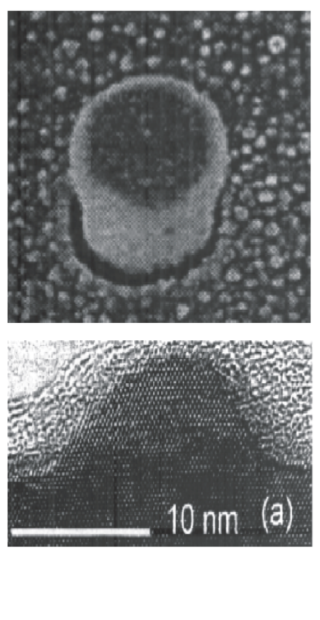
Unavoidable decoherence in QD

L. Jacak, P. Machnikowski, W. Jacak, M. Krzyżosiak

Institute of Physics, Wrocław University of Technology, Poland

Quantum dots – some experimental facts

3D confinement



- Strong coupling regime for carrier—LO-phonon interaction ($\hbar\,\Omega_{LO}\sim\Delta E$)
  Hameau *et al.* PRL **83** (1999) 4152
- Increased interaction with LO phonons (renormalization of Fröhlich constant)
  Hameau *et al.* PRB **65** (2002) 085316
- Relaxation bottleneck: relaxation times ~10-100 ps (1 order of magnitude slower than in bulk) or suppression of relaxation;
  Heitz *et al.* PRB **56** (1997) 10435; **64** (2001) 241305
- Partial dephasing on 1 ps timescale
  Borri *et al.* PRL **87** (2001) 157401
- Limited coherent control
  Zrenner *et al.* Nature **418** (2002) 612

Renormalization of the Fröhlich constant for electron in QD

$$\vec{P}_{lok}(\vec{r})=\vec{P}_0(\vec{r})-\vec{P}_\infty(\vec{r})$$

local crystal polarization acting on band electron

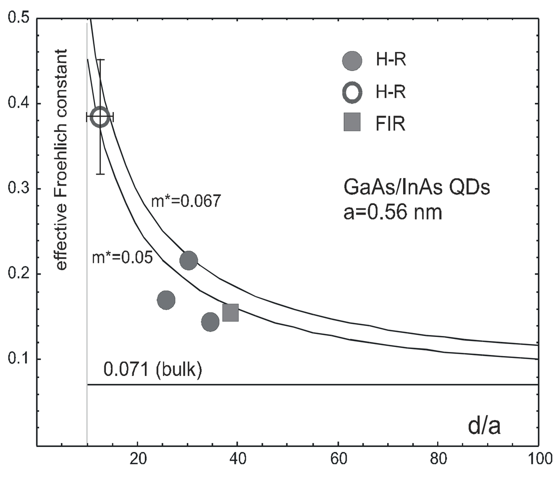
inertial part of local polarization

$$\alpha=\frac{e^2}{\tilde{\varepsilon}}\sqrt{\frac{m^*}{2\hbar\Omega}},\qquad \frac{1}{\tilde{\varepsilon}}=\frac{1}{\varepsilon_\infty}-\frac{1}{\varepsilon_0}$$

$$\vec{P}_{lok}(\vec{r})=\vec{P}_0(\vec{r})-\vec{P}_{QD}(\vec{r})$$

local crystal polarization acting on electron in QD

$$\frac{1}{\tilde{\varepsilon}}=\frac{1-\frac{a}{d}}{\varepsilon_\infty}-\frac{1}{\varepsilon_0}+\frac{a}{d}$$



L.Jacak, J.Krasnyj, W.Jacak, PLA 304, 168 (2002)

Amplitude decoherence - relaxation

Hamiltonian of : QD + LO +LA +TA + int$_{QD-LO}$ + int$_{QD-LA}$ + anh$_{LO-TA}$

$$H=H_0+H_1$$

$$H_0=\sum_n \varepsilon_n a_n^+a_n+\sum_{\vec{k}}\hbar\Omega_{\vec{k}}b_{\vec{k}}^+b_{\vec{k}}+\sum_{s,\vec{q}}\hbar\omega_{s,\vec{q}}c_{s,\vec{q}}^+c_{s,\vec{q}}$$

