Default Clustering and Valuation of CDOs

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Bachelier 2010

Outline

1. CDO and Review of Pricing Models
   - What is a CDO?
   - Review of Existing Pricing Models
2. The Conditional Survival (CS) Model for CDO Pricing
   - Motivation and Default Clustering
   - Conditional Survival (CS) Model
3. Calibration Results

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What is a CDO?

- Banks suffered tens of billion dollar losses due to subprime CDOs at the end of 2007
- What is a CDO?

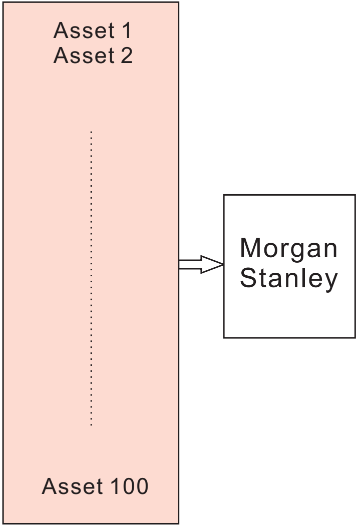
Collateralized Debt Obligation (CDO)

- A CDO is a debt security that is constructed from a portfolio of collateral (assets).

What is a CDO?

Morgan Stanley

What is a CDO?



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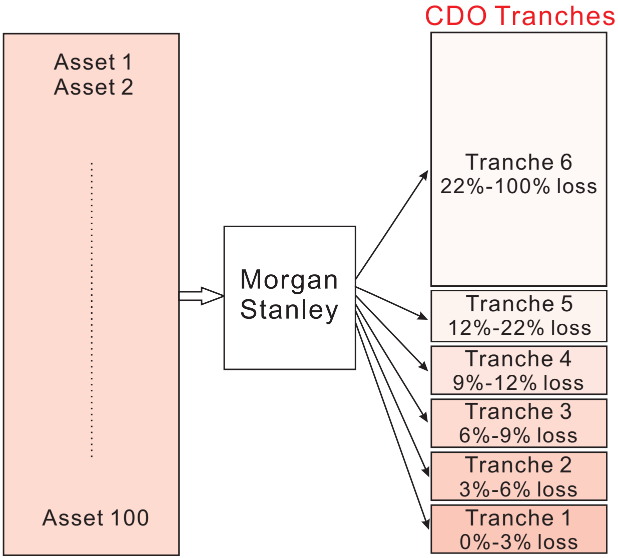
CDO Tranches
Asset 1
Asset 2
Asset 100
Morgan Stanley
Tranche 6
22%-100% loss
Tranche 5
12%-22% loss
Tranche 4
9%-12% loss
Tranche 3
6%-9% loss
Tranche 2
3%-6% loss
Tranche 1
0%-3% loss

Objective of a CDO Pricing Model

- τ_i: default time of the i-th name, $i = 1, \ldots, n$
- Cumulative portfolio loss at time t:

$$L_t = \sum_{i=1}^{n} c_i \cdot 1_{\{\tau_i \leq t\}}$$

- CDO tranche valuation reduces to calculation of $E[(L_t - K)^+]$
- Objective of CDO pricing model
  - calibrate to single name marginal default probability
  - calibrate to CDO tranche spreads

Two Approaches for CDO Modeling

- Top-down approach builds models for the portfolio cumulative loss process directly
  - Good fit for standard CDO portfolios
  - CANNOT calibrate to single name marginal default probability
  - Longstaff & Rajan, 2007, Giesecke, et al., 2007, Halperin, 2007, Cont & Minca, 2007
- Bottom-up approach builds models for single name default times
  - Consistent with single name marginal default probability
  - Has more difficulty in calibrating CDO tranche spreads
  - Examples: Static bottom-up models, e.g. copula models, dynamic intensity models

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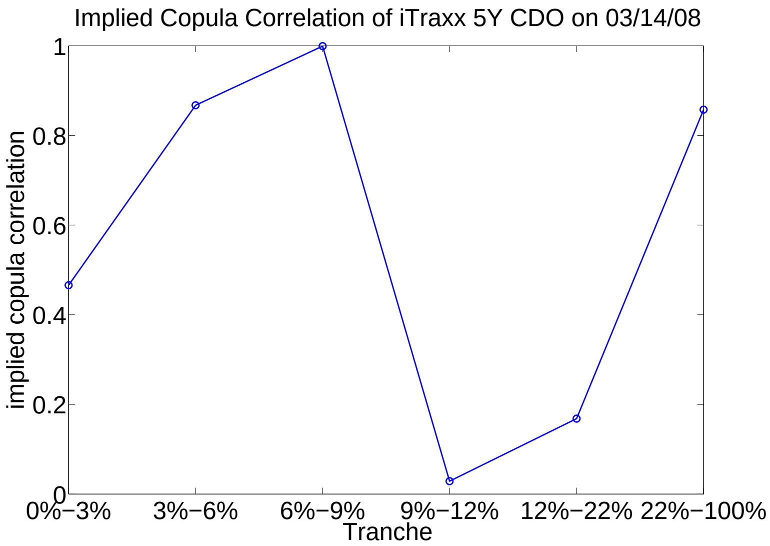
The Market Standard—Gaussian Copula Models

