ABSTRACTS 1.2

TIM AUSTIN
University of California, Los Angeles

A very brief look at the Density Hales-Jewett Theorem

In 1991 Furstenberg and Katznelson offered a proof of the density version of a classical result of Hales and Jewett in colouring Ramsey Theory, from which many other results in density Ramsey Theory (such as the multidimensional Szemeredi Theorem) can quickly be deduced. We will give a very brief overview of their approach, and discuss the possibility of turning it into a quantitative account and so extracting (more-or-less) explicit bounds.

ANTAL BALOG
Renyi Institute

Trigonometric sums with multiplicative coefficients

tba

VITALY BERGELSON
Ohio State University

Generalized polynomials and an Extension of the Polynomial Szemeredi Theorem

Polynomial Szemeredi Theorem (joint result with A Leibman) states that if p_i, $i = 1, 2, \ldots, k$ are polynomials with integer coefficients which satisfy $p_i(0) = 0$, then any set A in N which has positive upper density contains "many" polynomial configurations of the form $a, a + p_1(n), a + p_2(n), \ldots, a + p_k(n)$. (The classical Szemeredi theorem corresponds to the case where $p_i(n) = in$, $i = 1, 2, \ldots, k$).

We will discuss a new extension of the Polynomial Szemeredi Theorem which deals with the "upgrade" of the Polynomial Szemeredi Theorem to the so called generalized polynomials, namely functions which are obtained from regular polynomials via iterated use of the floor function (joint work with Randall McCutcheon).

JEAN BOURGAIN
Institute for Advanced Study

Invariant measures and stiffness for non Abelian actions on tori

We present an affirmative solution to some questions raised by Furstenberg and Margulis on the action of certain $SL^d(Z)$ subgroups on tori.The method involves additive combinatorics

BORIS BUKH
Princeton University

Sums of dilates

The λ-dilate of a set A is $\lambda \cdot A = \{\lambda a : a \in A\}$. I will show how to obtain sharp lower bounds on the size of sums of dilates, such as $2 \cdot A + 7 \cdot A + 12 \cdot A$ for integer sets A. I will also discuss Plünnecke-type inequalities for such sumsets.

MEI-CHU CHANG
UC Riverside

Sum-product and character sums

We use explicit sum-product estimates to obtain various new bounds on incomplete character sums in finite fields. In particular, we improve on earlier work of Davenport-Lewis and Karacuba.

ALINA COJOCARU
Illinois-Chicago

The Koblitz Conjecture on average

Let E be an elliptic curve over Q. For a prime p of good reduction, let E_p be the reduction of E modulo p. In 1988, Neal Koblitz formulated a conjecture regarding the number of primes p ¡ x for which the group of points of E_p has prime order. The question, still open, may be viewed as an analogue of the twin prime conjecture. I will present a result that says that Koblitz's conjecture is true on average over a two-dimensional family of elliptic curves. This is joint work with Antal Balog (Budapest) and Chantal David (Montreal).

ERNIE CROOT
Georgia Tech

On rich lines in grids

Using methods from arithmetic combinatorics, we prove the following.

Theorem. For every $\epsilon > 0$, there exists $\delta > 0$, such that the following holds for all pairs of sets A and B of real numbers with

$$|A| \;=\; |B| \;=\; n,$$

where $n > n_0(\epsilon)$: Suppose one has a family of lines such that

- There are at least n^ϵ different slopes among them; and,
- every line is parallel to at least n^ϵ others.

Then, at least one of the lines must hit the grid $A \times B$ in fewer than $n^{1-\delta}$ points; that is, not all the lines can be "$n^{1-\delta}$-rich".

If one applies the Szemeredi-Trotter inequality to this problem, one will see that the bounds it gives are nowhere near to what one would need in order to prove it. Furthermore, the theorem already implies some weak form of sum-product inequalities, so is at least at that level of depth.

We will mention some conjectures related to this theorem; and, if time permits, we will present a simple geometric argument that gives somewhat weaker (thought still non-trivial) results.

This is joint work with my student Evan Borenstein.

