Pricing CO_2 permits using approximation approaches

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Basic idea of cap-and-trade systems

- **At the beginning** of the compliance period, the regulator **allocates** permits to the companies
- **During** the compliance period, the companies can **trade** permits among each other
- **At the end** of the compliance period, a regulated company has to **hand in** one permit or **pay a penalty fee** per unit of emission

Permit price in the EU ETS during the first phase

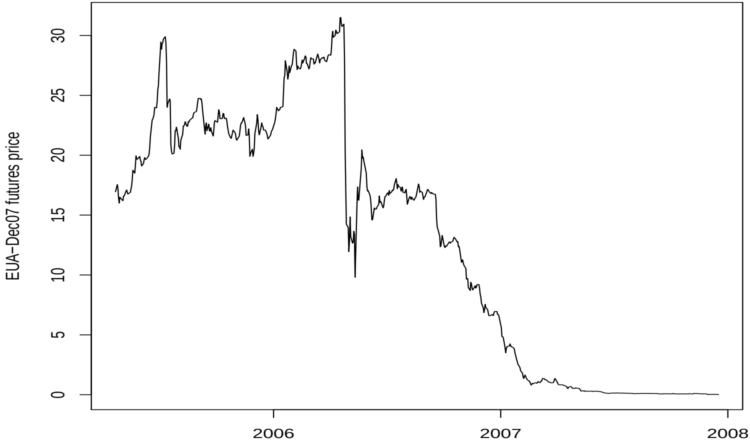

Figure: EUA-Dec07 futures price (22 April 2005 - 17 December 2007).

Analyze permit price jumps in an equilibrium model

- Equilibrium models have been widely used in literature with the aim of showing theoretical properties of emission trading systems, especially, its cost-effectiveness.
- EU ETS is by far the largest cap-and-trade system. Prices during the first phase exhibited jumpy behaviour and an extreme price drop in April/May 2006. Price dynamics have been analyzed using jump-diffusion models, GARCH-models and regime-switching models.

Motivation of our paper

- Modify an existing permit price model
- Explain the permit price slump in 2006 with an equilibrium model

Overview of selected stochastic equilibrium models (1/3)

Permit price in the model of Carmona et al. (2009)

$$F(t,T) = P \cdot \mathbb{P}\left(q_{[0,T]} > N | \mathcal{F}_t\right) \tag{1}$$

$$= \begin{cases} P & \text{if } q_{[0,t]} \geq N \\ P \cdot \mathbb{P}\left(q_{[t,T]} > N - q_{[0,t]} | \mathcal{F}_t\right) & \text{if } q_{[0,t]} < N \end{cases} \tag{2}$$

where

- P is the penalty fee that has to be paid for each emission unit not covered by an emission allowance at the compliance time T.
- T is the compliance time
- N is the amount of emission allowances handed out by the regulator
- $q_{[0,t]}$ models the cumulative emissions in the time period $[0, t]$

Overview of selected stochastic equilibrium models (2/3)

Permit price in the model of Chesney and Taschini (2008)

specifies the process for the cumulative emissions in the framework of Carmona et al. (2009) by

$$q_{[0,t]} = \int_0^t Q_s ds \tag{3}$$

where the emission rate Q_t follows a geometric Brownian motion.

There is no closed-form density for $q_{[0,t]}$ available.

Linear approximation approach of Chesney and Taschini (2008)

$$q_{[t_1,t_2]} \approx \tilde{q}^{Lin}_{[t_1,t_2]} = Q_{t_2}(t_2 - t_1) \tag{4}$$

Overview of selected stochastic equilibrium models (3/3)

Moment matching approaches of Grüll and Kiesel (2009)

(a) Log-normal (moment matching)

$$q_{[t_1,t_2]} \approx \tilde{q}^{Log}_{[t_1,t_2]} = logN\left(\mu_L(t_1,t_2), \sigma^2_L(t_1,t_2)\right) \tag{5}$$

(b) Reciprocal gamma (moment matching)

$$q_{[t_1,t_2]} \approx \tilde{q}^{IG}_{[t_1,t_2]} = IG(\alpha_{IG}, \beta_{IG}) \tag{6}$$

where the parameters $\mu_L(t_1,t_2)$, $\sigma_L(t_1,t_2)$ and α_{IG} and β_{IG} are chosen such that the first two moments of $\tilde{q}^{Log}_{[t_1,t_2]}$ and $\tilde{q}^{IG}_{[t_1,t_2]}$, respectively, match those of $q_{[t_1,t_2]}$.

