Medical Image Processing Using Transforms

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Outlines

- Image Quality
- Gray value transforms
- Histogram processing
- Transforms in image space
- Transforms in Fourier space
- Transforms in Time-frequency space

4. Transform in image space

4.1. Mechanics of spatial filters

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The mechanics of spatial filters

A spatial filter consists of

1. A neighborhood (typically a small rectangle or square)
2. A predefined operation that is performed on the image pixels encompassed by the neighborhood

   If the operation is linear, then it is called linear spatial filter; otherwise, it is nonlinear.

The mechanics of spatial filters

Let $f(x,y)$ be an original image. At any point (x,y) in the image, the filtered image $g(x,y)$ of a linear spatial filter of size 3×3 is

$$g(x,y) = w(-1,-1)\,f(x-1,y-1) + w(-1,0)\,f(x-1,y) + \ldots$$
$$+ w(0,0)\,f(x,y) + \ldots + w(1,1)\,f(x+1,y+1).$$

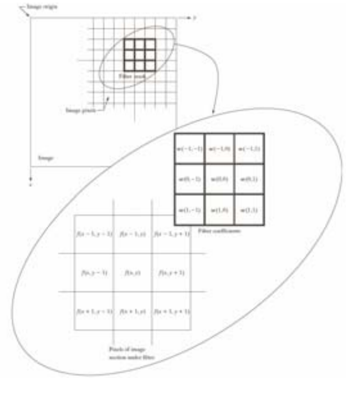

FIGURE 3.28 The mechanics of linear spatial filtering using a 3 × 3 filter mask. The form chosen to denote the coordinates of the filter mask coefficients simplifies writing expressions for linear filtering. (p146)

The mechanics of spatial filters

For a mask of an odd size $m \times n$, where $m = 2a+1$ and $n = 2b+1$.

In general, the linear spatial filtering of an image $f(x,y)$ with the filter of size $m \times n$ is

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

for every pixel (x,y) in f.

The mechanics of spatial filters

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for every pixel (x,y) in f.

Correlation and Convolution

The filtered image can be expressed as the correlation of $w(x,y)$ and $f(x,y)$

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) = w(x,y) * f(x,y).$$

Note that the convolution of $w(x,y)$ and $f(x,y)$ is defined as

$$\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t) = w(x,y) \otimes f(x,y).$$

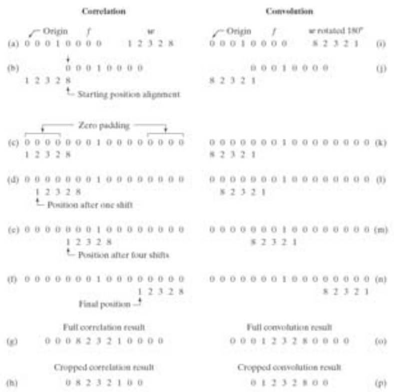

FIGURE 3.29 Illustration of 1-D correlation and convolution of a filter with a discrete unit impulse. Note that correlation and convolution are functions of *displacement*.

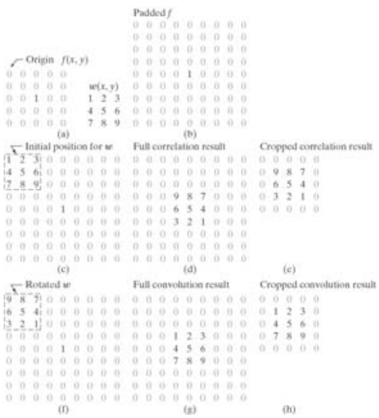

FIGURE 3.30 Correlation (middle row) and convolution (last row) of a 2-D filter with a 2-D discrete, unit impulse. The 0s are shown in gray to simplify visual analysis.