Fröhlich constant

$$H_1=\frac{1}{\sqrt{N}}\sum_{n_1,n_2,\vec{k}}F_{n_1,n_2}^{(o)}(\vec{k})a_{n_1}^+a_{n_2}(b_{\vec{k}}+b_{-\vec{k}}^+)+\frac{1}{\sqrt{N}}\sum_{n_1,n_2,\vec{q}}F_{n_1,n_2}^{(a)}(\vec{q})\,a_{n_1}^+a_{n_2}(c_{l,\vec{q}}+c_{l,-\vec{q}}^+)+\sum_{\vec{k}_1,\vec{k}_2,\vec{q}}W(\vec{k}_1,\vec{k}_2,\vec{q})\delta_{\vec{k}_1,\vec{k}_2+\vec{q}}b_{\vec{k}_1}^+b_{\vec{k}_2}(c_{t,\vec{q}}+c_{t,-\vec{q}}^+)$$

$$F_{n_1,n_2}^{(o)}(\vec{k})=\sqrt{4\pi\alpha\frac{\hbar^2}{2m}}\ldots f_{n_1,n_2}(\vec{k})$$

$$F_{n_1,n_2}^{(a)}(\vec{q})=\sigma\sqrt{\frac{\hbar q}{2MC_l}}f_{n_1,n_2}(\vec{q})$$

$$f_{n_1,n_2}(\vec{k})=\int d^3r\Psi_{n_1}^*(\vec{r})e^{i\vec{k}\cdot\vec{r}}\Psi_{n_2}(\vec{r})$$

bottle-neck

anharmonism LO-TA

conservation of energy and „momentum"

Davydov diagonalization – calculation method

$$\alpha=e^S a e^{-S},\qquad \beta=e^S b e^{-S}$$

where (Davydov, Pestryakov 1972)

$$S=\frac{1}{\sqrt{N}}\sum_{n_1,n_2,\vec{q}}\Phi_{n_1,n_2}(\vec{q})\,a_{n_1}^+a_{n_2}(b_{l,\vec{q}}-b_{l,-\vec{q}}^+)$$

H_0 non-perturbatively diagonalized ⇒ polaron

H_1 ⇒ polaron relaxation (perturbatively)

$$H_0=\sum_n E_n\alpha_n^+\alpha_n+\sum_{\vec{k}}\hbar\Omega\beta_{\vec{k}}^+\beta_{\vec{k}}+\sum_{s,\vec{q}}\hbar\omega_{s,\vec{q}}c_{s,\vec{q}}^+c_{s,\vec{q}}$$

$$E_n=\varepsilon_n+\frac{1}{N}\sum_{\vec{k},l}\frac{|F_{n,l}^{(o)}(\vec{k})|^2}{E_n-E_l-\hbar\Omega}$$

$$H_1=\frac{1}{\sqrt{N}}\sum_{n_1,n_2,\vec{q}}F_{n_1,n_2}^{(a)}(\vec{q})\,\alpha_{n_1}^+\alpha_{n_2}(c_{l,\vec{q}}+c_{l,-\vec{q}}^+)+\sum_{\vec{k}_1,\vec{k}_2,\vec{q}}W(\vec{k}_1,\vec{k}_2,\vec{q})\delta_{\vec{k}_1,\vec{k}_2+\vec{q}}\beta_{\vec{k}_1}^+\beta_{\vec{k}_2}(c_{t,\vec{q}}+c_{t,-\vec{q}}^+)+\sum_{n_1,n_2,\vec{q},\vec{k}}\tilde{W}_{n_1,n_2}^s(\vec{q},\vec{k})\,\alpha_{n_1}^+\alpha_{n_2}\beta_{\vec{k}}(c_{s,\vec{q}}+c_{s,\vec{q}}^+)+h.c.$$

s=t

$$\tilde{W}_{n_1,n_2}^{t}(\vec{q},\vec{k})=\frac{1}{\sqrt{N}}\frac{F_{n_1,n_2}^{(o)}(\vec{k}+\vec{q})W(\vec{k}+\vec{q},\vec{k},\vec{q})}{E_{n_1}-E_{n_2}+\hbar\Omega}$$

