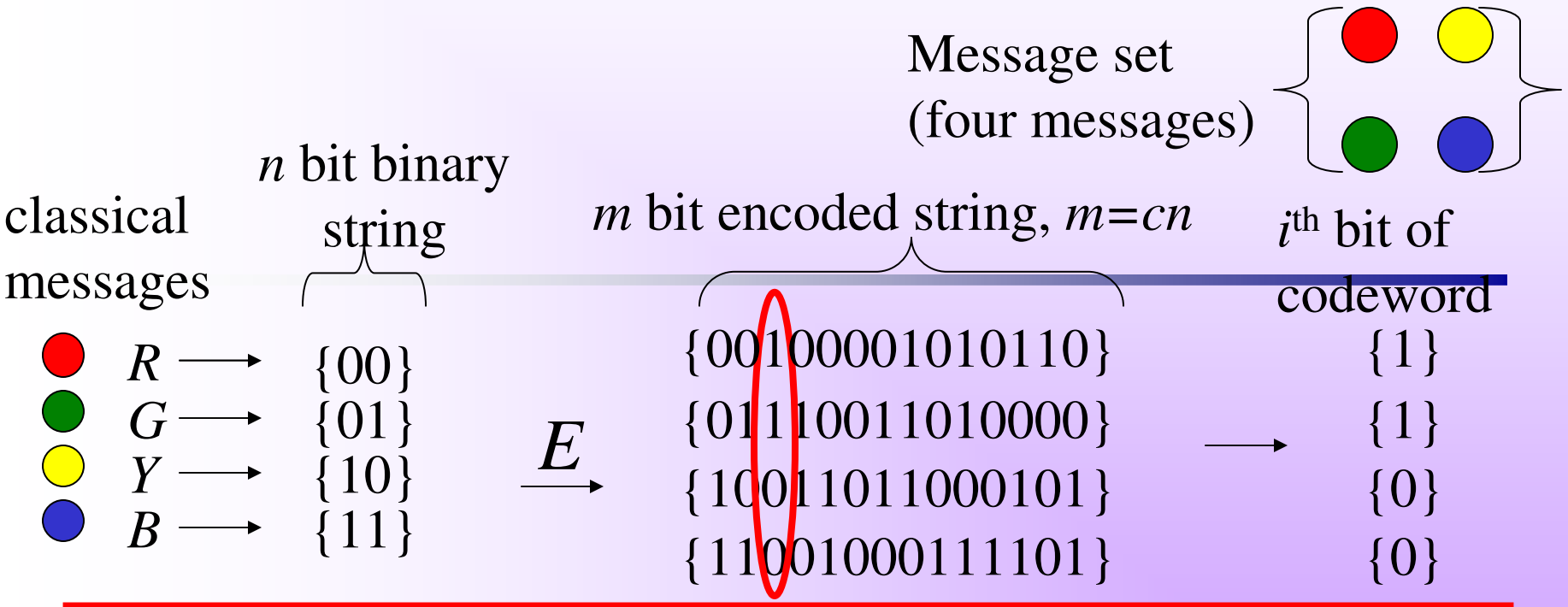
The indistinguishability of any two unequal states is determined by

$$\delta(\Omega', \Omega) \equiv |\langle\Omega'|\Omega\rangle|^2 = \left|\cos\frac{\theta'}{2}\cos\frac{\theta}{2} + \mathrm{e}^{\mathrm{i}(\phi-\phi')}\sin\frac{\theta'}{2}\sin\frac{\theta}{2}\right|^2,$$

which is *1/3* for the tetrahedral states. Thus,

$$p_{\text{success}} = 1 - p_{\text{err}} = 1/3$$

cf: classical WCS one-bit fingerprinting



$a=b \Rightarrow E_i(a) = E_i(b)$ but not converse so, in the worst case scenario (WCS), referee always fails.