NIKOS FRANTZIKINAKIS
University of Memphis

Szemerédi's theorem, Hardy sequences, and nilmanifolds

We say that a sequence $a(n)$ is *good* for Szemerédi's theorem, if every subset of the integers with positive density contains arbitrarily long arithmetic progressions of the form $m, m + a(n), ..., m + ka(n)$. A result of Bergelson and Leibman (1996) shows that if p is a polynomial with integer coefficients and zero constant term, then the sequence $p(n)$ is good. We will give several new examples of good sequences that are not polynomial. To name a few, $a(n) = [n^{\sqrt{2008}}]$, $b(n) = [n \log n]$, or $c(n) = [n^3/\log\log n]$, are all good sequences. In fact, it turns out that if $f(x)$ is a function that belongs to a Hardy field and satisfies some mild growth conditions, then the sequence $[f(n)]$ is going to be good (joint work with M. Wierdl). For general sequences, it seems that certain equidistribution properties on nilmanifolds suffice to guarantee that a sequence is good. We will mention a related conjecture and a partial result (joint work with M. Wierdl and E. Lesigne).

GREGORY FREIMAN
Tel Aviv University

On a detailed structure of sumsets and difference sets

It is known that a set of k intergers with small doubling (small —A+A—) satisfying the condition —A+A—=2k-1+b,b in [0,k-3] is a part of arithmetic progression of k+b terms. It appeared that the structure of A may be described in a much more detailed way. The similar result may be obtained for difference A-A and for the sum A+B of two different sets A and B.

WEIDONG GAO
Nankai University

Zero-sum Problems in Abelian Groups

Let G be an additive finite abelian group and let $S = a_1 \cdot \ldots \cdot a_l$ be a sequence over G. We say that S is a zero-sum sequence if $a_1 + \ldots + a_l = 0$. A typical direct zero-sum problem studies conditions which ensure that a sequence contains a zero-sum subsequence of prescribed length (usual conditions require the length to be $|G|$, or $\exp(G)$ or at most $\exp(G)$). The associated inverse zero-sum problem studies the structure of extremal sequences which have no such zero-sum subsequences.

These investigations were initiated by a result of P. Erdős, A. Ginzburg and A. Ziv in 1961, who proved that $2n-1$ is the smallest integer $l \in \mathbb{N}$ such that every sequence S over a cyclic group of order n with length $|S| \geq l$ has a zero-sum subsequence of length n. A classical invariant is the Davenport constant $\mathsf{D}(G)$ which is defined as the smallest integer $l \in \mathbb{N}$ such that every sequence S over G of length $|S| \geq l$ has a zero-sum subsequence. Zero-sum problems arise naturally in various branches of combinatorics, graph theory, classical number theory and finite geometry. Moreover, zero-sum theory has been promoted by applications in the theory of non-unique factorizations.

We give a survey on the variety of zero-sum problems and present some recent results.

ALEXEY GLIBICHUK
Moscow State University

Additive properties of product sets in finite fields

We shall study the following problem: given $n \geqslant 2$ subsets $A_1, A_2, \ldots, A_n$ of an arbitrary finite field $\mathbb{F}_q$ with q elements. One should define when there exists a finite natural number N such that N-fold sumset of the set $A_1 \cdot A_2 \cdot \ldots \cdot A_n$ covers all the field $\mathbb{F}_q$. This problem is not solved in general, but some special cases the problem is studied. The purpose of this talk is to present these partial results and discuss possible difficulties that may occur in study of the problem.

RON GRAHAM
University of California, San Diego

Some Ramsey results for the n-cube

In this talk I will describe some recent results obtained with Jozsef Solymosi concerning partitions of the diagonals on an n-cube. I will also mention several applications of these results for obtaining monochromatic corners in various structures.

BEN GREEN
Cambridge University

Exponential sums and Gowers Norms in finite field models

I will discuss some recent joint work with T. Tao on the distribution of degree d polynomials $f(X_1, .., X_N)$ with coefficients in some fixed finite field k (such as F_2 or F_3). If the values taken by such a polynomial are not close to being equidistributed then it turns out that f must be expressible in terms of lower degree polynomials, if the characteristic of k is large enough. When the characteristic becomes very small (and in particular if k = F_2) these issues become a lot less clear and remain unsolved in some cases. I will also discuss a counterexample to the so-called "Inverse Gowers Conjecture" in finite characteristic. I'll also describe what can be salvaged from the conjecture, and explain why this counterexample does not leave us unduly worried about our programme to count solutions to linear equations in primes.