Moment matching requires two steps

(1) Compute the first two moments m_k of a log-normal and a reciprocal gamma random variable and solve for the parameters.

In the log-normal case we have that $m_k = e^{k\mu + k^2 \frac{\sigma^2}{2}}$ and

$$\sigma^2 = \ln\left(\frac{m_2}{m_1^2}\right) \qquad \mu = \ln(m_1) - \frac{1}{2}\sigma^2$$

(2) Compute the first two moments of the integral over a geometric Brownian motion

$$\mathbb{E}\left[q_{[t_1,t_2]}\right] = Q_{t_1}\alpha_{t_2-t_1} \tag{7}$$

$$\mathbb{E}\left[\left(q_{[t_1,t_2]}\right)^2\right] = 2Q_{t_1}^2\beta_{t_2-t_1} \tag{8}$$

and plug those into the above equation.

Permit price - linear approximation

The permit price at time t is given by

$$S_t^{Lin} = \begin{cases} Pe^{-r\tau} & \text{if } q_{[0,t]} \geq N \\ Pe^{-r\tau} \cdot \Phi\left(\frac{-\ln\left(\frac{1}{\tau}\left[\frac{N-q_{[0,t]}}{Q_t}\right]\right)+\left(\mu-\frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}} \right) & \text{if } q_{[0,t]} < N \end{cases} \quad (9)$$

where
$\tau = T - t$ is the time to compliance.
$\Phi(\cdot)$ denotes the c.d.f. of a standard normal random variable.

Permit price - log-normal moment matching

The permit price at time t is given by

$$S_t^{Log} = \begin{cases} Pe^{-r\tau} & \text{if } q_{[0,t]} \geq N \\ Pe^{-r\tau} \cdot \Phi\left(\frac{-\ln\left(\frac{N-q_{[0,t]}}{Q_t}\right) + 2\ln(\alpha_\tau) - \frac{1}{2}\ln(2\beta_\tau)}{\sqrt{\ln(2\beta_\tau) - 2\ln(\alpha_\tau)}} \right) & \text{if } q_{[0,t]} < N \end{cases} \tag{10}$$

where
$\tau = T - t$ is the time to compliance and
α_τ, β_τ are obtained by calculating the first and the second moment of the integral over a geometric Brownian motion.
$\Phi(\cdot)$ denotes the c.d.f. of a standard normal random variable.

Permit price - reciprocal gamma moment matching

The permit price at time t is given by

$$S_t^{IG} = \begin{cases} Pe^{-r\tau} & \text{if } q_{[0,t]} \geq N \\ Pe^{-r\tau} \cdot G\left(\frac{Q_t}{N - q_{[0,t]}} \middle| \frac{4\beta_\tau - \alpha_\tau^2}{2\beta_\tau - \alpha_\tau^2}, \frac{2\beta_\tau - \alpha_\tau^2}{2\alpha_\tau \beta_\tau}\right) & \text{if } q_{[0,t]} < N \end{cases} \tag{11}$$

where

$\tau = T - t$ is the time to compliance and

α_τ, β_τ are obtained by calculating the first and the second moment of the integral over a geometric Brownian motion.

$G(x|a, b)$ denotes the c.d.f. of a gamma random variable with shape parameter a and scale parameter b.

Relating theoretical permit prices to x_t

We introduce the following two random variables that are very easy to interpret

Time needed to exhaust the remaining permits

$$x_t := \frac{N - q_{[0,t]}}{Q_t} \tag{12}$$

and

Over-/Underallocation in years

$$x_t - (T - t) \tag{13}$$

Numerical illustrations (1/2)