- Idea: using copula functions to model default time correlation
- Literature: Gaussian copula model (Li, 2000)
- What is wrong with Gaussian copula?
  - Gaussian copula cannot generate tail dependence

$$\lim_{q \to 0} P(\tau_2 < F_2^{-1}(q) | \tau_1 < F_1^{-1}(q)) = 0$$

  - Gaussian copula does not work during crisis, when the default correlation is strong.

Gaussian Copula Does Not Work During Crisis



Dynamic Intensity Models

- General idea
  - Single name default intensity $\lambda_i(t)$: (Jarrow & Turnbull, 1995)

$$P(\tau_i \leq t + \Delta t | \mathcal{F}_t) = \lambda_i(t)\Delta t + o(\Delta t), \text{on } \{\tau_i > t\}$$

  - Building correlation among default intensities $\lambda_1(t), \ldots, \lambda_n(t)$
- Dynamic intensity model for CDO pricing (Duffie & Gârleanu, 2001, Mortensen, 2006)
  - $\lambda_i(t) = a_i \lambda^M(t) + \lambda_i^{id}(t)$, $i = 1, \ldots, n$
  - $\lambda^M(t)$ and $\lambda_i^{id}(t)$ are independent affine jump diffusion processes
- Drawback: cannot incorporate strong default correlation
- Other dynamic models: Hull & White (2008), Hurd & Kuznetsov (2006), Joshi & Stacey (2006), Papageorgiou & Sircar (2007), Schönbucher (2007), Tsui (2010)

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Motivation

Default clustering effect: cross-sectional and dynamic (across time)

- The recent demise of major financial institutions
- The iTraxx 5Y index tranche spreads

Tranches	0-3%	3-6%	6-9%	9-12%	12-22%	22-100%
09/20/07	1812	84	37	23	15	7
03/14/08	5150	649	401	255	143	70
09/16/08	4598	618	375	215	102	59

- Empirical evidence of default clustering (Das, Duffie, Kapadia, & Saita, 2007, Longstaff & Rajan, 2007, Azizpour & Giesecke, 2008)

Motivation (continued)

- Dynamic intensity model (Doubly stochastic model):

$$\tau_i = \inf\{t \geq 0 : \Lambda_i(t) \geq E_i\}, E_i \overset{d}{\sim} \exp(1), i.i.d.$$

$$\Lambda_i(t) = \int_0^t \lambda_i(s)ds \quad (\Lambda_i(t) \text{ is continuous!})$$

- Drawback: It cannot generate simultaneous defaults of several names.
- Empirical observation of simultaneous default: 24 railway firms defaulted on June 21, 1970 (Azizpour & Giesecke, 2008)
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The New Model: Conditional Survival (CS) Model

Our new model: conditional survival (CS) model is based on cumulative default intensity

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$$\Lambda_i(t) = \sum_{j=1}^{J} a_{i,j} M_j(t) + X_i^{id}(t), i = 1, \ldots, n.$$

- $M(t) = (M_1(t), \ldots, M_J(t))$: market factor processes that may have jumps
- $X_i^{id}(t)$: idiosyncratic part of the cumulative default intensity

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Conditional Survival Probability

Conditional survival probability

$$q_i^c(t) := P(\tau_i > t \,|\, M(t)) = E\left[e^{-X_i^{id}(t)}\right] e^{-\sum_{j=1}^{J} a_{i,j} M_j(t)}$$

Survival probability

$$q_i(t) := P(\tau_i > t) = E\left[e^{-X_i^{id}(t)}\right] E\left[e^{-\sum_{j=1}^{J} a_{i,j} M_j(t)}\right]$$

- Conditional survival probability is the building block

$$q_i^c(t) = q_i(t) \cdot \frac{e^{-\sum_{j=1}^{J} a_{i,j} M_j(t)}}{E\left[e^{-\sum_{j=1}^{J} a_{i,j} M_j(t)}\right]}$$

- Use $q_i(t)$ as input: the model automatically calibrates to single name default probability
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Specifying Dynamics of Market Factors

- Pólya process $M(t)$
  - A Pólya process is a mixed Poisson process
  - Clustering jumps: a Pólya process has positive correlation between increments

$$\mathrm{Cov}(M(t), M(t+h) - M(t)) > 0$$

- Discrete integral of CIR process: $M(t) = \int_0^t V(s)\,ds$

$$dV(t) = \kappa(\theta - V(t))dt + \sigma\sqrt{V(t)}dW(t)$$

- Laplace transforms of both processes have closed form.