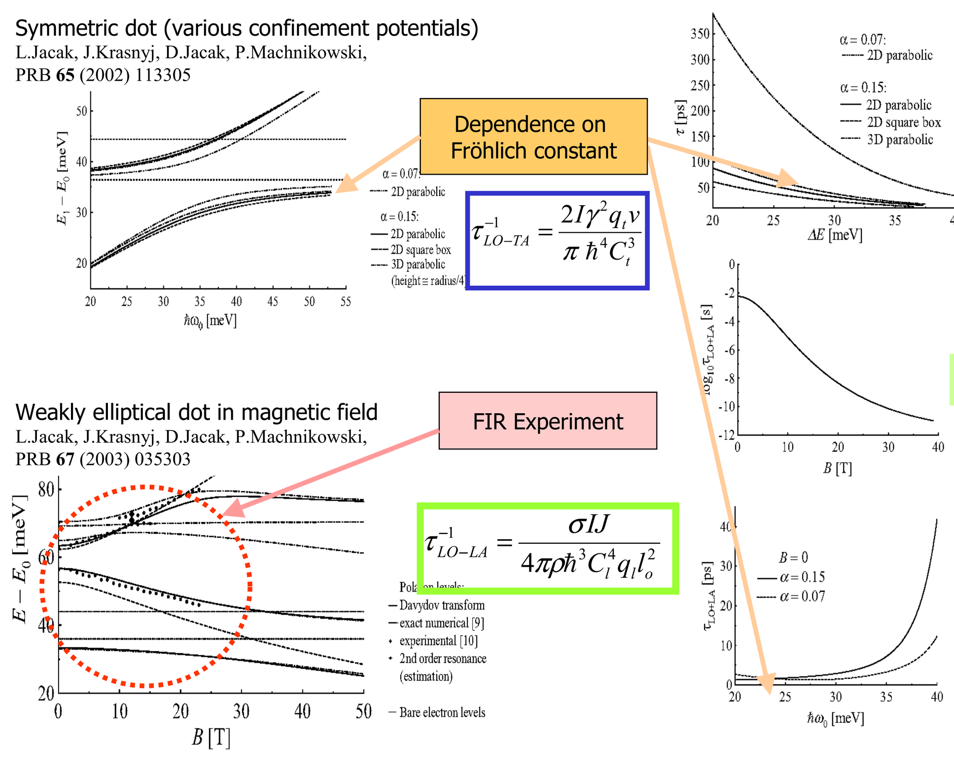
LO-TA channel

s=l

$$\tilde{W}_{n_1,n_2}^{l}(\vec{q},\vec{k})=-\frac{1}{N}\sum_l\left[\frac{F_{n_1,l}^{(o)}(\vec{k})F_{l,n_2}^{(a)}(\vec{q})}{E_{n_1}-E_l+\hbar\Omega}-\frac{F_{n_1,l}^{(a)}(\vec{k})F_{l,n_2}^{(o)}(\vec{q})}{E_l-E_{n_2}+\hbar\Omega}\right]$$

LO-LA channel

Polaron spectrum and relaxation rates



Relaxation-related conclusions

relaxation in QDs can be diminished by suitable choice of geometrical/energy QD parameters (up to 50 – 100 ps)

3 classes of effects decide on relaxation times of orbital degrees of freedom in QDs ⟶