4. Transform in image space

4.2 Blurring

Image Blurring

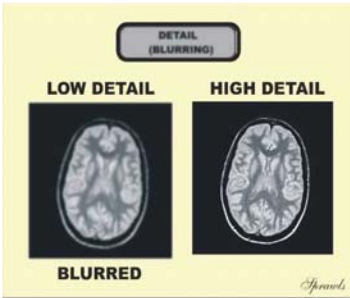

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Image Blurring

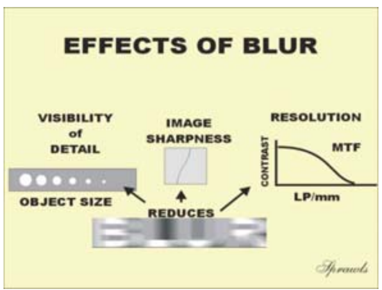

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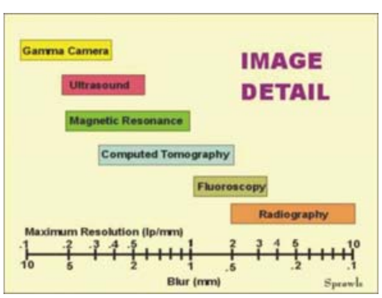
Image Blurring



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But, sometimes one prefers to have blurred images…

4. Transform in image space

4.3 Blurring filters

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Averaging (or box) filter

$\frac{1}{9} \times$

1	1	1
1	1	1
1	1	1

For a mask of size 3×3, the value of the filtered image g at the pixel (x, y) is

$$g_{(x,y)} = \frac{1}{9}\sum_{k=1}^{9} f_k.$$

Weighted Averaging filter

$\frac{1}{16} \times$

1	2	1
2	4	2
1	2	1

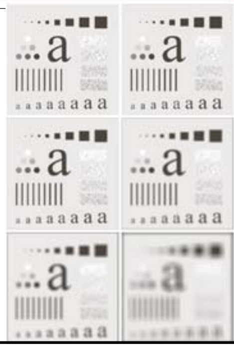
For a mask of size 3×3, the value of the filtered image g at the pixel (x,y) is

$$g_{(x,y)} = \frac{1}{16}\left(f_1 + 2f_2 + f_3 + 2f_4 + 4f_5 + 2f_6 + f_7 + 2f_8 + f_9\right).$$

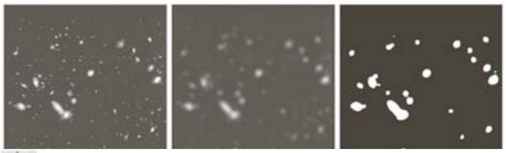
Effect of average filtering

FIGURE 3.33 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $m = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

a b
c d
e f



Get a gross representation of objects of interest



a b c

FIGURE 3.34 (a) Image of size 528 × 485 pixels from the Hubble Space Telescope. (b) Image filtered with a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

Gaussian filter

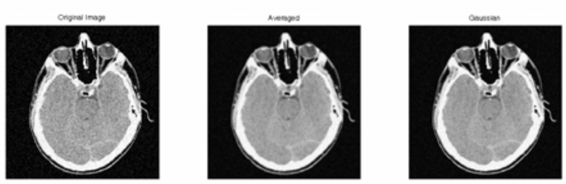
Gaussian filter is a sample of the Gaussian function which has the basic form:

$$w(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}.$$

For example,

$$W = \begin{pmatrix} 0.0113 & 0.0838 & 0.0113 \\ 0.0838 & 0.6193 & 0.0838 \\ 0.0113 & 0.0838 & 0.0113 \end{pmatrix}$$

Gaussian filter



4. Transform in image space

4.4 Sharpening

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Sharpening an image

- **Sharpening**: the principal objective of sharpening is to highlight transitions in intensity
- While smoothing is accomplished in the spatial domain by pixel averaging in a neighborhood (or spatial integration), sharpening can be done by spatial differentiation

Derivatives and Differences

The derivatives of a digital image are usually defined in terms of differences.

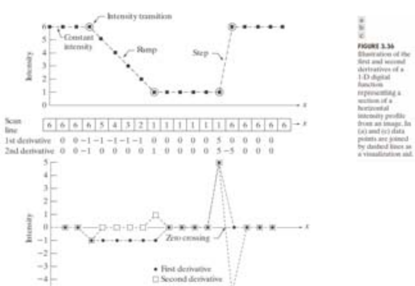
We approximate the 1st order derivatives by

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

and the 2nd order derivatives by

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) - 2f(x) + f(x-1)$$

Illustration of derivatives in a 1-D digital function



Properties of derivatives

- **The 1st derivatives are**
  a) zero in areas of constant
  b) nonzero at the onset of an intensity step or ramp
  c) nonzero along ramps
- **The 2nd derivatives are**
  a) zero in areas of constant
  b) nonzero at the onset of an intensity step or ramp
  c) zero along ramps if constant slopes

