Fields Institute Workshop Jordan Algebras and Related Fields
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Symmetric compositions and trisotopies

Following the approach of Professor Petersson to cyclic compositions (cf. [5]), symmetric compositions will be studied by means of it symmetric trisotopies, defined as triples (C, η, u), C a composition algebra over an arbitrary field k, $u \in C^{\times}$ and η a k-isomorphism from C onto its (u, u^{-1})-isotope (cf. [4]) that fixes u and satisfies $\eta^3(x) = uxu^{-1}$ for all $x \in C$. It will be shown that pointed symmetric compositions and symmetric trisotopies are equivalent notions. We determine symmetric trisotopies of dimension $r \leq 2$ and show that for dimension $4r$, $(r \leq 2)$, symmetric trisotopies can always be obtained from a hermitian space of rank 3 over a quadratic étale k-algebra equipped with an endomorphism ϕ satisfying conditions linked to the ones of η. The accompanying symmetric compositions will be characterized as being of para-Hurwitz- or Okubo-type (cf. [1], [2] and [3]). This approach provides a unified way of enumerating symmetric compositions in all characteristics.

[1] A. Elduque *Symmetric composition algebras* Journal of Algebra **196**, 283-300 (1997)

[2] A. Elduque, J. M. Perez *Composition algebras with associative bilinear form*, Comm. Algebra **24** (1996), 1091-1116.

[3] M.-A. Knus, A. Merkurjev, M. Rost and J.-P. Tignol *The book of involutions*, vol. 44 of American Mathematical Society Colloquium Publications, American Mathematical Society, Providence, RI, 1998.

[4] K. McCrimmon *Homotopes of alternative algebras*, Math. Ann. **191** (1971), 253–262.

[5] H. P. Petersson *Cyclic compositions and trisotopies*, Submitted, (2005).