HARALD HELFGOTT
University of Bristol

Growth in SL_3

Let G = $SL_3(Z/pZ)$. Then, for every subset A of G not contained in any proper subgroup of G, $|A{*}A{*}A| > |A|^{1+epsilon}$,unless A is very large ($|A| > |G|^{1-delta}$). Here epsilon depends only on delta.

BERNARD HOST
Marne-la-Vallee

Nilsequences in ergodic theory (joint work with B. kra)

Nilsequences were introduced in ergodic theory a few years ago in order to describe higher order correlations. Since then, they have been used by Green and Tao inadditive combinatorics. In this talk, we first present some recent results in ergodic theory that can be proved using nilsequences. we then formulate some questions aimed to understand the role played by nilsequences in very different contexts. We believe that a better understanding of these objects in termes of harmonic analysis could lead to more applications.

ALEX IOSEVICH
Missouri-Columbia

Distribution of dot products in vector spaces in finite fields and applications to problems in additive number theory and geometric combinatorics

We shall discuss some recent results on the distribution of dot products in vector spaces over finite fields and will use them to deduce sum-product estimate and results related to the Erdos distance problem. We shall also make explicit connections with the extension theory for the Fourier transform.

MIHALIS KOLOUNTZAKIS
University of Crete

The discrepancy of a needle on a checkerboard

Consider the plane as a checkerboard, with each unit square coloured black or white in an arbitrary manner. We show that for any such colouring there are straight line segments, of arbitrarily large length, such that the difference of their white length minus their black length, in absolute value, is at least the square root of their length, up to a multiplicative constant. For the corresponding “finite” problem (NxN checkerboard) we also prove that we can color it in such a way that the above quantity is at most the square root of N log N, for any placement of the line segment.

VSEVOLOD LEV
University of Haifa at Oranim

On the number of popular differences

We prove that there exists an absolute constant $c > 0$ such that for any finite integer set A with $|A| > 1$ and any positive integer $m < c|A|/\ln|A|$ there are at most m positive integers b satisfying $|(A+b) \setminus A| \leq m$; equivalently, there are at most m positive integers possessing $|A| - m$ (or more) representations as a difference of two elements of A. This is best possible in the sense that for every positive integer m there exists a finite integer set A with $|A| > m\log_2(m/2)$ such that $|(A+b) \setminus A| \leq m$ holds for $b = 1, ..., m+1$. Joint work with Sergei Konyagin.

YU-RU LIU
University of Waterloo

Vinogradov's mean value theorem in function fields

Let $\mathbb{F}_q[t]$ be the ring of polynomials over the finite field $\mathbb{F}_q$. In this talk, we will discuss a generalization of Vinogradov's mean value theorem in $\mathbb{F}_q[t]$. We will apply our result to obtain an upper bound for $\tilde{G}_q(k)$, which is the least integer s such that every polynomial in $\mathbb{F}_q[t]$ of sufficiently large degree, the expected asymptotic formula in Waring's problem holds. This is a joint work with Trevor Wooley.

NEIL LYALL
University of Georgia

Optimal polynomial return times

MATE MATOLCSI
Renyi Institute

Low degree tests at large distances

This is joint work with K. Gyarmati and I. Z. Ruzsa.