Replacing the cSWAP

- No cSWAP in deterministic linear optics.
- Discriminating the tetrahedral states is possible via the Hong-Ou-Mandel dip: the two photons are directed into two ports of a 1:1 beam splitters, and the output is guaranteed *not* to produce a coincidence if the two photons are indistinguishable.
- Seeing a coincidence guarantees they are different with same error as cSWAP:

$$p_{\text{success}} = 1/3$$

Shared Entanglement

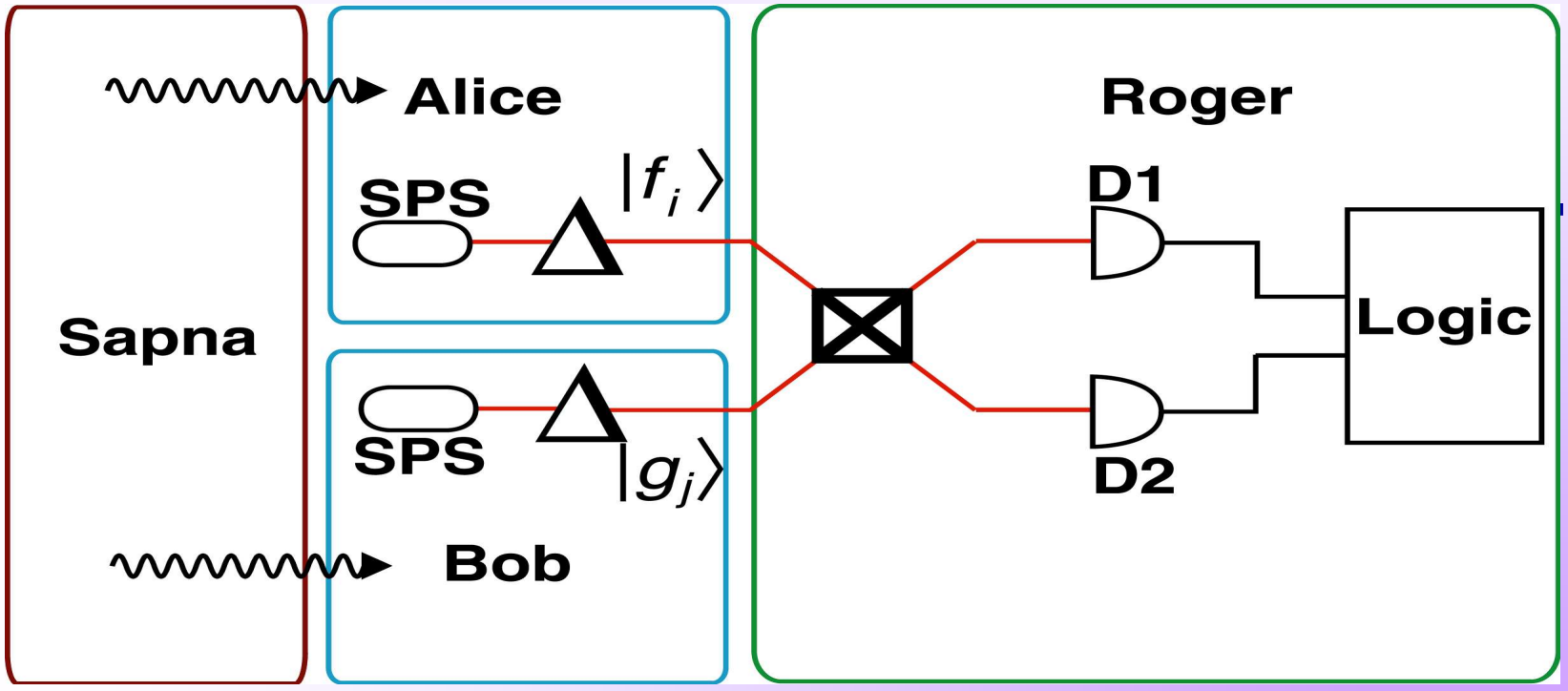
- Suppose Alice and Bob share a pure maximally-entangled two photon Bell 'singlet' state