- *coherent electron-LO phonon interaction (polaron)* – polaron dispersion of polaron is pushing out of resonaces (the larger Fröhlich constant the stronger pushing out)
- *bottle-neck* – strongly dumps 2-phonon (anharmonic) and 1-phonon (direct) relaxations out of resonances
- *finite width of LA(TA) phonon band* – (relatively small for TA phonons in GaAs)

Exciton dressing with phonon modes

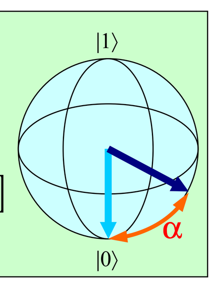
$$H=H_{ex}+H_{ph}+H_{ex-ph}$$

(LA, LO phonons)

Relation to QIP → fidelity:

$$F=1-\delta$$

$$\delta=\frac{1}{2}\sin^2 2\alpha\left[1-\mathrm{Re}\left(G_c(t)e^{iE}\right)\right]$$



Causal Green function:

$$G_c(n_1,n_2,t)=-\frac{i}{\hbar}\left\langle T\{a_{n_1}(t)a_{n_2}^+\}\right\rangle$$

$$\Delta_n(\omega)=\frac{1}{N}\sum_{\vec{k},n_1}|F_o(n,n_1,\vec{k})|^2\left[\frac{1+N_{\vec{k},o}}{\hbar\omega-E_{n_1}-\Delta_{n_1}(\omega-\Omega)-\hbar\Omega}+\frac{N_{\vec{k},o}}{\hbar\omega-E_{n_1}-\Delta_{n_1}(\omega-\Omega)+\hbar\Omega}\right]$$

$$+\frac{1}{N}\sum_{\vec{k},n_1}|F_a(n,n_1,\vec{k})|^2\left[\frac{1+N_{\vec{k},a}}{\hbar\omega-E_{n_1}-\Delta_{n_1}(\omega-C_ak)-\hbar C_ak}+\frac{N_{\vec{k},a}}{\hbar\omega-E_{n_1}-\Delta_{n_1}(\omega-C_ak)+\hbar C_ak}\right]$$

$$\gamma_n(\omega)=\frac{\pi}{N}\sum_{\vec{k},n_1}|F_o(n,n_1,\vec{k})|^2\left[(1+N_{\vec{k},o})\delta(\hbar\omega-E_{n_1}-\Delta_{n_1}-\hbar\Omega_1)+N_{\vec{k},o}\delta(\hbar\omega-E_{n_1}-\Delta_{n_1}+\hbar\Omega_1)\right]$$

$$+\frac{\pi}{N}\sum_{\vec{k},n_1}|F_a(n,n_1,\vec{k})|^2\left[(1+N_{\vec{k},a})\delta(\hbar\omega-E_{n_1}-\Delta_{n_1}-\hbar C_ak)+N_{\vec{k},a}\delta(\hbar\omega-E_{n_1}-\Delta_{n_1}+\hbar C_ak)\right]$$

Exciton Green function in QD – analytical properties

$$G_c(0,0,\omega)=\frac{1}{\hbar\omega-E_0-\Delta(\omega)+i\gamma(\omega)+i\varepsilon}$$

$$x=\hbar\omega-E_0-\Delta$$

$$I(t)=-2\hbar\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{Im}\,G_c(0,0,\omega)e^{-i\omega t}d\omega$$

