Trapping states of light in a micromaser

V. I. Koroli
Institute of Applied Physics, Academy of Sciences of Moldova,
Academiei str. 5, Kishinau MD-2028, Moldova

Abstract

We examine the interaction between the cavity field and a string of equidistant three-level atoms of cascade type. The model with the intensity-dependent coupling is proposed. The matrix elements of the dipole transitions between adjacent levels are equal. Trapping conditions for the flying time in the cavity as well as the explicit state of the field are found. The properties of cavity fields are examined. In various limits, the state exhibits sub-Poissonian and super-Poissonian statistics and squeezing properties.

1 Introduction

The creation of one-atom micromasers has prompted experimental studies of a single atom interacting with the resonant mode of cavity fields in recent years (Rempe et al 1987, Brune et al 1987). Much interest in quantum optics has been generated by such devices, especially, in connection with the trapping states in the micromaser. As an example, the trapping effect occurs (Slosser et al 1989) when a two-level atom enters the cavity in the coherent state $\alpha|+\rangle + \beta|-\rangle$, where we have defined the ground and excited states of the two-level atom as $|-\rangle$ and $|+\rangle$, respectively. The state will no longer be able to change as long as the flying time τ in the cavity satisfies the trapping conditions $\sqrt{N_d}k\tau = q\pi$ for even integer q and $\sqrt{N_u+1}k\tau = p\pi$ for odd integer p. The cavity field is assumed to be in a pure state $\sum_{n=N_d}^{N_u} s_n|n\rangle$ with N_u and N_d representing the upper and lower boundaries of the Fock states.

We consider a three-level atom of cascade type with the ground state, first and second excited states denoted by $|g\rangle$, $|i\rangle$ and $|e\rangle$, respectively. In the case the levels are equally spaced, the trapping condition in a cavity is already found when the dipole transition matrix elements between adjacent levels are assumed to be the same (Enaki and Koroli 1999). The initial state $\alpha|e\rangle+\beta|i\rangle+\gamma|g\rangle$ differs from the final state $\alpha|e\rangle-\beta|i\rangle+\gamma|g\rangle$ by a phase change. The trapping conditions for the flying time τ are given by $\sqrt{2}\tau k\sqrt{2N_u+3} = p\pi$ for even integers p and $\sqrt{2}\tau k\sqrt{2N_d-1} = q\pi$ for even integers q. It is shown that such a three-level atom is equivalent to a pair of indistinguishable two-level atoms (Enaki and Koroli 1999, Wehner et al 1994).

It has been shown that under certain circumstances, a simple quantum harmonic oscillator driven by a quantum current evolves to unique pure states even if it starts as a mixed state (Slosser et al 1989). It has also been shown that in various limits, these states exhibit nonclassical properties such as sub-Poissonian statistics (Short and Mandel 1983, Rempe et al 1990), or more interestingly resemble a macroscopic superposition. Moreover, it is found from the analysis of coherent trapping states in a lossless two-photon micromaser that the field evolves to a pure state, which may be a superposition of even or odd photon number states (Orszag et al 1992). These states may exhibit perfect second-order squeezing behavior as the upper limit when the trapping increases.

The trapping-state solution is obtained in a two-channel Raman micromaser, in which three-level atoms of Λ-type are injected through the two-mode cavity (Puri et al 1994). It is noted that the problem is greatly simplified in paper of Puri et al 1994 by considering the interaction of two-level atoms with a single-mode field. Some important results connecting with the trapping states in a micromaser are also obtained (Enaki et al 2003).

We consider in this paper the interaction of the single-mode cavity with a string of three-level atoms of equally spaced energy levels. It should be noted that we investigate the three-level model with the intensity-dependent coupling (Buzek 1989). This model to a marked degree differs from the ordinary three-level one (Enaki and Koroli 1999). The atoms are supposed to have equal transition matrix elements between levels in the cascade configuration. As a result, the problem is equivalent to the interaction of pairs of indistinguishable two-level atoms with a single-mode cavity field, provided that the separation between the pair is smaller than the wavelength of the cavity field. It is clear that the beam of pairs of two-level atoms must satisfy super-Poissonian statistics rather than Poissonian statistics (Wehner et al 1994, Orszag et al 1994).