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Outline

- Laplacian filter
- Unsharp masking, highboost filtering
- Gradient filter

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Laplacian filters

The Laplacian for a function $f(x,y)$ is defined as

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}$$

where

$$\frac{\partial^2 f}{\partial x^2}=f(x+1,y)+f(x-1,y)-2f(x,y)$$

$$\frac{\partial^2 f}{\partial y^2}=f(x,y+1)+f(x,y-1)-2f(x,y).$$

The discrete Laplacian is

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=f(x+1,y)+f(x-1,y)+f(x,y+1)+f(x,y-1)-4f(x,y)$$

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Laplacian filters

0	1	0	1	1	1
1	−4	1	1	−8	1
0	1	0	1	1	1
0	−1	0	−1	−1	−1
−1	4	−1	−1	8	−1
0	−1	0	−1	−1	−1

a b
c d

FIGURE 3.37 (a) Filter mask used to implement Eq. (3.6-6). (b) Mask used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other implementations of the Laplacian found frequently in practice.

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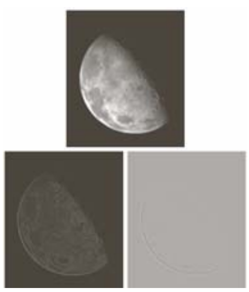
Laplacian filters



a
b c
d e

FIGURE 3.38 (a) Blurred image of the North Pole of the moon. (b) Laplacian without scaling. (c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)

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Laplacian filters

Because the Laplacian is a derivative operator, it highlights the intensity discontinuous in an image and de-emphasizes the regions with slowing varying intensity levels. It produces grayish edge lines and discontinuities, superimposed on a dark, featureless background. Thus, we can get a better sharpened image by

$$g(x,y) = f(x,y) + c\left(\frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}\right)$$

where c = -1 if the center value in the filter is negative; otherwise, c = 1;

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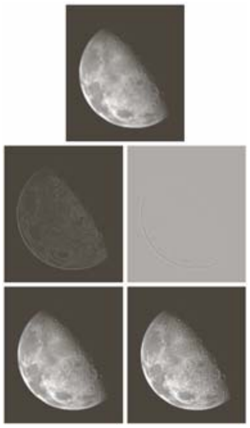


a
b c
d e

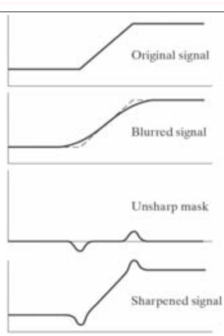
FIGURE 3.38 (a) Blurred image of the North Pole of the moon. (b) Laplacian without scaling. (c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)

Unsharpen Masking (k=1) and Highboost Filtering (k>1)

A process is often used for the printing and publishing industry to sharpen images, involving subtracting an unsharp (smoothed) version of an image from its original:

- Blur the original image $\overline{f}(x,y)$
- Subtract the blurred image from the original (unsharp mask) $g_{mask}(x,y) = f(x,y) - \overline{f}(x,y)$
- Add the mask to the original $g(x,y) = f(x,y) + k\, g_{mask}(x,y)$

Mechanics of Unsharpen Masking



a
b
c
d

FIGURE 3.39 1-D illustration of the mechanics of unsharp masking. (a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).

Example

a
b
c
d
e

FIGURE 3.40 (a) Original image. (b) Result of blurring with a Gaussian filter. (c) Unsharp mask. (d) Result of using unsharp masking. (e) Result of using highboost filtering.

Gradient

Another tool to finding edges at a location (x, y) of an image $f(x,y)$ is the gradient

$$\nabla f = grad(f) \equiv \begin{bmatrix} f_x \\ f_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

which points in the direction of the greatest rate of change of f at (x,y).

The magnitude of the vector grad(f) can be defined as

$$M(x, y) = \sqrt{f_x^2 + f_y^2},$$

i.e., the value of the rate of change in the direction of the gradient.