$$|F_o(0,0,\vec{k})|^2\cong g_0\frac{k^2}{k_m}e^{-\alpha k^2}$$

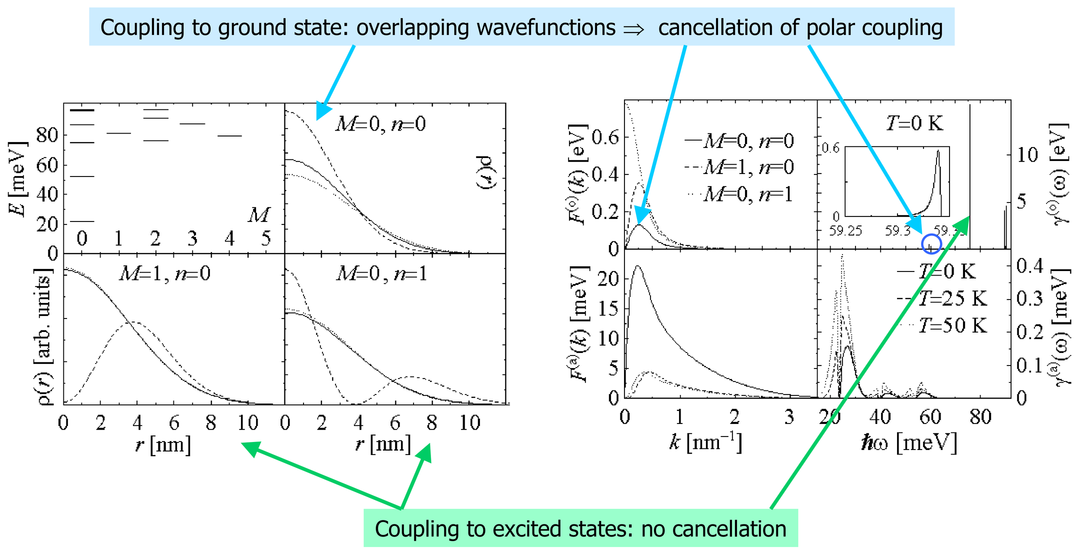
$$|F_a(0,0,\vec{k})|^2\cong g_1\frac{k}{k_m}e^{-\alpha k^2}$$

$$\mathrm{Im}\,G_c(0,0,\omega)=-a^{-1}\pi\delta(x)+\frac{a^{-1}\gamma'(x)/x^2}{1+(\gamma'(x)/x)^2}$$

$$\gamma(x)=Ax^3e^{-\frac{\alpha x^2}{\hbar^2C_a^2}}\left[\Theta(x)(1+N(x))-\Theta(-x)N(-x)\right]$$

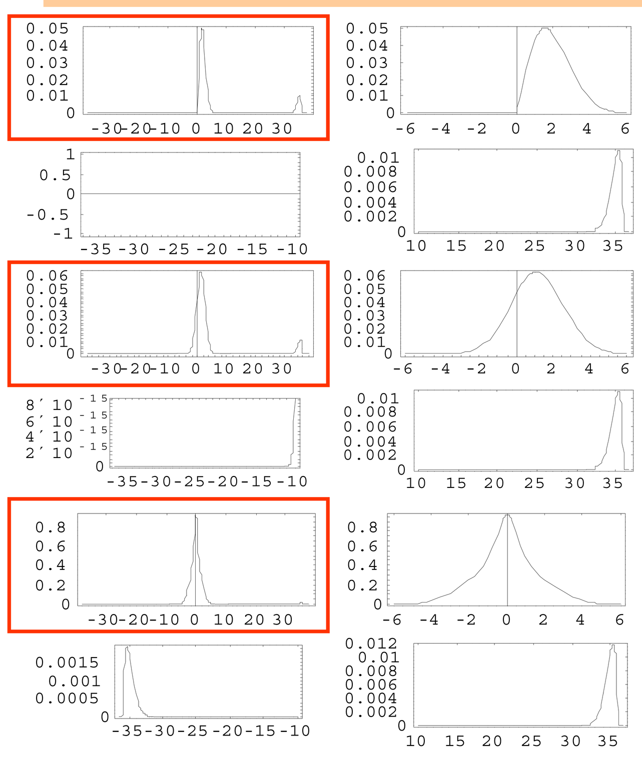
$$+B\left[\begin{array}{l}\Theta(x)(\hbar\Omega-x)(\hbar\Omega-x)^{3/2}e^{-\frac{\alpha(\hbar\Omega-x)}{\hbar\beta}}\Theta(-0.9\hbar\Omega+x)(1+N(x))\\+\Theta(\hbar\Omega+x)(\hbar\Omega+x)^{3/2}e^{-\frac{\alpha(\hbar\Omega+x)}{\hbar\beta}}\Theta(-0.9\hbar\Omega-x)N(-x)\end{array}\right]$$