In the present work, we obtain the trapping-state solution for a three-level atom in the cavity without further simplification, and investigate the properties of the field in the trapping state. The state exhibits sub-Poissonian and super-Poissonian statistics and squeezing properties.

2 Trapping-state solution

We consider, as usual, a micromaser in which a very high-Q cavity is pumped by a string of atoms at a rate so low that there is at most one atom at a time in the cavity (Meschede et al 1985a, Meschede et al 1985b). The incident beam consists of three-level atoms of cascade type with equally spaced levels but different dipole transition matrix elements. Suppose that the pumping is regular so that the time interval τ_p between the arrival of two successive atoms remains unchanged (Puri et al 1994). Assume further that the cavity is lossless. Then the evolution of the coupled atom-field system during the time interval t_{int} for an atom flying through the cavity is described by the unitary operator $U(t_{int})$. If ρ_a is the density matrix for an atom entering the cavity, then the density matrix $\rho(k+1)$ for the field at the instant when the $(k+1)th$ atom exits is given by (Filipowicz et al 1986a, Meystre et al 1988)

$$\rho(k+1) = tr_a[U(t_{int})\rho_a U_0(\tau_p - t_{int})\rho(k)]. \tag{1}$$

Here tr_a indicates the trace taking over atomic states and U_0 is a unitary operator describing the free field evolution from the instant one atom leaves the cavity to the arrival of the next one.

The system of the electron interacting with a cavity field in the case of the intensity-dependent coupling is described by the following Hamiltonian in the rotating-wave approximation ($\hbar = 1$)

$$H = \omega_0 S_z + \omega a^\dagger a + \lambda(R^\dagger S^- + RS^+), \tag{2}$$

where

$$\begin{aligned} S^+ &= \sqrt{2}\{|e_2\rangle\langle e_1| + |e_1\rangle\langle g|\}, \\ S^- &= \sqrt{2}\{|e_1\rangle\langle e_2| + |g\rangle\langle e_1|\}, \\ S_z &= |e_2\rangle\langle e_2| - |g\rangle\langle g|. \end{aligned} \tag{3}$$

The constant λ is a real number; ω and ω_0 are the frequences of the atom and the field, respectively. The oparators R and $R^\dagger$ are

$$R = a\sqrt{N}, R^\dagger = \sqrt{N}a^\dagger, \tag{4}$$

with the commutation relations

$$[R, R^\dagger] = 2N + 1,\ [R, N] = R,\ [R^\dagger, N] = -R^\dagger, \tag{5}$$

where $N = a^\dagger a$ is the photon number operator. It should be noted that the Hamiltonian (2) effectively describes the intensity-dependent coupling between the atom and the single-mode cavity field.

Taking into account that the SU(1,1) Lie algebra for a single-mode field may be realized as

$$K_+ = R^\dagger, K_- = R, K_0 = N + \frac{1}{2}, \tag{6}$$

with the commutation relations for the generators K_0 and $K_\pm$,

$$[K_-, K_0] = 2K_0, [K_0, K_\pm] = \pm K_\pm, \tag{7}$$

the Hamiltonian (2) may be represented by the following:

$$H = \omega_0\sigma_3 + \omega(K_0 - \frac{1}{2}) + \lambda(K_+S^- + K_-S^+). \tag{8}$$

We emphasize that the Hamiltonian (8) has the same form (up to constant factors) as that in Gerry's paper (Gerry 1988). But in our model the realization of the operators K_0 and $K_\pm$ is fundamentally different, because Gerry has analyzed the case with

$$K_0 = (a^\dagger a + \frac{1}{2})/2, K_+ = (a^\dagger)^2/2, K_- = a^2/2. \tag{9}$$