$$|\Psi^-\rangle \equiv 2^{-1/2}[|01\rangle - |10\rangle]$$

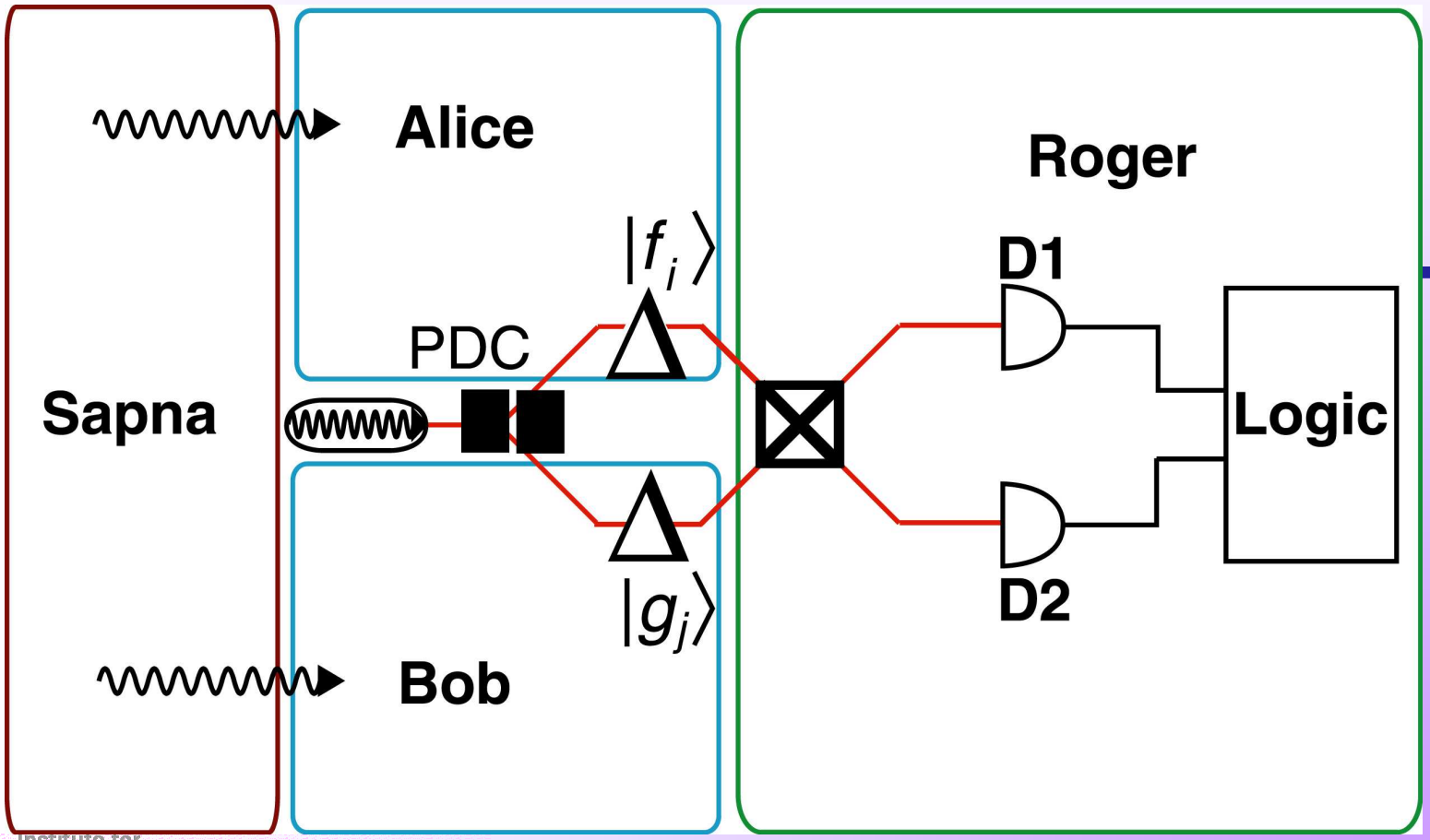
- Alice and Bob each perform one of the Pauli operations σ_X, σ_Y, σ_Z, σ_I depending on message received: the Bell singlet state is invariant if they perform same operation; otherwise uields a different Bell state.
- Shared entanglement produces 100% successful q. fingerprinting, which is unachievable for single-bit fingerprinting regardless of amount of shared random bits: classical bound is $p_{\text{success}} = 2/3$.

E. Proposed Experiment

With Independent Single-Photon Sources



With Shared Parametric Down Converter



F Conclusions

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Conclusions

- *M=4,* single qubit q. fingerprinting is feasible in linear quantum optics.
- Optical q. fingerprinting gives one-sided error success rate of 1/3 in WCS compared to zero for one-bit fingerprinting in WCS.
- Entangled resource produces 100% success rate, which is better than success rate of 2/3 for classical scheme with arbitrarily large shared randomness (Cleve, Horn, Lvovsky, Sanders).
- Scaling to more qubits is possible using postselected cSWAP gate.