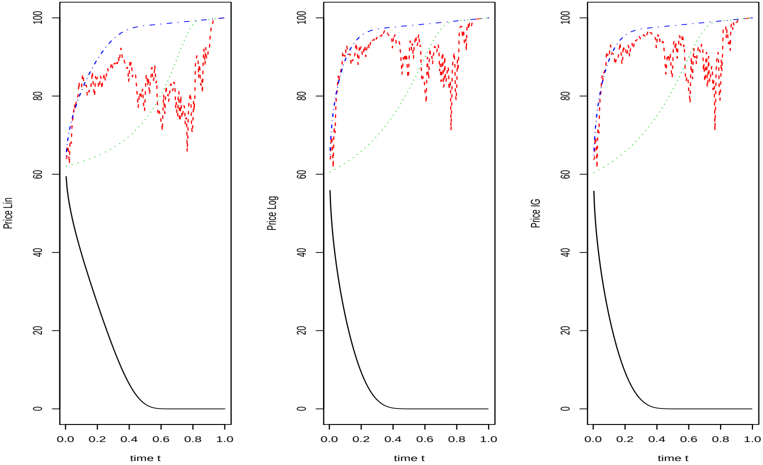

Figure: Trajectory of $S_t^{Lin}(x_t)$ (left), $S_t^{Log}(x_t)$ (middle) and $S_t^{IG}(x_t)$ (right) for $t \in [0, 1]$, $N = Q_0 = 100$, $\mu = 0.02$ and $\sigma = 0.05$.

Numerical illustrations (2/2)

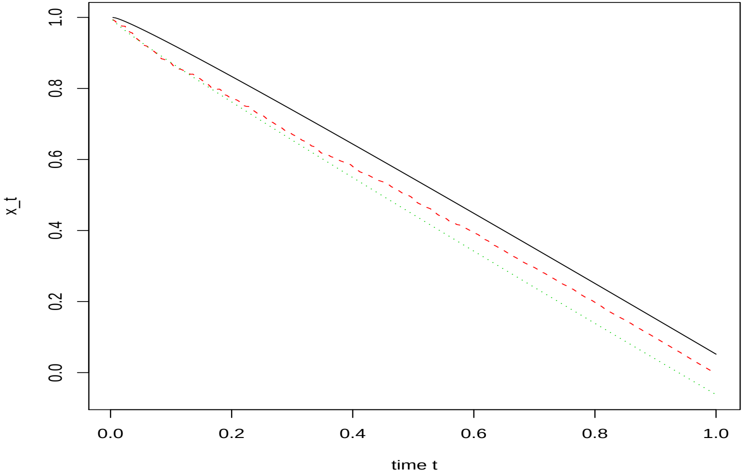

Figure: Trajectory of x_t for $t \in [0, 1]$, $N = Q_0 = 100$, $\mu = 0.02$ and $\sigma = 0.05$.

Price slump of more than 50% can be related to an implicit change in x_t of less than 5% (1/2)

For the analysis we introduce the following notation

- $t - \Delta$ is the date before the publication of verified emissions that affected the permit price (28 April 2006)
- t is the date of the announcement of cumulative emissions (15 May 2006)

Using

- the cumulative emissions until t denoted by $q_{[0,t]}$
- the futures permit price at and before publication of emission data denoted by $F(t, T)$ and $F(t - \Delta, T)$, respectively

the implicit time needed to exhaust the remaining permits before the announcement was $h(\sigma)$ per cent larger where

$$h(\sigma) = \frac{F(t,T) - F(t-\Delta,T)}{P\phi\left(\Phi^{-1}\left(\frac{F(t,T)}{P}\right)\right)} \cdot f^{approx}(\sigma, t, x_t) \tag{14}$$

Price slump of more than 50% can be related to an implicit change in x_t of less than 5% (2/2)

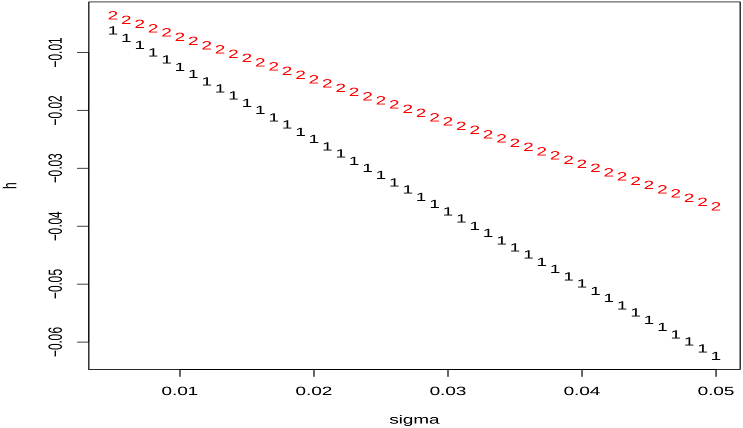

Figure: Linear approximation ("1"), log-normal moment matching ("2").