The state-vector of the system at time t is given by

$$|\Psi(t)\rangle = \sum_{n=N_d}^{N_u} s_n|n\rangle(\alpha|e\rangle + \beta|i\rangle + \gamma|g\rangle), \tag{10}$$

in which we have expanded the one-mode cavity field in the Fock space $\sum_{n=N_d}^{N_u} s_n|n\rangle$. When the atom enters the cavity at time t', the state evolves according to the $Schr\ddot{o}dinger$ equation

$$i\hbar\frac{\partial}{\partial t'}|\Psi(t')\rangle = H|\Psi(t')\rangle. \tag{11}$$

We consider the resonance case when $\omega = \omega_0$. The detuning case results in a cumbersome expression and is not very interesting. The state for the coupled system is

$$\begin{aligned}
|\Psi(t+\tau)\rangle &= \exp(-iH\tau/\hbar)|\Psi(t)\rangle = \exp[-i\omega_0(S_z + K_0 - 1/2)\tau] \\
&\times \Bigg\{ \sum_{n=N_d}^{N_u} s_n\Big[\alpha\left(1 + \frac{1}{2}\frac{(n+1)^2}{n^2+1+3(n+\frac{1}{2})}\left\{\cos\left(2\tau k\sqrt{n^2+1+3(n+\frac{1}{2})}\right) - 1\right\}\right)|n\rangle \\
&- \frac{i\beta}{\sqrt{2}}\frac{n}{\sqrt{n^2+n+\frac{1}{2}}}\sin\left(2\tau k\sqrt{n^2+n+\frac{1}{2}}\right)|n-1\rangle \\
&+ \frac{\gamma}{2}\frac{n(n-1)}{n^2+1-(n+\frac{1}{2})}\left\{\cos\left(2\tau k\sqrt{n^2+1-(n+\frac{1}{2})}\right) - 1\right\}|n-2\rangle\Big]|e_2\rangle \\
&+ \sum_{n=N_d}^{N_u} s_n\Big[\beta\cos\left(2\tau k\sqrt{n^2+n+\frac{1}{2}}\right)|n\rangle \\
&- \frac{i\alpha}{\sqrt{2}}\frac{n+1}{\sqrt{n^2+1+3(n+\frac{1}{2})}}\sin\left(2\tau k\sqrt{n^2+1+3(n+\frac{1}{2})}\right)|n+1\rangle \\
&- \frac{i\gamma}{\sqrt{2}}\frac{n}{\sqrt{n^2+1-(n+\frac{1}{2})}}\sin\left(2\tau k\sqrt{n^2+1-(n+\frac{1}{2})}\right)|n-1\rangle\Big]|e_1\rangle \\
&+ \sum_{n=N_d}^{N_u} s_n\Big[\left(\gamma + \frac{1}{2}\frac{\gamma n^2}{n^2+1-(n+\frac{1}{2})}\left\{\cos\left(2\tau k\sqrt{n^2+1-(n+\frac{1}{2})}\right) - 1\right\}\right)|n\rangle \\
&+ \frac{\alpha}{2}\frac{(n+1)(n+2)}{n^2+1+3(n+\frac{1}{2})}\left\{\cos\left(2\tau k\sqrt{n^2+1+3(n+\frac{1}{2})}\right) - 1\right\}|n+2\rangle \\
&- \frac{i\beta}{\sqrt{2}}\frac{n+1}{\sqrt{n^2+n+\frac{1}{2}}}\sin\left(2\tau k\sqrt{n^2+n+\frac{1}{2}}\right)|n+1\rangle\Big]|g\rangle\Bigg\}.
\end{aligned} \tag{12}$$

