Medical Image Processing Using Transforms

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Outline

- Image Quality
- Gray value transforms
- Histogram processing
- Filters in image space
- Filters in Fourier space
- Filters in Time-frequency space

5. Filters in time-frequency space

5.1. Time-Frequency analysis

The Fourier Transform (1807)

$$F(k) = \int_{-\infty}^{+\infty} f(t)\,exp(-i2\pi kt)\,dt$$

$$f(t) = \int_{-\infty}^{+\infty} F(k)\,exp(i2\pi kt)\,dt$$

- Joseph Fourier

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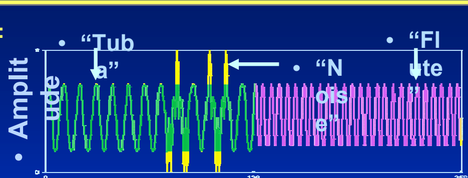
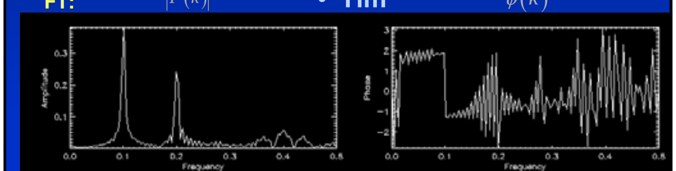
The Fourier Transform (FT)

music :



FT: $|F(k)|$ $\phi(k)$



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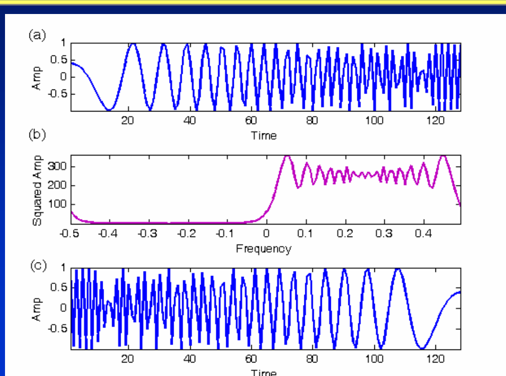
The Fourier Transform (FT)



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Most signals are non-stationary

Finite duration

Time/Spatial varying

Corrupted by noise

How can we characterize a signal simultaneously in time and frequency?

---- the aim of time-frequency analysis

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Atomic decomposition

Linearly decompose a signal over a set of elementary "building blocks" which would be reasonably 'localized' in both time and frequency

$$\lambda_f(\tau,k)=\int_{-\infty}^{+\infty} f(t)\tilde{b}^{*}_{\tau,k}(t)\,dk$$

$$f(t)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \lambda_f(\tau,k)b_{\tau,k}(t)\,d\tau\,dk$$

where $\tilde{b}^{*}_{\tau,k}(\bullet)$ is some analysis fucntion deduced from the "synthesis" function $b_{\tau,k}(\bullet)$, making $\lambda_f(\tau,k)$ a (linear) time-frequency representation of $f(t)$.

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The Gabor Transform (1946)

Also called the short-time or windowed FT

$$G(\tau,k)=\int_{-\infty}^{+\infty} f(t)w^{*}(t-\tau)exp(-i2\pi kt)\,dt$$

where, e.g., w can be the Gaussian function

$$w(t-\tau)=\frac{1}{\sqrt{2\pi b^2}}exp\left(-\frac{(t-\tau)^2}{2b^2}\right)$$

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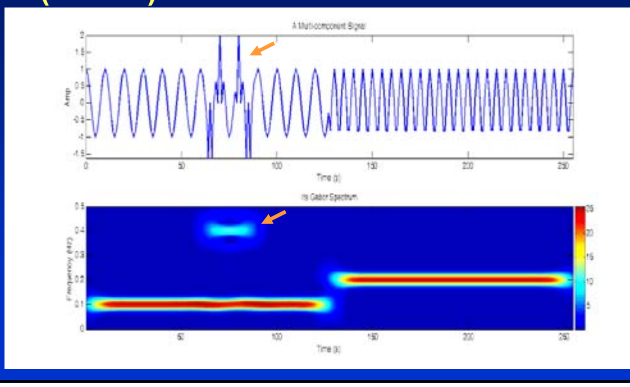
Time-Frequency Representation (Gabor)



