LAYER POTENTIALS FOR COMPLEX DIVERGENCE FORM EQUATIONS

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Abstract. We recall that if $B = B(x)$ is an accretive, L^∞, $n \times n$ complex matrix, then the Kato square root estimate

$$\|\nabla_x u\|_{L^2(\mathbb{R}^n)} \approx \|(-\operatorname{div}_x B(x)\, \nabla_x)^{1/2} u\|_{L^2(\mathbb{R}^n)},$$

may be re-interpreted in three different (but equivalent) ways as a statement about solutions to the elliptic equation

$$u_{tt} + \operatorname{div}_x B(x)\, \nabla_x\, u = 0 \quad \text{in } \mathbb{R}^{n+1}_+; \tag{1}$$

first, as a "Rellich identity" (with consequent solvability of the Neumann and Regularity problems with L^2 data); second, in terms of the boundedness of the associated layer potentials; and third, as identification of the domain of the generator of the Poisson semigroup. The equation (1) can be re-written as

$$\operatorname{div}_{x,t} A(x) \nabla_{x,t} u = 0, \tag{2}$$

where A is the $(n+1) \times (n+1)$ block diagonal matrix

$$\left[\begin{array}{c|c} & 0 \\ B & \vdots \\ & 0 \\ \hline 0 \cdots 0 & 1 \end{array}\right]. \tag{3}$$

From the perspective of elliptic PDE, the question then naturally arises whether (any of) these three alternative interpretations of the Kato problem remain true in the absence of the special "block" structure (3), i.e., when the coefficient matrix A in (2) is a "full" $(n+1) \times (n+1)$ elliptic, L^∞, complex, t-independent matrix. It turns out that in that case, at least two of these three interpretations need no longer always be true, nor need they all be equivalent. In this talk, we shall discuss to what extent they are known to continue to remain true.

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