The expansion coefficients s_n of the cavity field in the Fock space satisfy the recursion relation

$$s_n = -i\frac{\sqrt{2}\beta}{\gamma}\cot\left(\tau k\sqrt{n^2-n+\frac{1}{2}}\right)\frac{\sqrt{n^2-n+\frac{1}{2}}}{n}s_{n-1} - \frac{\alpha}{\gamma}\frac{n-1}{n}s_{n-2} \tag{13}$$

and the trapping conditions for the flying time τ are given by

$$2\tau k\sqrt{N_d^2 - N_d + \frac{1}{2}} = q\pi, \quad for\ even\ q \tag{14}$$

and

$$2\tau k\sqrt{N_u^2+3N_u+\frac{5}{2}}=p\pi, \quad for\ even\ p. \tag{15}$$

It is clear that Eq. (13) cannot be expressed as a cotangent or tangent function for the two-level atom (Slosser et al 1989, Li and Lin 1997, Hillery and Skvarcek 1998). Hence the atom or the pair remains in the coherent state and the state-vector of the coupled atom-field system will be

$$|\Psi(t+\tau)\rangle = \exp[-i\omega_0(S_z+K_0-\frac{1}{2})\tau]\sum_{n=N_d}^{N_u} s_n|n\rangle(\alpha|e\rangle-\beta|i\rangle+\gamma|g\rangle) \tag{16}$$

when the atom or the pair leaves the cavity at instant $t+\tau$. Equation (16) indicates that after a free evolution, the system has returned to its initial state before the atom enters the cavity. The atomic and cavity field states become separable if s_n satisfies the recursion relation (13).

Further we consider the most practical case, $N_d = 0$, in which τ satisfies the condition (15) only. The relevant initial conditions typically include the vacuum state $|0\rangle$ with the recursion relation given by Eq. (13).

3 The state-vector of the cavity field

The state-vector of the cavity field can be found analytically by using the recursion formula (13). It is observed from Eq. (13) that $s_{N_d+1} = P_{N_d+1}s_{N_d}$ with

$$P_{N_d+1} = -\frac{i\sqrt{2}\beta}{\gamma}\cot\left(\tau k\sqrt{(N_d+1)^2-(N_d+1)+\frac{1}{2}}\right)\frac{\sqrt{(N_d+1)^2-(N_d+1)+\frac{1}{2}}}{N_d+1}.$$

In a similar fashion, we find that $s_{N_d+2} = P_{N_d+2}s_{N_d+1}$. Here

$$\begin{aligned} P_{N_d+2} &= -\frac{i\sqrt{2}\beta}{\gamma}\cot\left(\tau k\sqrt{(N_d+2)^2-(N_d+2)+\frac{1}{2}}\right)\frac{\sqrt{(N_d+2)^2-(N_d+2)+\frac{1}{2}}}{N_d+2} \\ &- \frac{\frac{\alpha}{\gamma}\frac{N_d+1}{N_d+2}}{-\frac{i\sqrt{2}\beta}{\gamma}\cot\left(\tau k\sqrt{(N_d+1)^2-(N_d+1)+\frac{1}{2}}\right)\frac{\sqrt{(N_d+1)^2-(N_d+1)+\frac{1}{2}}}{N_d+1}}. \end{aligned}$$

In order to observe the dependence of the coefficients P_m on m, which enter in the relation $s_m = P_m s_{m-1}$, we give below the expression for P_{N_d+3}

$$\begin{aligned} P_{N_d+3} &= -\frac{i\sqrt{2}\beta}{\gamma}\cot\left(\tau k\sqrt{(N_d+3)^2-(N_d+3)+\frac{1}{2}}\right)\frac{\sqrt{(N_d+3)^2-(N_d+3)+\frac{1}{2}}}{N_d+3} \\ &- \frac{\frac{\alpha}{\gamma}\frac{N_d+2}{N_d+3}}{-\frac{i\sqrt{2}\beta}{\gamma}\cot\left(\tau k\sqrt{(N_d+2)^2-(N_d+2)+\frac{1}{2}}\right)\frac{\sqrt{(N_d+2)^2-(N_d+2)+\frac{1}{2}}}{N_d+2} - \frac{\frac{\alpha}{\gamma}\frac{N_d+1}{N_d+2}}{P_{N_d+1}}}. \end{aligned}$$