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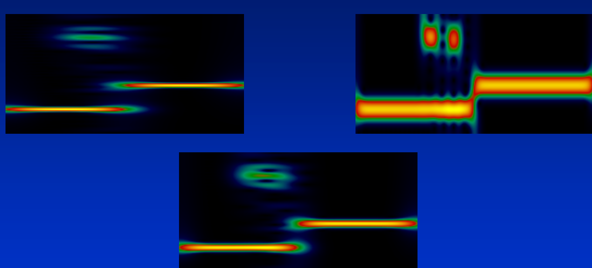
How accurate can the GT be ?



Width of time window → Smaller

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Heisenberg Inequality

Also called the Uncertainty Principle:

Resolution in time and frequency cannot be arbitrarily small, because their product is bounded below:

$$\Delta t \cdot \Delta k \geq \frac{1}{4\pi}$$

Here, given the window functions $w(t) \xleftrightarrow{FT} W(k)$**,**

$$\Delta t^2 = \frac{\int t^2 |w(t)|^2 dt}{\int |w(t)|^2 dt} \qquad \Delta k^2 = \frac{\int k^2 |W(k)|^2 dk}{\int |W(k)|^2 dk}$$

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Next Step

$$\Delta t \cdot \Delta k \geq \frac{1}{4\pi}$$

There always is a trade off between Δt and Δk.

Fortunately, many signals consist of
low frequencies of long duration and/or
high frequencies of short duration

The next logical step is to use a windowing technique with variable sizes:
long time window for better Δk at low frequencies,
short time window for better Δt at high-frequencies.

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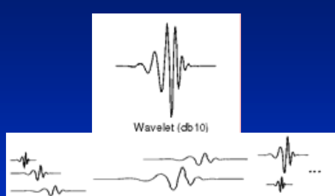
The Continuous Wavelet Transform (CWT)

Wavelets: small waves



The CWT decomposes a signal into the scaled and shifted replica of the Mother wavelet (a waveform of effectively limited duration and zero mean)

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The Continuous Wavelet Transform

Wavelets: small waves (1984)

$$CW(\tau, s) = \frac{1}{\sqrt{|s|}} \int_{-\infty}^{+\infty} f(t)\, w^{*}\left(\frac{t-\tau}{s}\right) dt$$

where $\frac{1}{\sqrt{|s|}} w\left(\frac{t-\tau}{s}\right)$ is the scaled and shifted replica of the Mother wavelet, a waveform satisfying

$$c_w = \int_{-\infty}^{+\infty} \frac{|W(k)|^2}{k} dk < \infty$$

Effectively, $W(0) = 0$ and $W(k) \to 0$ as $k \to \infty$

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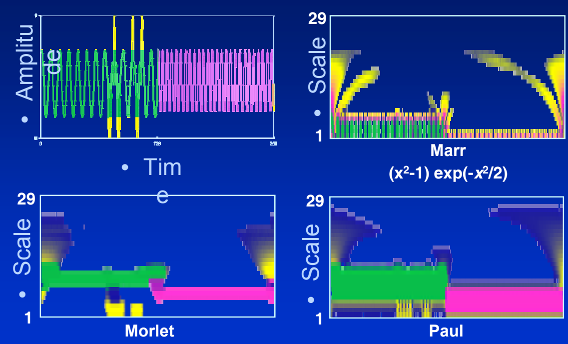
The Continuous Wavelet Transform



$(x^2\text{-}1)\ \exp(\text{-}x^2/2)$

$\pi(\text{-}1/4)s^{-1/2}\exp(j\ k\ x\ /s)\ \exp(\text{-}(x\ /\ s)^2/2)$

$(1\text{-}|x|^s)^{\text{-}m\text{-}1}$

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Gabor Transform: Time-frequency Representation

Wavelet Transform: Multi-scale Analysis

The ST is a Multi-scale Time-frequency Analysis

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The Stockwell Transform

$$S(\tau,k)=\int_{-\infty}^{+\infty} f(t)\,g\left(t-\tau,1/|k|\right)exp\left(-i2\pi kt\right)dt$$

where the window function g is the Gaussian function with frequency-dependent window width,

$$g(t-\tau,1/k)=\frac{|k|}{\sqrt{2\pi}}exp\left(-\frac{(t-\tau)^2k^2}{2}\right)$$