Gradient

The magnitude of the vector grad(f) can be defined as

$$M(x, y) = \sqrt{f_x^2 + f_y^2},$$

i.e., the value of the rate of change in the direction of the gradient.

The magnitude of the gradient is not a linear operator, but it is rotational invariant (i.e, isotropic).

In some implementation, it is easier to approximate the square root operations by absolution values

$$M(x, y) \approx |f_x| + |f_y|$$

which preserves the relative change in intensity, but the isotropic property is lost in general.

Gradient

a b
c d

FIGURE 10.16 (a) Original image of size 834 × 1114 pixels, with intensity values scaled to the range [0, 1]. (b) $|g_x|$, the component of the gradient in the x-direction, obtained using the Sobel mask in Fig. 10.14(f) to filter the image. (c) $|g_y|$, obtained using the mask in Fig. 10.14(g). (d) The gradient image, $|g_x| + |g_y|$.

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Laplacian and Gradient

- The Laplacian, being a 2nd order derivative operator, is superior in enhancing fine details.
- The gradient operator, being the 1st order derivative, has stronger average response for significant intensity transitions than does the Laplacian

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Gradient Operators

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0		0	−1
0	1		1	0

Roberts

−1	−1	−1		−1	0	1
0	0	0		−1	0	1
1	1	1		−1	0	1

Prewitt

−1	−2	−1		−1	0	1
0	0	0		−2	0	2
1	2	1		−1	0	1

Sobel

a
b c
d e
f g

FIGURE 10.14 A 3 × 3 region of an image (the z's are intensity values) and various masks used to compute the gradient at the point labeled z_5.

Roberts cross-gradient operators

Roberts (1965) developed the cross differences

$f_x = z_9 - z_5$ and $f_y = z_8 - z_6$

The magnitude of the gradient is approximated by

$M(x, y) \approx |z_9 - z_5| + |z_8 - z_6|$

which can be implemented using two linear filter masks

Prewitt Operators (1970)

Approximation to the gradient in a 3 by 3 neighborhood centered at z_5 are as follows:

$f_x = (z_7 + z_8 + z_9) - (z_1 + z_2 + z_3)$

and

$f_y = (z_3 + z_6 + z_9) - (z_1 + z_4 + z_7)$

The magnitude of the gradient is approximated by Prewitt operator

$$M(x, y) \approx |(z_7 + z_8 + z_9) - (z_1 + z_2 + z_3)| + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)|$$

Note that the weight value 2 in the center coefficient is to achieve some smoothing by giving some importance to the center point.

Sobel Operators

Approximation to the gradient in a 3 by 3 neighborhood centered at z_5 are as follows:

$f_x = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$

and

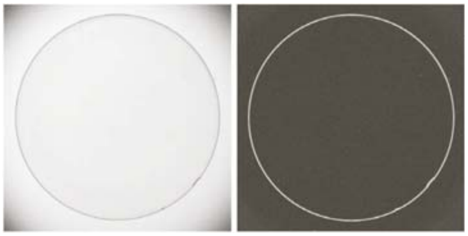
$f_y = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$

The magnitude of the gradient is approximated by Sobel operator

$$M(x, y) \approx |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)|$$

Note that the weight value 2 in the center coefficient is to achieve some smoothing by giving some importance to the center point.

Gradient often used for inspection



a b

FIGURE 3.42 (a) Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Pete Sites, Perceptics Corporation.)

4. Transform in image space

4.5 Combining spatial enhancement methods

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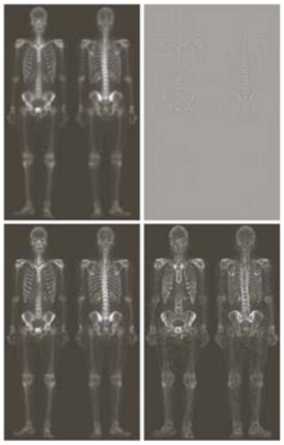
Combining different filters

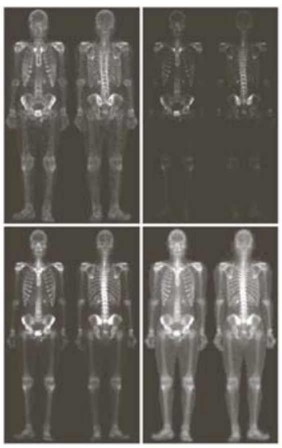
(a) Quite often one requires applications of several filtering techniques in order to achieve an acceptable result.