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CDO Pricing by Exact Simulation

- Simulation is fast: only need to simulate market factor processes and Bernoulli r.v.s
- Key fact: conditional on $M(t)$, default events $I_i = 1_{\{\tau_i \le t\}}, i = 1, \ldots, n$ are independent Bernoulli$(1 - q_i^c(t))$ r.v.
- Exact simulation of L_t at given time t:
  1. Generate market factors $M_1(t), M_2(t), \ldots, M_J(t)$.
  2. Calculate the conditional survival probability analytically

$$q_i^c(t) = q_i(t) \cdot \frac{e^{-\sum_{j=1}^{J} a_{i,j} M_j(t)}}{E\left[e^{-\sum_{j=1}^{J} a_{i,j} M_j(t)}\right]}$$

  3. Generate independent $I_i \overset{d}{\sim}$ Bernoulli$(1 - q_i^c(t)), i = 1, \ldots, n$.
  4. Calculate $L_t = \sum_{i=1}^{n} c_i \cdot I_i$.
- $E[(L_t - K)^+]$: leads to CDO tranche spreads
- Control variants: L_t

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Sensitivity of CDO tranches w.r.t Single Name CDS

- Sensitivity w.r.t. to single name survival probability

$$E[(L_t - K)^+] = E\left[A_i(t)q_i(t) + B_i(t)\right]$$

$$\frac{\partial E[(L_t - K)^+]}{\partial q_i(t)} = E[A_i(t)]$$

- $A_i(t)$ can be obtained as a byproduct in each simulation of L_t.
- The sensitivities w.r.t. each of the n single name CDS are obtained concurrently with CDO tranche pricing.

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Calibration to iTraxx 5Y Tranche Spreads on 03/14/08

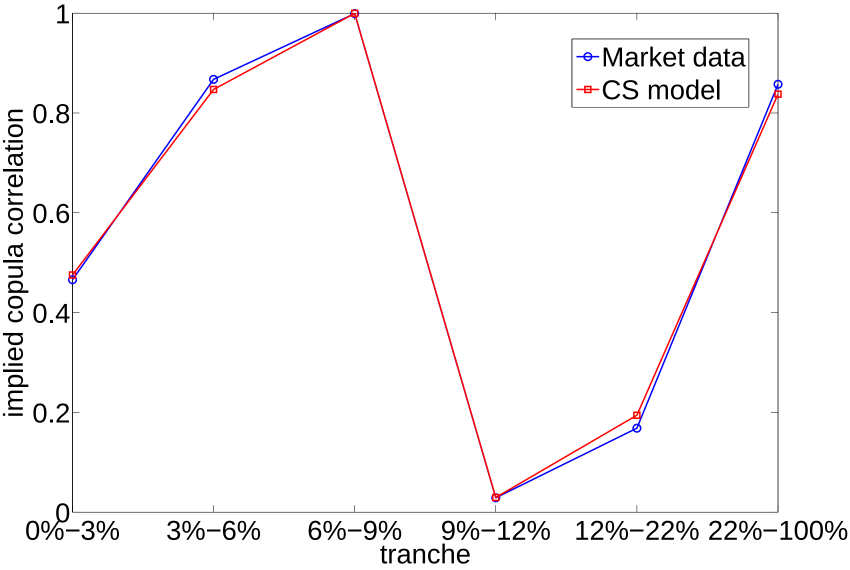
- CDO and CDS data on March 14, 2008, right before the collapse of Bear Stern
- Calibration results by using 2 Polya process and 1 discrete integral of CIR process

Tranche(%)	0-3	3-6	6-9	9-12	12-22	22-100
Market spread	5150	649	401	255	143	70
Model spreads	5071	689	394	258	164	67
B-A spread	158	24	25	20	12	3

- Pricing error: Chi-square = 6.48(p-value = 0.26), RMSE = 1.11

$$\text{CHISQ} = \sum_{k=1}^{6} \frac{(s_k - s_k^o)^2}{s_k}, \quad \text{RMSE} = \sqrt{\frac{1}{6}\sum_{k=1}^{6}\left(\frac{s_k - s_k^o}{s_k^{o,a} - s_k^{o,b}}\right)^2}$$