Stockwell (1996) IEEE T Signal Processing, V44

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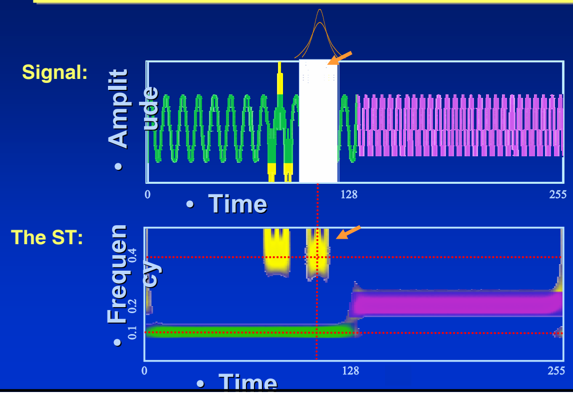
The Stockwell Transform (ST)



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The ST and Morlet wavelets

With the complex Morlet mother wavelet

$$\psi^{\nu_0}(t) = \frac{1}{\sqrt{2\pi}} exp\left(-\frac{t^2}{2}\right) exp(i2\pi\nu_0 t),$$

the Morlet wavelet transform (MWT) is defined as

$$MW(\tau, a) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{+\infty} f(t)\psi^{\nu_0 *}\left(\frac{t-\tau}{a}\right) dt.$$

where $a = \frac{\nu_0}{k}$. We can show that

$$S(\tau, k) = \sqrt{k}\, e^{i2\pi k\tau} M_{\psi^1}\left(\tau, \frac{1}{k}\right).$$

Du, Wong, Zhu (2006)
Gibson, Lamoureux, Margrave (2006)

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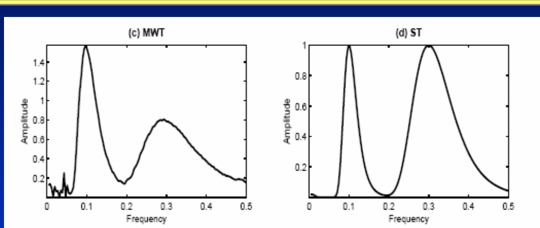
The ST and Morlet wavelets



(a) Small oscillations occur for small frequencies
(b) The absolute referenced phase information is retained in the ST, while the MWT gives relative referenced phase information.

Liu, Zhu (2007)

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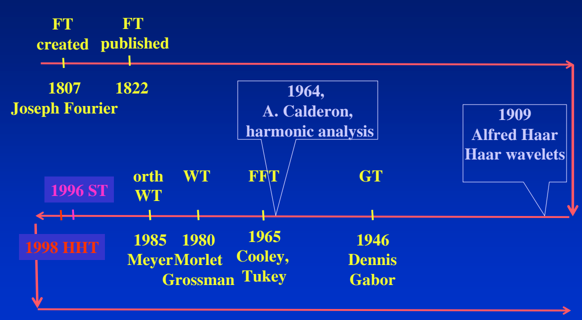
Many different time-frequency transforms



Applications of Time-frequency Representations

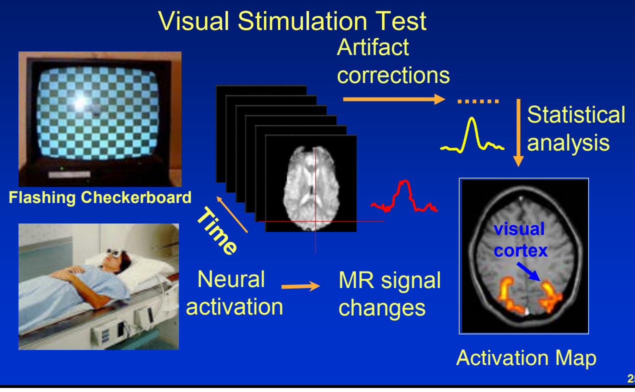
$$f(t) \rightleftarrows TF(\tau, k)$$

Object **Time-frequency**

- Analyze the raw signal in the (τ, k) domain to identify its local characteristics
- Remove noise from signal or separate and analyze specific components
- Extract Features from its time-frequency representation
- Extension to two or higher dimensions; …

Correct motion artifacts in fMRI

Functional MRI (fMRI)



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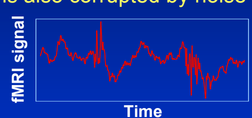
fMRI Signal

- ≤ 5% of collected MR data is related to neural activities triggered by fMRI experiment
- Limited data is also corrupted by noise



Problem

How can we correct unpredictable motion artifacts to improve the accuracy and reproducibility in fMRI analysis?