Exciton wavefunctions and phonon coupling

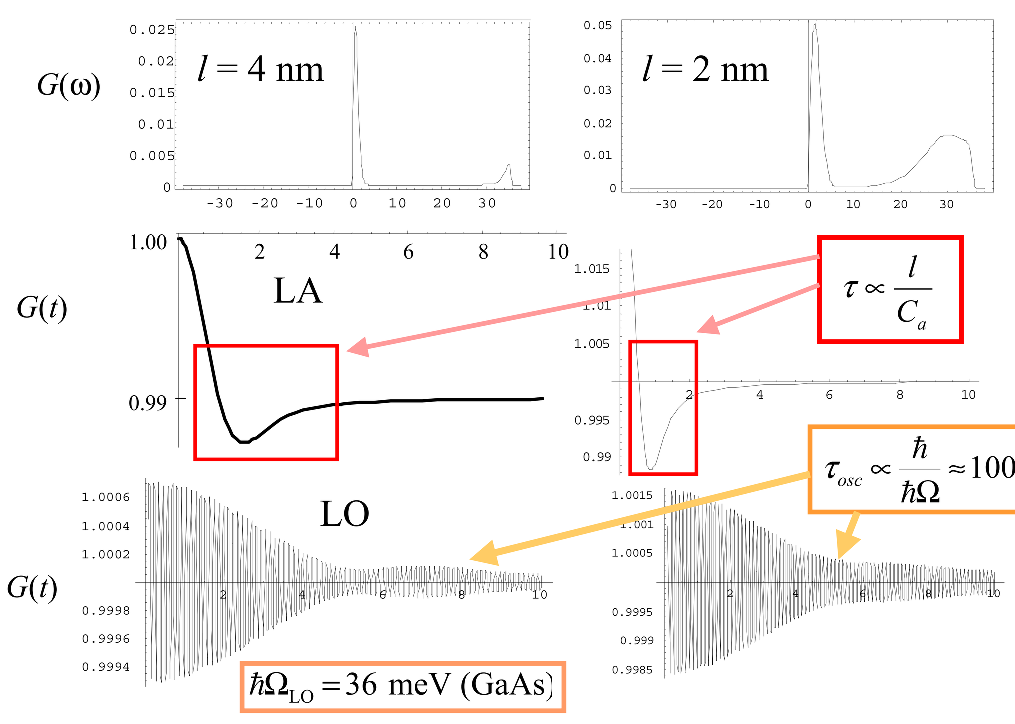


L.Jacak, J.Krasnyj, P.Machnikowski, P. Zoller, Eur. Phys. J. D **22** (2003) 319

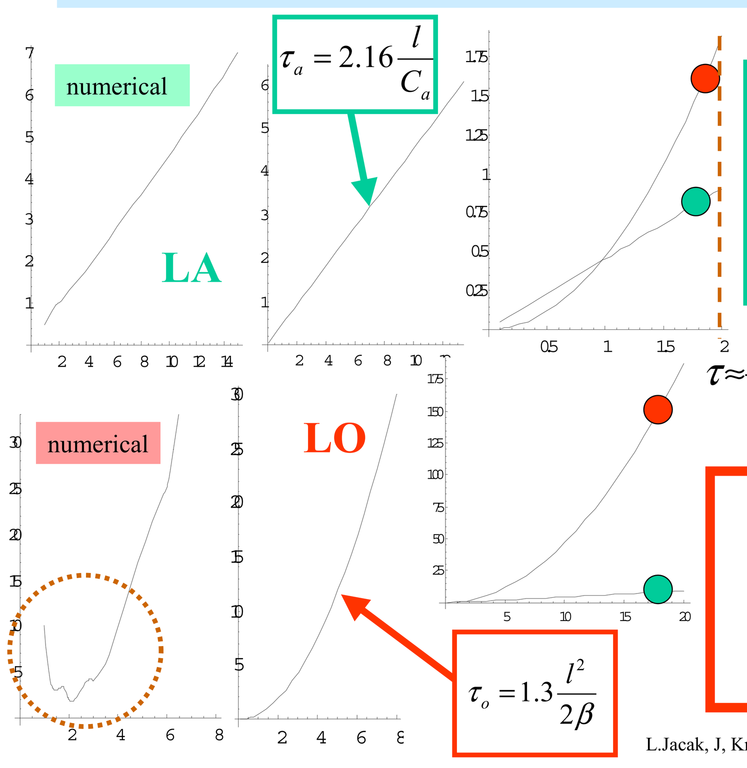
Exciton Green function $G(\omega)$



Exciton dephasing: $G(t)$ (correlation function)



Phase decoherence of exciton in QDs



$$\tau_a\approx\frac{l}{C_a}$$

$$\tau\approx\frac{l}{v_o},\quad v_o=\frac{\partial\omega}{\partial p}\approx 2\beta p,\quad p\approx\frac{1}{l}$$

$$\tau_o\approx\frac{l^2}{2\beta}$$

L.Jacak, J. Krasnyj, P. Machnikowski, P. Zoller (in prep.)

Orbital degrees of freedom in QD

Phonons LA

unavoidable quick (ps) phase decoherence in all nanostructures in crystalline