From these results, it is not difficult to write down the product

$$s_n = \prod_{m=N_d+1}^{n} P_m s_{N_d}. \tag{17}$$

Here the coefficient P_m can be expressed through continued fraction of the form

$$\begin{aligned} P_m &= -\frac{i\sqrt{2}\beta}{\gamma}\cot\left(\tau k\sqrt{m^2-m+\frac{1}{2}}\right)\frac{\sqrt{m^2-m+\frac{1}{2}}}{m} \\ &- \cfrac{\frac{\alpha}{\gamma}\frac{m-1}{m}}{b(\tau)-\cfrac{\frac{\alpha}{\gamma}\frac{m-2}{m-1}}{c(\tau)-\cfrac{}{\ddots - d(\tau)}}}, \end{aligned} \tag{18}$$

where

$$b(\tau) = -\frac{i\sqrt{2}\beta}{\gamma}\cot\left(\tau k\sqrt{(m-1)^2-(m-1)+\frac{1}{2}}\right)\frac{\sqrt{(m-1)^2-(m-1)+\frac{1}{2}}}{m-1},$$

$$c(\tau) = -\frac{i\sqrt{2}\beta}{\gamma}\cot\left(\tau k\sqrt{(m-2)^2-(m-2)+\frac{1}{2}}\right)\frac{\sqrt{(m-2)^2-(m-2)+\frac{1}{2}}}{m-2}$$

and

$$d(\tau) = \frac{i\sqrt{2}\beta}{\gamma}\cot\left(\tau k\sqrt{(N_d+1)^2-(N_d+1)+\frac{1}{2}}\right)\frac{\sqrt{(N_d+1)^2-(N_d+1)+\frac{1}{2}}}{N_d+1}.$$

In this situation three-level atom (or pair of two-level atoms) entering the cavity in the coherent state $\alpha|e\rangle + \beta|i\rangle + \gamma|g\rangle$ will no longer be able to change it provided that the flying time τ satisfies the conditions (14) and (15). That's why we call this state as trapping state. As the coefficients s_n are related by the cotangent we also call this state as the cotangent state for the proposed systems. The constant s_{N_d} in equation (17) is determined by the normalization.

To obtain the analytical expression for the coefficient s_n through s_{N_u}, we observe that s_{N_u-1} can be represented through s_{N_u} in the form $s_{N_u-1} = Q_{N_u-1}s_{N_u}$, where

$$Q_{N_u-1} = -i\frac{\sqrt{2}\beta}{\alpha}\cot\left(\tau k\sqrt{(N_u+1)^2-(N_u+1)+\frac{1}{2}}\right)\frac{\sqrt{(N_u+1)^2-(N_u+1)+\frac{1}{2}}}{N_u}.$$

In the same way one can find that $s_{N_u-2} = Q_{N_u-2}s_{N_u-1}$. Here

$$\begin{aligned} Q_{N_u-2} &= -i\frac{\sqrt{2}\beta}{\alpha}\cot\left(\tau k\sqrt{N_u^2-N_u+\frac{1}{2}}\right)\frac{\sqrt{N_u^2-N_u+\frac{1}{2}}}{N_u-1} \\ &- \frac{\frac{\gamma}{\alpha}\frac{N_u}{N_u-1}}{-i\frac{\sqrt{2}\beta}{\alpha}\cot\left(\tau k\sqrt{(N_u+1)^2-(N_u+1)+\frac{1}{2}}\right)\frac{\sqrt{(N_u+1)^2-(N_u+1)+\frac{1}{2}}}{N_u}}. \end{aligned}$$