1. Introduction
2. Equilibrium models
3. Permit prices for different approximation approaches
4. Theoretical discussion of permit price slump in 2006
5. **Conclusion**

Conclusion

- Our model overcomes the shortcoming of the model of Chesney and Taschini (2008) that the moments of the approximated cumulative emissions do not match the true ones
- Permit prices are inherently prone to jumps
- Price jumps of the magnitude of 2006 are unlikely to occur again as the measurement of the emission data has been improved significantly

BACKUP

Implied over-/underallocation during the first phase of the EU ETS

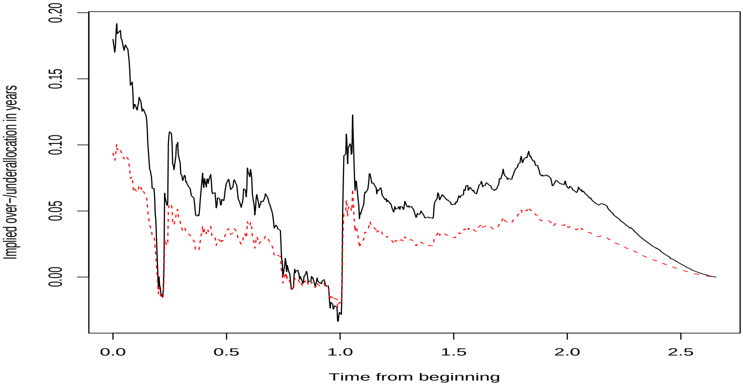

Figure: Implied $x_t - (T - t)$ for first phase for fixed $\mu = 0.02$ and $\sigma = 0.05$. Linear approximation approach (straight line), log-normal moment matching (dashed line). Positive values correspond to overallocation.

Auxiliary functions for moments of integral over GBM

$$\alpha_{t_2-t_1} = \begin{cases} \frac{1}{\mu}\left(e^{\mu(t_2-t_1)} - 1\right) & \text{if } \mu \neq 0 \\ t_2 - t_1 & \text{if } \mu = 0 \end{cases} \tag{15}$$

$$\beta_{t_2-t_1} = \begin{cases} \frac{\mu e^{(2\mu+\sigma^2)(t_2-t_1)} + \mu + \sigma^2 - \left(2\mu+\sigma^2\right)e^{\mu(t_2-t_1)}}{\mu(\mu+\sigma^2)(2\mu+\sigma^2)} & \text{if } \mu \neq 0 \\ \frac{1}{\sigma^4}\left(e^{\sigma^2(t_2-t_1)} - 1 - \sigma^2(t_2-t_1)\right) & \text{if } \mu = 0 \end{cases} \tag{16}$$

CO_2 emission allowance spot price - Fitting spot price to standard stochastic processes

Literature: [Wag07], [DPM07], [BT08], [PT08]

- **Geometric Brownian motion with drift** [Wag07], [DPM07] $dS_t = S_t[\mu dt + \sigma dW_t]$
- **Mean reverting processes**
  (a) Ornstein-Uhlenbeck process [Wag07] $dS_t = \kappa(\mu - S_t)dt + \sigma dW_t$
  (b) Mean Reverting Logarithmic process[DPM07] $d(\ln(S_t)) = \kappa(\mu - \ln(S_t))dt + \sigma dW_t$
  (c) Constant Elasticity Variance [DPM07] $dS_t = \kappa(\mu - S_t)dt + S_t^{\gamma}\sigma dW_t$
  (d) Mean Reverting Square-Root process [DPM07] $dS_t = \kappa(\mu - S_t)dt + \sqrt{S_t}\sigma dW_t$
  (e) OU-process in the interval (a, b) as in Ingersoll (1997) [Wag07]
- **Jump diffusion models**
  (a) JD of Merton (1976) [Wag07] $\frac{dS_t}{S_t} = \left(\mu - \lambda k - \frac{\sigma^2}{2}\right) dt + \sigma dW_t + (Y_t - 1)dN_t$
  (b) Mean Reverting Square-Root process augmented by Jumps [DPM07]
  $dS_t = \kappa(\mu - S_t)dt + \sqrt{S_t}\sigma dW_t + \sigma dW_t + (Y_t - 1)dN_t$
- **GARCH-Models**
  (a) AR(1)-GARCH(1,1) with normal innovation distribution [BT08]
  (b) AR(1)-GARCH(1,1) with generalized asymmetric t innovation distribution [PT08]
  (c) AR(1)-MiXN(r,s)-GARCH(1,1) [PT08]
  (d) Heston-GARCH(1,1) as in Heston and Nandi (2000) [Wag07]
- **Regime-Switching models**
  (a) RS-Model of Wagner [Wag07] $S_t = S_{t-1} + \mu_{R_t} + \sigma_{R_t}\varepsilon_t$ where R_t is a Markov chain
  (b) RS-Model of Benz and Trück on page 15 [BT08]