Model and Market Implied Correlation on 03/14/08



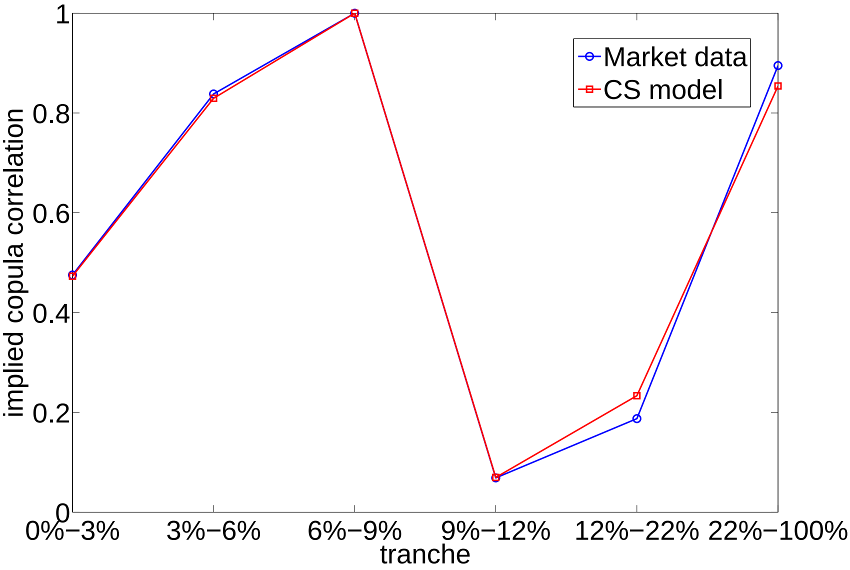
Calibration to iTraxx 5Y Tranche Spread on 09/16/08

- CDO and CDS data on September 16, 2008, right after Lehman Brothers went bankruptcy
- Calibration results by using 2 Polya process and 1 discrete integral of CIR processes

Tranche(%)	0-3	3-6	6-9	9-12	12-22	22-100
Market spread	4598	618	375	215	102	59
Model spread	4617	631	347	217	131	53
Bid-ask spread	118	14	13	11	5	3

- Pricing error: Chi-square = 9.25(p-value = 0.10), RMSE = 2.55

Model and Market Implied Correlation on 09/16/08



Summary

We propose the conditional survival (CS) model:

- It is based on cumulative default intensities instead of intensities.
- It is able to generate a substantially high degree of default clustering.
- It does not specify any dynamics for idiosyncratic default risk component.
- It automatically calibrates to single name marginal default probability.

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- It is able to generate a substantially high degree of default clustering.
- It does not specify any dynamics for idiosyncratic default risk component.
- It automatically calibrates to single name marginal default probability.

Summary

We propose the conditional survival (CS) model:

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- It automatically calibrates to single name marginal default probability.

Thank you!

Parameter Stability

Table: Compare parameters for 03/14/08 and 09/16/08

	03/14/08	09/16/08
α_1	0.5321	0.6871
β_1	0.0301	0.0179
α_2	0.0013	0.0058
β_2	8.2619	9.0245
κ	0.0526	
σ	1.6837	
$\lambda(0)$	1.9176	

Implicit Constraints on Model Parameters

- The idiosyncratic cumulative intensities $X_i^{id}(t) \geq 0$ and increasing

$$E\left[e^{-X_i^{id}(T_m)}\right] \leq E\left[e^{-X_i^{id}(T_{m-1})}\right] \leq \cdots \leq E\left[e^{-X_i^{id}(T_1)}\right] \leq 1, \forall 1 \leq i \leq n$$

- Recall that

$$E\left[e^{-X_i^{id}(t)}\right] = \frac{q_i(t)}{E\left[e^{-\sum_{j=1}^{J} a_{i,j} M_j(t)}\right]}$$

- This imposes parameter constraints:

$$\frac{q_i(T_m)}{E\left[e^{-\sum_{j=1}^{J} a_{i,j} M_j(T_m)}\right]} \leq \cdots \leq \frac{q_i(T_1)}{E\left[e^{-\sum_{j=1}^{J} a_{i,j} M_j(T_1)}\right]} \leq 1, \forall 1 \leq i \leq n$$

Calibration Algorithm

Pricing error function: $F(\Theta)$

1. Initialization: set market factor parameter Θ_0, and set $s = 0$.
2. Iteration: $s \to s + 1$
   - For given Θ_s, determine loading coefficients $a_{i,j}$ by solving a constrained optimization problem:

$$\begin{array}{ll} \min & E\left[e^{-\sum_{j=1}^{J} a_{i,j} M_j(T_m)}\right] - q_i(T_m) \\ \text{s.}t. & \dfrac{q_i(T_m)}{E\left[e^{-\sum_{j=1}^{J} a_{i,j} M_j(T_m)}\right]} \leq \cdots \leq \dfrac{q_i(T_1)}{E\left[e^{-\sum_{j=1}^{J} a_{i,j} M_j(T_1)}\right]} \leq 1 \\ & 0 \leq a_{i,j} \end{array}$$

   - Calculate the tranche spreads and pricing error $F(\Theta_s)$
   - Update the market factor parameter $\Theta_s \to \Theta_{s+1}$ by some optimization routine, e.g. Powell's direction-set algorithm
   - Repeat