For finite sets of integers $A_1, A_2 \dots A_n$ we first study the cardinality of the n-fold sumset $S = A_1 + \cdots + A_n$ compared to those of $n-1$-fold sumsets $S_i = A_1 + \cdots + A_{i-1} + A_{i+1} + \dots A_n$. We prove a superadditivity and a submultiplicativity property for these quantities, namely:

$$(n-1)|S| \geq -1 + \sum_{j=1}^{n} |S_j|, \qquad \text{and} \tag{1}$$

$$|S| \leq \left(\prod_{i=1}^{n} |S_i| \right)^{\frac{1}{n-1}}. \quad (2)$$

We next prove the following version of Plünnecke's inequality for different summands: assume that for finite sets A, $B_1, \ldots B_n$ we have information on the size of the sumsets $A + B_{i_1} + \cdots + B_{i_l}$ for all choices of indices $i_1, \ldots i_l$. Then there exists a non-empty subset X of A such that we have 'good control' over the size of the sumset $X + B_1 + \cdots + B_n$. This leads us to a generalization of inequality

ALEX SAMORODNITSKY
Hebrew University

Low degree tests at large distances

Consider the following problem: given a function from $(F_2)^n$ to F_2 we need to decide whether it is "highly structured", which in our case means representable by a low-degree n-variate polynomial over F_2, or "pseudorandom", which we take to mean being far from all low-degree polynomials. Our decision has to be based on a very restricted randomized local view of the function.

This question arises naturally in 'property testing', a sub-area of theoretical computer science which deals with deciding whether a given combinatorial object has a certain global property or, alternatively, is far from all objects with this property, based on viewing some randomized local samples from the object. It also has some connections with other areas of theoretical cs, such as probabilistically checkable proofs and derandomization.

We will discuss some recent developments regarding this question. In particular, it turns out that an attempt to measure pseudorandomness of a function analytically, via its 'Gowers norm', does not work so well.

Based in part on joint work with Shachar Lovett, Roy Meshulam, and Luca Trevisan

TOM SANDERS
Cambridge-IAS

Chowla's cosine problem in abelian groups

Suppose that A is a finite symmetric subset of an abelian group G and write $M_G(A)$ for the magnitude of the largest negative Fourier coefficient of 1_A. Chowla asked for a lower bound on $M_{\mathbf{R}Z}(A)$uniform in $|A|$. We consider the question for an arbitrary group in which case one has a dichotomy: either A is close to a union of subgroups of G, or $M_G(A)$ is large. We shall emphasize the case when G is finite and $|A| = \Omega(|G|)$, where we are able to achieve particularly strong lower bounds for $M_G(A)$.

CHUN-YEN SHEN
Indiana University Bloomington

Quantitative sum-product estimates

The sum product phenomenon has received a great deal of attention, since Erdös and Szemerèdi made their well known conjecture that

$$\max(|A+A|, |AA|) \geq C_\epsilon |A|^{2-\epsilon} \forall \epsilon > 0.$$

where A is a finite subset of integers and

$$A + A = \{a + b : a \in A, b \in A\},$$

and

$$AA = \{ab : a \in A, b \in A\}.$$

In this talk, we will present that if A is a subset in a finite field F_p, p prime, with $|A| < p^{\frac{1}{2}}$ then

$$\max(|A+A|, |F(A,A)|) \gtrapprox |A|^{\frac{13}{12}}.$$

where F : $F_p \times F_p$ to F_p , $(x, y) \rightarrow x(f(x) + by)$, f is any function and $b \in F_p^*$. For the case f=0 and $b = 1$, it corresponds to the well known sum product theorem by Bourgain, Katz and Tao. This is joint work with Nets Katz.

MATTHEW SMITH
University of Georgia

On solution-free sets for simultaneous additive equations

We use a combination of the Hardy-Littlewood circle method and the methods developed by Gowers in his recent proof of Szemerédi's Theorem on long arithmetic progressions to obtain quantitative estimates for the upper density of a set of integers containing no solutions to a translation and dilation invariant system of diagonal polynomials of degrees $1, 2, \ldots, k$. We will also explore the question of finding a bound for the upper density of a subset of the primes containing no solutions to a system of this type in the case $k = 2$.

YONUTZ V. STANCHESCU
Afeka College and Open University

On the exact structure of multidimensional sets with small doubling property

We will formulate and discuss various results about the exact structure of multidimensional sets K, assuming that the doubling constant $\sigma = \frac{|K+K|}{|K|}$ is very small.

BALAZS SZEGEDY
University of Toronto

The symmetry preserving regularity and removal lemmas

We prove versions of the well known Regularity a subgraph removal lemmas which are symmetric under the automorphism group of the graph. Using them we prove generalizations of theorems of Ben Green about abelian groups.

