Bounds on Kolmogorov spectra for the Navier - Stokes equations

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Abstract: Let $u(x,t)$ be a (possibly weak) solution of the Navier - Stokes equations on all of $\mathbb{R}^3$, or on the torus $\mathbb{R}^3/\mathbb{Z}^3$. The *energy spectrum* of $u(\cdot,t)$ is the spherical integral

$$E(\kappa,t) = \int_{|k|=\kappa} |\hat{u}(k,t)|^2 dS(k), \qquad 0 \leq \kappa < \infty,$$

or alternatively, a suitable approximate sum. An argument involking scale invariance and dimensional analysis given by Kolmogorov in 1941, and subsequently refined by Obukov, predicts that large Reynolds number solutions of the Navier - Stokes equations in three dimensions should obey

$$E(\kappa,t) \sim C\kappa^{-5/3},$$

at least in an average sense. I will explain a global estimate on weak solutions in the norm $|\mathscr{F}\partial_x u(\cdot,t)|_\infty$ which gives bounds on a solution's ability to satisfy the Kolmogorov law. The result gives rigorous upper and lower bounds on the inertial range, and an upper bound on the time of validity of this regime. This is joint work with Andrei Biryuk.