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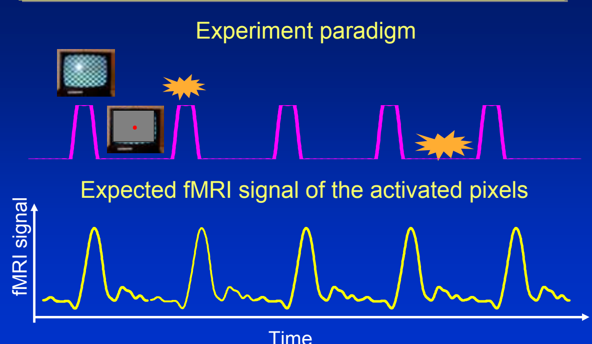
fMRI Visual Stimulation Experiment



Zhu, Goodyear, et al. Med Phys 30:1134-1141(2003)

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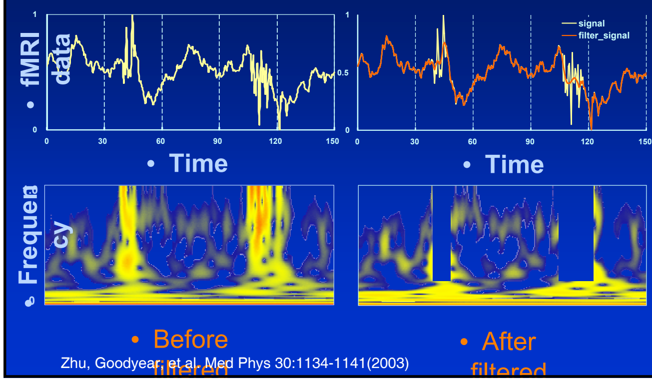
ST Correction for Motion Artifacts



Zhu, Goodyear, et al. Med Phys 30:1134-1141(2003)

Filtering using wavelet transforms

Discrete Wavelet Transforms

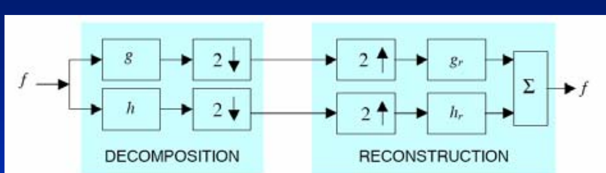
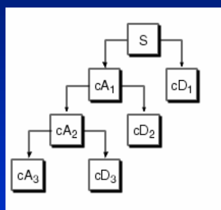


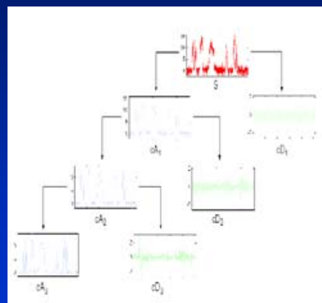
Fig. 4.2 Illustration of wavelet-based signal decomposition and reconstruction g, g_r – lowpass filters; h, h_r – highpass filters; (2↓) – downsampling (decimation by 2); (2↑) – upsampling.