In order to observe the dependence of the coefficients Q_m on m, which appear in the relation $s_m = Q_m s_{m+1}$, we represent the expression for Q_{N_u-3}

$$\begin{aligned} Q_{N_u-3} &= -i\frac{\sqrt{2}\beta}{\alpha}\cot\left(\tau k\sqrt{(N_u-1)^2-(N_u-1)+\frac{1}{2}}\right)\frac{\sqrt{(N_u-1)^2-(N_u-1)+\frac{1}{2}}}{N_u-2} \\ &- \frac{\frac{\gamma}{\alpha}\frac{N_u-1}{N_u-2}}{-i\frac{\sqrt{2}\beta}{\alpha}\cot\left(\tau k\sqrt{N_u^2-N_u+\frac{1}{2}}\right)\frac{\sqrt{N_u^2-N_u+\frac{1}{2}}}{N_u-1}-\frac{\frac{\gamma}{\alpha}\frac{N_u}{N_u-1}}{Q_{N_u-1}}}. \end{aligned}$$

Now one can observe that s_n can be represented through s_{N_u} in the following manner

$$s_n = \prod_{m=n}^{N_u-1} Q_m s_{N_u}, \tag{19}$$

where the coefficient Q_m can be expressed through continued fraction by the following

$$\begin{aligned} Q_m &= -i\frac{\sqrt{2}\beta}{\alpha}\cot\left(\tau k\sqrt{(m+2)^2-(m+2)+\frac{1}{2}}\right)\frac{\sqrt{(m+2)^2-(m+2)+\frac{1}{2}}}{m+1} \\ &- \frac{\frac{\gamma}{\alpha}\frac{m+2}{m+1}}{b_Q(\tau)-\frac{\frac{\gamma}{\alpha}\frac{m+3}{m+2}}{c_Q(\tau)-\ddots-d_Q(\tau)}}, \end{aligned} \tag{20}$$

where

$$b_Q(\tau) = -i\frac{\sqrt{2}\beta}{\alpha}\cot\left(\tau k\sqrt{(m+3)^2-(m+3)+\frac{1}{2}}\right)\frac{\sqrt{(m+3)^2-(m+3)+\frac{1}{2}}}{m+2},$$

$$c_Q(\tau) = -i\frac{\sqrt{2}\beta}{\alpha}\cot\left(\tau k\sqrt{(m+4)^2-(m+4)+\frac{1}{2}}\right)\frac{\sqrt{(m+4)^2-(m+4)+\frac{1}{2}}}{m+3}$$

and

$$d_Q(\tau) = i\frac{\sqrt{2}\beta}{\alpha}\cot\left(\tau k\sqrt{(N_u+1)^2-(N_u+1)+\frac{1}{2}}\right)\frac{\sqrt{(N_u+1)^2-(N_u+1)+\frac{1}{2}}}{N_u}.$$

The next section is devoted to the statistical properties of the single-mode cavity field for the trapping-state solution.

4 Numerical results

Let us now consider the statistical properties of the cavity field for the trapping-state solution. It should be noted that our results for the coefficients of the decomposition of a cavity field on the Fock space block (13) and (18) differ from those for a two-level atom, interacting with the one-mode cavity field (Slosser et al 1989, Li and Lin 1997) and it is necessary to emphasize that in our case these coefficients depend on the three parameters α, β and γ, which satisfy the relation $|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1$. In this case the electromagnetic field fluctuations depend on the two independent parameters α and γ. In the case of a single two-level atom this dependence was described by one parameter (Slosser et al 1989, Li and Lin 1997) α ($|\alpha|^2 + |\beta|^2 = 1$). As a function of these two parameters α and γ the photon statistics in a cavity drastically changes in comparison with the two-level atom (Slosser et al 1989). Further we calculate the mean number of photons and their fluctuations for various values of $N_u = 3,\ 20$ for two different atomic inversions $|\gamma|^2 = 0.25,\ 0.65$ in the case when α, β and γ are real by using the following formulae