Bibliography (1/2)

Benz, E. and Trück, S. (2008) Modeling the Price Dynamics of CO_2 Emission Allowances. *Energy Economics,* 31(1), 4–15.

Carmona, R., Fehr, M., Hinz J., Porchet A. (2009) Market design for emission trading schemes. *SIAM Review,* 2009.

Cetin, U. and Verschuere, M. (2009) Pricing and hedging in carbon emissions markets. *International Journal of Theoretical and Applied Finance,* forthcoming.

Chesney, M. and Taschini, L. (2008) The Endogenous Price Dynamics of Emission Allowances and An Application to CO_2 Option Pricing. *Preprint.*

Cronshaw, M. and Kruse, J. (1996) Regulated Firms in Pollution Permit Markets with Banking. *Journal of Regulatory Economics,* 9(2), 179–189.

Daskalakis, G., Psychoyios, D., Markellos, R.N. (2009) Modeling CO_2 Emission Allowance Prices and Derivatives: Evidence from the European Trading Scheme. *Journal of Banking & Finance,* 33(7), 1230–1241

European Union (2006) EU emissions trading scheme delivers first verified emissions data for installations. *Press release IP/06/612.* Brussels.

Hintermann, B. (2009). An Options Pricing Approach for CO2 Allowances in the EU ETS. *CEPE Working Paper 64*, Centre for Energy Policy and Economics (CEPE), ETH Zurich.

Kling, C. and Rubin, J. (1997) Bankable permits for the control of environmental pollution. *Journal of Public Economic Theory,* 64, 101–115.

Mansanet-Bataller, M., Tornero, A.P., Valor i Micó, E. (2007) CO_2 prices, energy and weather. *The Energy Journal.* 28(3), 73–92.

Bibliography (2/2)

Milevsky, M.A. and Posner, S.E. (1998). Asian Options, the Sum of Lognormals, and the Reciprocal Gamma Distribuation. *Journal of Financial and Quantitative Analysis,* 33(3), 409–421.

Milevsky, M.A. and Posner, S.E. (1998). A closed-form approximation for valuing basket options. *Journal of Derivatives,* 5(4), 54–61.

Montgomery, W. D. (1972) Markets in Licenses and Efficient Pollution Control Programs. *Journal of Economic Theory,* 5, 395–418.

Paolella, M. and Taschini, L. (2008). An econometric analysis of emission allowance prices. *Journal of Banking and Finance,* 32, 2022–2032.

Posner, S.E. and Milevsky, M.A. (1998). Valuing Exotic Options by Approximating the SPD with Higher Moments. *Journal of Financial Engineering,* 7(2), 109–125.

Rubin, J. (1996) A Model of Intertemporal Emission Trading, Banking and Borrowing. *Journal of Environmental Economics and Management,* 31, 269–286.

Schennach, S. (2000) The Economics of Pollution Permit Banking in the Context of Title IV of the 1990 Clean Air Act Amendments. *Journal of Environmental Economics and Management,* 40, 189–210.

Seifert, J. et al. (2008) Dynamic Behavior of CO2 Spot Prices - Theory and Empirical Evidence. *Journal of Environmental Economics and Management,* 56(2), 180–194.

Wagner, M. W. (2007) CO2-Emissionszertifikate - Preismodellierung und Derivatebewertung. *Universität Karlsruhe Universitätsbibliothek.*