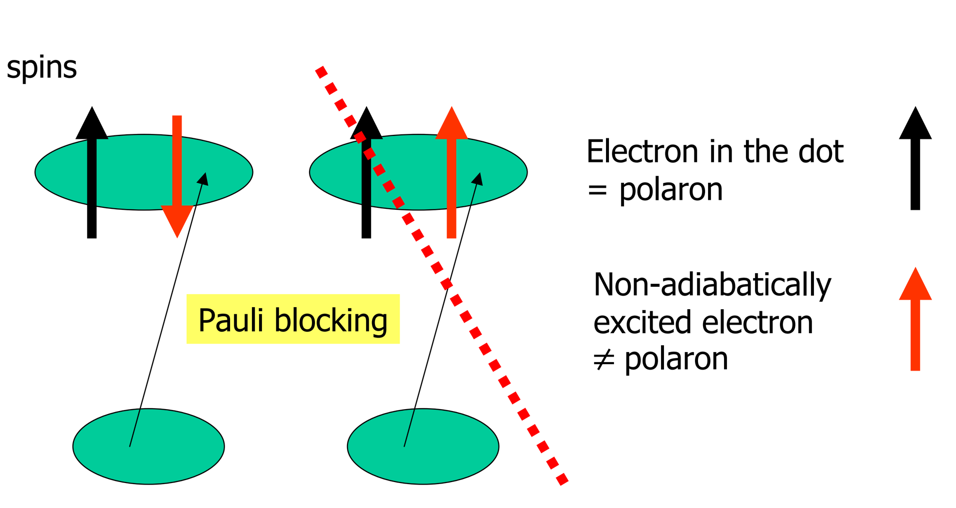
Phonons LO

strong polaronic effects

anharmonism (LO-TA) also quick dephasing

broadening of exciton line – anharmonism and geometrical effects

Violation of Pauli blockade under non-adiabatic switching



Different particles ⟶ violation of Pauli blockade

Conclusions – related to phase decoherence

- Dephasing (~ 1ps) precludes sub-picosecond optical-type coherent control of orbital degreees of freedom in QDs
- Spin degrees of freedom manipulated by Pauli blocking - slower than ps scale (limit for spin-charge conversion)
- Dephasing causes limits for Rabi oscillations in QDs
- QDs spectroscopy is completely different than atomic spectroscopy