$$\begin{aligned} \langle n \rangle &= \sum_{n,n'=0}^{N_u} s_n^* s_{n'} \langle n|a^\dagger a|n'\rangle \\ &= \sum_{n=0}^{N_u} |s_n|^2 n, \\ \langle n^2 \rangle &= \sum_{n=0}^{N_u} |s_n|^2 n^2, \end{aligned} \tag{21}$$

$$\sigma = \frac{\langle n^2\rangle - \langle n\rangle^2}{\langle n\rangle}.$$

In figures 1-4 we represent our numerical results for $N_u = 3$ and 20. In the case of $N_u = 3$ we observe that the mean photon number $\langle n\rangle$ mainly decreases with the increasing of the atomic inversion $|\alpha|^2$, while the fluctuations in the mean photon number σ increase with the increasing of $|\alpha|^2$. It should be noted that in the case of small inversion $|\gamma|^2 = 0.65$ takes place mainly super-Poissonian statistics. With the increasing of the atomic inversion $|\gamma|^2 = 0.25$ occurs mainly sub-Poissonian statistics.

With the increasing of N_u ($N_u = 20$) the mean number of photons and fluctuations σ exhibit more interesting behavior. For $N_u = 20$, $|\gamma|^2 = 0.25$ in contrast with the case $N_u = 20$ and $|\gamma|^2 = 0.65$ both $\langle n\rangle$ and σ oscillate beginning from the region $|\alpha|^2 \approx 0.22$. In this case $\langle n\rangle$ has some sharp decreases, which are simultaneously accompanied with sharp jumps of σ. For example, in the closed region $|\alpha|^2 \approx 0.33$ the fluctuations take their maximal value $\sigma \approx 5.0$. These points describe the strong energy exchange between the atomic and photon subsystems. One can see that in these points takes place the quick collapses and revivals of the mean photon number.

In the points of sharp jumps we observe the super-Poissonian statistics $\sigma > 1$. It should be noted that such sharp jumps where also observed in the paper of Enaki and Koroli 1999 devoted to the three-level equidistant model with the two ordinary cascade transitions. One can see that the photon statistics for the three-level model with the intensity-dependent coupling to a marked degree differs from the ordinary three-level model (Enaki and Koroli 1999).

Below we will study the second-order and amplitude-squared squeezing. The phase components of the field amplitude in two quadrants are defined by

$$d_1 = (A + A^\dagger)/2, \tag{22}$$

$$d_2 = (A - A^\dagger)/2i, \tag{23}$$

where $A = a\exp(i\omega_0 t)$, $A^\dagger = a^\dagger \exp(-i\omega_0 t)$. Whenever the condition

$$\langle(\Delta d_i)^N\rangle < (N-1)!!/2^N \tag{24}$$

is satisfied, the corresponding state is called the Nth-order squeezing state (Hong and Mandel 1985a, Hong and Mandel 1985b). The degree of squeezing is measured by the parameter

$$D_i^{(N)} = \frac{2^N\langle(\Delta d_i)^N\rangle}{(N-1)!!} - 1, \qquad i = 1, 2. \tag{25}$$

It is clear that squeezing appears when $-1 < D_i^{(N)} < 0$ according to the definition in Eq. (24).

