THE FIELDS INSTITUTE
FOR RESEARCH IN MATHEMATICAL SCIENCES

ABSTRACTS 1.2

YURI BAHTURIN
Memorial University of Newfoundland and Moscow State University

Exchange Theorem and its Applications to the Gradings of Algebras and Superalgebras

In this talk we present a tool that proved to be useful in describing gradings by finite abelian groups on finite-dimensional simple and (super)involution simple associative algebras and superalgebras. Since many simple Lie and Jordan algebras or superalgebras arise as sets of skew-symmetric or symmetric elements on such associative algebras, the results are helpful in finding abelian group gradings in any of these cases.

GEORGIA BENKART
University of Wisconsin

The Tetrahedron Algebra and Its Friends and Relatives

The tetrahedron Lie algebra has connections with three-point loop algebras and quantum groups. It arises naturally in the study of tridiagonal pairs of linear transformations. This talk will survey these topics.

IVAN DIMITROV
Queen's University

Borel-Weil-Bott theorem for diagonal direct limits of algebraic groups

We study the cohomoly groups of homogeneous line bundles over G/B, where G is a diagonal direct limit of simple algebraic groups and B is a Borel subgroup of G. The main difficulty in studying this problem is that the naive definition of the Weyl group of G produces elements of infinite length only, yet there are homogeneous line bundles with nonzero higher cohomology groups. We introduce the appropriate notion of a Weyl group and prove the analog of the classical Borel-Weil-Bott theorem.

ANDREW DOUGLAS
University of Toronto

Finite dimensional representations of the Euclidean algebra $\mathfrak{e}(2)$

We will discuss the finite dimensional representations of the Euclidean algebra $\mathfrak{e}(2)$ and briefly consider the classification of certain classes of these representations. We will then investigate the finite dimensional representations of $\mathfrak{e}(2)$ that are obtained by embedding $\mathfrak{e}(2)$ into $\mathfrak{sl}_3$. We will show that the finite dimensional, irreducible representations of $\mathfrak{sl}_3$ restricted to $\mathfrak{e}(2)$ are indecomposable and, when possible, we will give a graphical description of these $\mathfrak{e}(2)$ representations.

YUN GAO
York University

Spin representations and BC-graded Lie algebras

In this talk, I will present a spin construction of certain BC-graded Lie algebras coordinatized by quantum tori with nontrivial central extensions.

JACOB GREENSTEIN
University of California, Riverside

Graded level zero integrable representations of affine Lie algebras

We study the structure of the category of integrable level zero representations with finite dimensional weight spaces of affine Lie algebras. We show that this category possesses a weaker version of the finite length property, namely that an indecomposable object has finitely many simple constituents which are non-trivial as modules over the corresponding loop algebra. Moreover, any object in this category is a direct sum of indecomposables only finitely many of which are non-trivial. We obtain a parametrization of blocks in this category. That result depends crucially on the conjecture on the dimension of Weyl modules over loop algebras, which was recently proved by G. Fourier and P. Littelmann for all simply laced types.

J. LANDSBERG
Texas A and M

Beyond Lie algebras - and back

I will describe how considerations in geometry led L. Manivel and myself to study proposed categorical generalizations of Lie algebras due to Deligne and Vogel. After explaining these tensor categories, I will explain some of their consequences for representation theory and the geometry of rational homogeneous varieties.

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KAILASH C. MISRA
North Carolina State University

Perfect Crystals and Demazure crystals for $U_q(D_4^{(3)})$

Let $U_q(g)$ be a quantum affine algebra and $V(\lambda)$ denote the integrable highest weight $U_q(g)$-module of level l. The crystal $B(\lambda)$ associated with $V(\lambda)$ provides a combinatorial tool to study its properties. The crystal $B(\lambda)$ can be realized as semi-infinite sequence of elements (called "paths") in a perfect crystal B of level l. The perfect crystal B of level l is a crystal for a suitable level zero $U_q'(g)$-module. For a Weyl group element $w \in W$ let $V_w(\lambda)$ denote the Demazure module generated by the extremal weight vector $u_{w\lambda}$ and $B_w(\lambda)$ denote its crystal. It is known that under certain conditions $B_w(\lambda)$ has a tensor product like structure. Recently, jointly with Kashiwara, Okado and Yamada we constructed a coherent family of perfect crystals B_l of level l for the quantum affine algebra $U_q(D_4^{(3)})$. In this talk we will present this perfect crystals and show that for certain family of Weyl group elements $\{w_k\}$ the Demazure crystals $B_{w_k}(\lambda)$ have tensor product like structure.