TERRY TAO
University of California, Los Angeles

The Mobius-Nilsequences conjecture

A well-known heuristic in number theory is that the Mobius function mu(n) oscillates so randomly that it is asymptotically unbiased with respect to any "low-complexity" sequence. For instance, the prime number theorem asserts that mu(n) is unbiased with respect to 1, the prime number theorem in arithmetic progressions asserts that mu(n) is unbiased with respect to any periodic sequence, and a classical result of Davenport asserts that mu(n) is unbiased with respect to any almost periodic sequence. The Mobius-nilsequences conjecture asserts that mu(n) is in fact unbiased with respect to any nilsequence - a sequence arising from a flow on a nilpotent manifold. This conjecture, if true, would (in conjunction with another conjecture, the Inverse Conjecture for the Gowers norm) imply a general asymptotic for the number of solutions of (finite complexity) systems of linear equations with prime variables. Recently, Ben Green and I have established the Mobius-nilsequences conjecture, by combining a classical method of Vaughan with an equidistribution result for nilsequences. We will sketch the proof of this conjecture and its connections with linear equations of primes in this talk.

AKSHAY VENKATESH
Courant Institute of Mathematical Sciences

Random permutations in number theory

It is well-known that certain aspects of the Riemann zeta-function are modelled by a randomly chosen element of a large matrix group. In fact, many natural phenomena in number theory are modelled by randomly chosen elements from "large" finite groups, e.g. permutation groups or large matrix groups over a fixed finite field, or, for that matter, more interesting groups. I would like to discuss several fairly down-to-earth examples of this. The example I will eventually discuss in detail will be related to the Cohen-Lenstra heuristics, but I will explain precisely what this is and how we would like to generalize it. This all represents joint work with Jordan Ellenberg.

MATE WIERDL
University of Memphis

Bases of integers and Hardy fields

We extend Waring's problem to a wide class of functions. This wide class of functions include Hardy's logarithmico exponential functions of polynomial growth, ie functions we can get by using the symbols log, exp, the real variable x, real constants, addition and multiplication. One of the consequences of our result is that if $a(x)$ is a logarithmoco exponential function of polynomial growth, then the integer parts $[a(1)], [a(2)], [a(3)] \dots$ always form an asymptotic basis for the natural numbers unless $a(x)$ is a rational polynomial with $\gcd([a(1)], [a(2)], [a(3)] \dots) > 1$.

PHILIP MATCHETT WOOD
Rutgers University

Polynomial Frieman isomorphisms

A Freiman isomorphism is a fundamental object in additive combinatorics that allows one to move an additive problem from one group to another, all while preserving the salient additive properties. In this talk, we will discuss a new mapping result that lets one move an algebraic problem—that is, one with additive and multiplicative properties—from any characteristic zero integral domain to the field $\mathbb{Z}/p$ of prime order, all while preserving the salient additive and multiplicative properties. This new mapping result allows us to obtain several combinatorial results for any characteristic zero integral domain, including the Szemeredi-Trotter Theorem, sum-product estimates, and bounding the singularity probability of a discrete random matrix. Since the complex numbers are a characteristic zero integral domain, we have obtained new proofs of these combinatorial results without relying on the topology of the plane.

TAMAR ZIEGLER
Technion - Israel Institute of Technology

Polynomial patterns in primes

In 1995, Bergelson and Leibman proved, using ergodic theoretic methods developed by Furstenberg, a vast generalization of Szemeredi's theorem on arithmetic progressions establishing the existence of arbitrarily long polynomial progression in subsets of the integers of positive density.

In a breakthrough in 2004, Green and Tao proved that the question of finding arithmetic progressions in the prime numbers - which are of zero density - can be reduced to that of finding arithmetic progressions in subsets of positive density. In recent work with

T. Tao we show that one can make a similar reduction for polynomial progressions, thus establishing, through the Bergelson-Leibman theorem, the existence of arbitrarily long polynomial progressions in the prime numbers. We discuss a general strategy for finding patterns in primes via ergodic Ramsey theory.