If the high-pass and low-pass filters satisfy certain conditions, we can downsample the details by two. This is because max freq is halved according to Nyquist's rule

DWT: Multi-level decomposition

wavelet decomposition tree





The DWT gives samples of the CWT

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2D-DWT

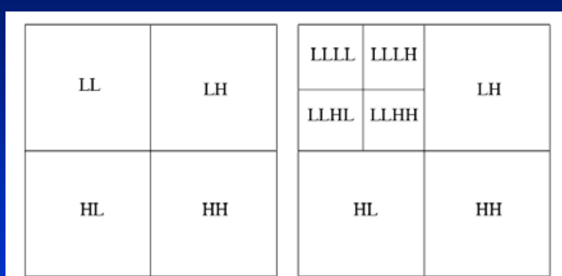

Fig. 4.4 One-scale decomposition (left), two-scale decomposition (right)

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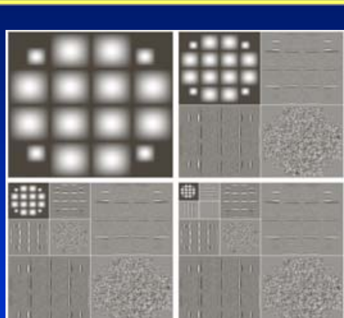
2D-DWT



a b
c d

FIGURE 7.25 Computing a 2-D three-scale FWT: (a) the original image; (b) a one-scale FWT; (c) a two-scale FWT; and (d) a three-scale FWT.

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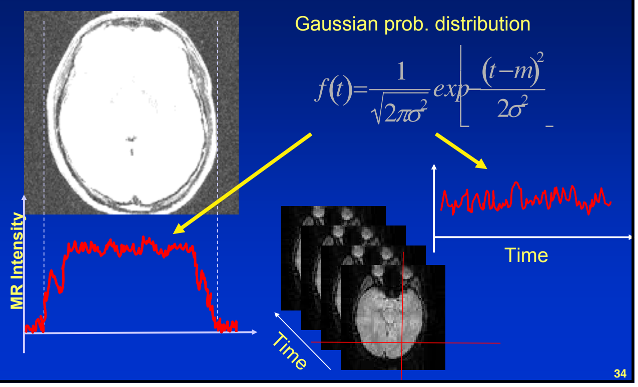
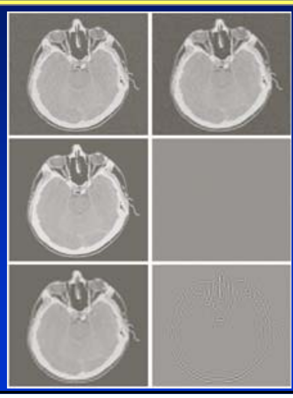
Thermal Noise ≈ White Noise

Gaussian prob. distribution

$$f(t)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{(t-m)^2}{2\sigma^2}\right]$$



Denoising



a b
c d
e f
FIGURE 7.28 Modifying a DWT for noise removal: (a) a noisy CT of a human head; (b), (c) and (e) various reconstructions after thresholding the detail coefficients; (d) and (f) the information removed during the reconstruction of (c) and (e). (Original image courtesy Vanderbilt University Medical Center.)

Wavelet-based Wiener Filter

Magnetic Resonance in Medicine 44:746–757 (2000)

Wavelet Transform-Based Wiener Filtering of Event-Related fMRI Data

Stephen M. LaConte, Shing-Chung Ngan, and Xiaoping Hu*

The advent of event-related functional magnetic resonance imaging (fMRI) has resulted in many exciting studies that have exploited its unique capability. However, the utility of event-related fMRI is still limited by several technical difficulties. One significant limitation in event-related fMRI is the low signal-to-noise ratio (SNR). In this work, a method based on Wiener filtering in the wavelet domain is developed and demonstrated for denoising event-related fMRI data. Application of the technique to simulated and experimental data demonstrates that the technique is effective in reducing noise while preserving neuronal activity-induced response. Magn Reson Med 44: 746–757, 2000. © 2000 Wiley-Liss, Inc.

Key words: event-related fMRI; denoising; stationary wavelet transform; Wiener filter

Wavelet-based Wiener Filter

In the wavelet domain, the desired Wiener filter takes the form:

$$H(j, n) = \frac{P_x(j, n)}{P_x(j, n) + P_v(j, n)} \qquad [13]$$

where $P_x(j,n)$ is the power density corresponding to the detailed component of true signal x in location n at resolution level j. $P_v(j,n)$ is the corresponding term of the noise.