A state of the field is said to be amplitude-squared squeezed whenever one of the two operators Y_1 and Y_2 satisfies the relation (Hillery 1987a, Hillery 1987b)

$$\langle(\Delta Y_i)^2\rangle < \langle\hat{N} + \frac{1}{2}\rangle, \qquad i = 1 \ or \ 2, \tag{26}$$

where

$$Y_1 = (A^2 + A^{\dagger 2})/2, \qquad Y_2 = (A^2 - A^{\dagger 2})/2i. \tag{27}$$

In this equation $\hat{N} = a^\dagger a$ represents the photon number operator. Let

$$Q_1 = \frac{\langle(\Delta Y_1)^2\rangle - \langle\hat{N} + \frac{1}{2}\rangle}{\langle\hat{N} + \frac{1}{2}\rangle} \tag{28}$$

and

$$Q_2 = \frac{\langle(\Delta Y_2)^2\rangle - \langle\hat{N} + \frac{1}{2}\rangle}{\langle\hat{N} + \frac{1}{2}\rangle}. \tag{29}$$

From (26) one can see that the field is in an amplitude-squared squeezed state when $-1 < Q_i < 0$. The smaller Q_i is, the stronger the amplitude-squared squeezing is.

In figs 5-8 we represent our numerical calculations for real values of α, β and γ in the case of $N_u = 3$ and 20 performed for the second-order and amplitude-squared squeezing. One can see that with the increase of N_u both the second-order and amplitude-squared squeezing increase. In the case of $N_u = 20$ we observe that with the increasing of the population of the ground state $|\gamma|^2$ the second-order and amplitude-squared squeezing increase. The maximal value of the second-order squeezing takes place in the case $|\gamma|^2 = 0.65$ and $D_1^{(2)}$ can reach almost 36% in the region $|\alpha|^2 \approx 0.06$. From figures 7 and 8 one can see that for $N_u = 20$ the maximal value of the amplitude-squared squeezing is $Q_1 \approx 45\%$ for $|\gamma|^2 = 0.65$ in the region $|\alpha|^2 \approx 0.14$.

5 SUMMARY

In this paper we have proposed the model, which exhibits the trapping effect and obtained the cotangent state of the electromagnetic field in a microcavity. Such trapping effect can be obtained for higher excited Rydberg states of atoms, for which the energy distance between the states $|n, s\rangle \rightarrow |n-1, p\rangle$ and $|n-1, p\rangle \rightarrow |n-1, s\rangle$ is approximately equal. The dependence of the detuning $\omega_{32} - \omega_{21}$ on the excitation number for Rydberg atoms is described in papers (Goy et al 1982, Brune et al 1987) and tends to zero value in the region of $n \approx 60$ and 40.

For trapping state we investigated the features of the one-mode cavity field. It should be noted that in the case when $N_u = 3,\ 20$ we observed both sub-Poissonian and super-Poissonian statistics for given $|\gamma|^2 = 0.25,\ 0.65$. It should be noted that with the increasing of Nu the properties of the single-mode cavity field become more interesting. For example, for $Nu = 20$ and $|\gamma|^2 = 0.25$ with the increasing of the atomic inversion $|\alpha|^2$ takes place sub-Poissonian statistics $\sigma < 1$ and beginning from the region $|\alpha|^2 \approx 0.2$ in the points of sharp jumps super-Poissonian statistics occurs $\sigma > 1$. Such sharp jumps are not observed in the case of $Nu = 3$.

We have also investigated the second-order and amplitude-squared squeezing of the field in a cotangent state. It is found that optimal squeezing can be achieved by properly choosing the atomic inversion and upper boundary of the Fock space block N_u.

In conclusion, it should be noted that although a model with a lossless cavity might seem academic, actual experiments in micromasers have been performed with extremely high Q values, and so this model might not be unrealistic (Rempe et al 1987).

An experimental verification of results requires an extremely low temperature, such that the number of thermal photons is much less than 1, which can be achieved experimentally (Rempe et al 1987, Meschede et al 1985). On the other hand, one would except these states to be robust to cavity dumping (Slosser and Meystre 1990). In any event, with present-day technology, very-high-Q cavities are available (Rempe et al 1987) ($Q \simeq 10^{11}$). The existence of the trapping states does not seem to be very sensitive to small velocity fluctuations in the beam (Orszag et al 1992).

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