JIRI PATERA
University of Montreal

New C-, S-, and E-transforms of a compact simple Lie algebra.

The three transforms are built on three families of unconventional special functions (C-, S-, and E-functions). Their continuous and discrete orthogonalities will be shown. In n-dimensions the underlying symmetry is a compact (semi)simple Lie algebra/Lie group of rank n. They are n-dimensional generalizations of cosine, sine, and the exponential function respectively. Numerous applications to digital data processing will be pointed out.

ARTURO PIANZOLA
University of Alberta

Descent considerations in infinite dimensional Lie theory

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ABSTRACTS 1.2

HADI SALMASIAN
Queen's University

Minimal and Singular Unitary Representations, Kirillov's Orbit Method and Quantization

Let G be a semisimple group over complex numbers. In the 70's Anthony Joseph defined a notion of a minimal unitary representation of G based on quantization of the minimal nilpotent orbit of the Lie algebra of G. However, despite significant contributions by B. Kostant, D. Kazhdan, R. Brylinski and S. Sahi, a complete classification and construction of these representations was not achieved until very recently. We explain various ways to define these representations and their connection to Roger Howe's theories of rank and theta correspondence. Finally, the status of these representations in the unitary dual of G are analyzed. This is recent work by the speaker.

HAMID USEFI
University of Western Ontario

Lie algebras and their universal envelopes

In this talk we discuss the so called isomorphism problem for enveloping algebras. Let L be a Lie algebra with universal enveloping algebra $U(L)$ over a field F. The isomorphism problem asks whether or not (the isomorphism type of) every Lie algebra L is determined by the algebra structure of $U(L)$. We show that there is not much hope if the characteristic of $\mathbb{F}$ is positive. In connection with the isomorphism problem, we will mention an open problem which asks whether or not there exists a non-free Lie algebra L over a field of characteristic zero such that $U(L)$ is a free associative algebra.

KAIMING ZHAO
Wilfrid Laurier University

Irreducible weight modules over the twisted Heisenberg-Virasoro algebra

Let C be the field of complex numbers. The twisted Heisenberg-Virasoro algebra L is the universal central extension of the Lie algebra $\{f(t)\frac{d}{dt} + g(t) | f, g \in C[t, t^{-1}]\}$ of differential operators of order at most one. More precisely, we have the following definition.

Definition 1.1. *The* **twisted Heisenbeg-Virasoro algebra** *L is a Lie algebra over C with the basis*

$$\{x_n, I(n), C_D, C_{DI}, C_I | n \in Z\}$$

and the Lie bracket given by

$$[x_n, x_m] = (m-n)x_{n+m} + \delta_{n,-m}\frac{n^3 - n}{12}C_D,$$

$$[x_n, I(m)] = mI(n+m) + \delta_{n,-m}(n^2+n)C_{DI},$$
$$[I(n), I(m)] = n\delta_{n,-m}C_I,$$
$$[L, C_D] = [L, C_{DI}] = [L, C_I] = 0.$$

The Lie algebra L was introduced and studied by mathematicians and physicists. The paper [E. Arbarello, C. De Concini, V. G. Kac and C. Procesi, Moduli spaces of curves and representation theory, *Comm. Math. Phys.*, (1)117, 1-36(1988)] precisely determined the determinant formula of the Shapovalov form for Verma modules.

In [Y. Billig, Representations of the twisted Heisenberg-Virasoro algebra at level zero, *Canad. Math. Bull.*, 46(2003), no.4, 529-537], Billig obtained the character formula for irreducible highest weight modules with trivial action of C_I.

In my talk, I will give all irreducible weight L-modules with finite dimensional weight spaces. More precisely, there are two different classes of them. One class is formed by simple modules of intermediate series, whose weight spaces are all 1-dimensional; the other class consists of the highest(or lowest) weight modules.

This talk is based on a joint paper [R. Lu, K. Zhao, Classification of irreducible weight modules over the twisted Heisenberg-Virasoro algebra, math.RT/0510194]