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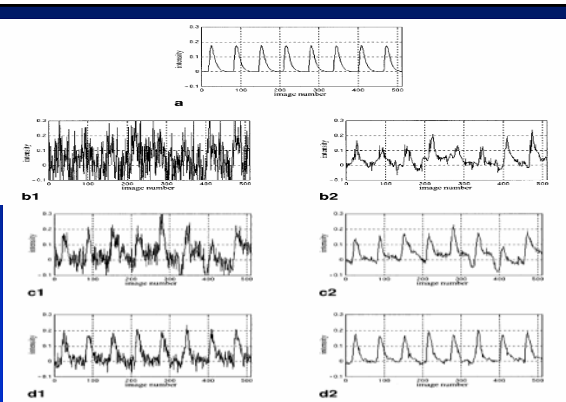

FIG. 1. Simulated time courses before and after denoising. The horizontal axis represents image number and the vertical axis corresponds to normalized signal intensity. a: The uncorrupted (true) signal. Baseline data were added to a to yield time courses with SNR of 0.5 (b1), 1.0 (c1), and 2.0 (d1), respectively. Their denoised counterparts are shown in b2, c2, and d2.

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Contrast Enhancement by Multiscale and Nonlinear Operators

Jian Fan and Andrew Laine

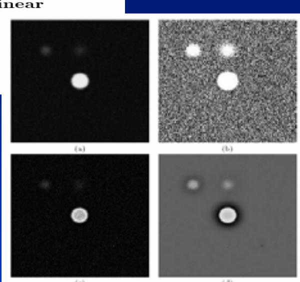

Figure 7.13: (a) Noisy image (white Gaussian noise contaminated). (b) Histogram equalized. (c) Nonlinear enhancement by Beghdadi and Negrate's algorithm. (d) Nonlinear enhancement with adaptive wavelet shrinkage denoising, $G_m = 20$, $N = 4$, $t = 0.1$.

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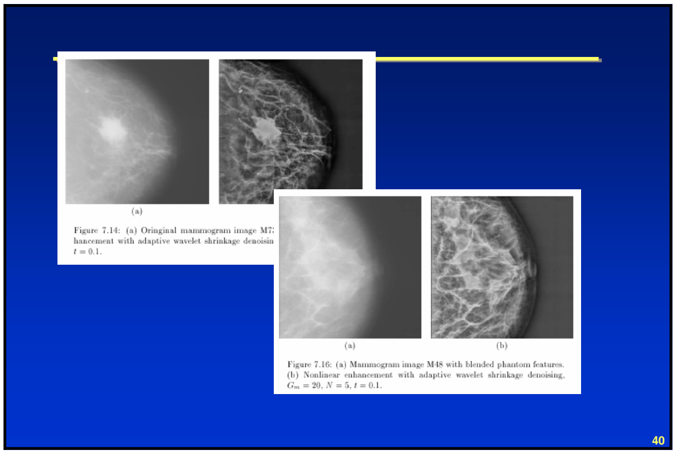
(a)
(a)
(b)
Figure 7.16: (a) Mammogram image M48 with blended phantom features. (b) Nonlinear enhancement with adaptive wavelet shrinkage denoising, $G_m = 20$, $N = 5$, $t = 0.1$.
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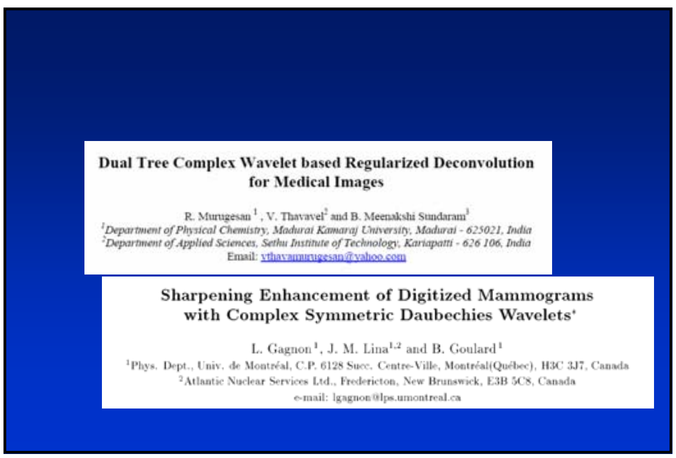
Dual Tree Complex Wavelet based Regularized Deconvolution for Medical Images
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Sharpening Enhancement of Digitized Mammograms with Complex Symmetric Daubechies Wavelets*
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