(b) We illustrate that via a nuclear whole body scan which is noisy and low-contrast. Our objectives is to sharpen it and to bring out more skeletal details.



a b
c d

FIGURE 3.43 (a) Image of whole body bone scan. (b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel gradient of (a).



c f
g h

FIGURE 3.43 *(Continued)* (e) Sobel image smoothed with a 5×5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)

4. Transform in image space

4.6 Anisotropic Diffusion Filter

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Assumption and Goals

Assumption

An image is a piecewise constant function that has been corrupted by zero-mean Gaussian noise with small variance

Goals

Efficiently remove noise in homogeneous regions

Preserve object boundaries, discontinuities, and detailed structures

Overview

IEEE TRANSACTIONS ON MEDICAL IMAGING, VOL. 11, NO. 2, JUNE 1992

Nonlinear Anisotropic Filtering of MRI Data

Guido Gerig, Olaf Kübler, Ron Kikinis, and Ferenc A. Jolesz

- To perform edge preserving smoothing iteratively
- To preverse edges
- First introduced by Perona & Malik (1990)
- Later used to enhance medical images

Isotropic diffusion

$$I_t(x,y,t) = \nabla \cdot \left(c\nabla I(x,y,t)\right) = c\left(I_{xx} + I_{yy}\right)$$

time or iteration

diffusion coefficient

- Blurring the original image by a Gaussian
- The Gaussian blurring is spatial invariant, i.e., it doesn't respect the natural boundaries of objects

Anisotropic diffusion

flow function Φ

$$I_t(x,y,t) = \nabla \cdot \left(c(x,y,t) \nabla I(x,y,t) \right)$$

iteration — diffusion (edge-stopping) function

- To encourage smoothing within a region while inhibiting smoothing cross regions (or piecewise smoothing)
- Function c depends on edge information
- c(x,y,t) can be defined as monotonically decreasing function of the image grad magnitude:

$$c(x,y,t) = g\left(\left| \nabla I(x,y,t) \right| \right)$$

Diffusion function c(x,y)

$$c_1 = exp\left(-\frac{|\nabla I|^2}{K^2} \right) \qquad c_2 = \frac{1}{1 + \left(\frac{|\nabla I|}{K} \right)^{1+\alpha}}, \qquad \alpha > 0$$

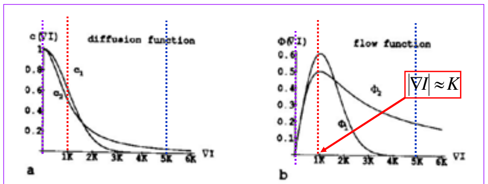

Fig. 1. (a) Diffusion functions (diffusion strength c versus gradient ∇I). (b) Flow functions (flow $\Phi(\nabla I)$ versus gradient ∇I). ∇I is given in scales of the parameter K.

Diffusion constant K

- Max flow occurs when $|\nabla I| \approx K$
- To reduce noise in an image, choose K corresponding to the grad magnitude produced by noise
- To enhance edges, choose K slightly < the grad magnitude of the edges
- Since most medical images contain a significant amount of low-contrast edges, we choose K based on noise estimate
  - Canny (1986) proposed to estimate K by the 90% value of integral of histogram of abs(grad)
  - Fix windowing technique: The window with min SD of all the regions can be used to estimate noise. 1.5 * SD < K < 2.0 * SD

Experiments: 1D (remove noise)

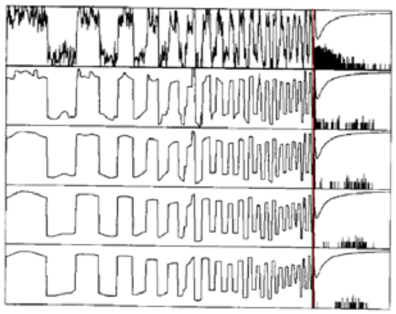

Fig. 5. One-dimensional iterated anisotropic smoothing. Top to bottom: original function and different stages of smoothing. Right-hand column: histogram of gradients and flow function (vertically flipped).

Experiments: 3D (track brain edges)