Pricing and Hedging Strategies for Contingent Claims in an Incomplete Hybrid Emissions Market

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General Introduction

Context

- Kyoto Protocol (1997): Emissions in developed countries
  - ▷ Reduced
  - ▷ Stabilized
- Regulator: Control their national emissions
  - ▷ Corporate: Additional risk
  - ▷ Consumer: Erosion in purchasing power
- Most implemented instruments policies:
  - ▷ Emissions tax: Finland (1990), Sweden (1991), Quebec (2007),...
  - ▷ Cap-and-Trade Market: EU ETS (2005), WCI (2007), RGGI (2006),...

General Introduction

Cap-and-Trade Market Mechanism

- Ceiling for emissions
- Compliance period
- Market: price to comply with emission target
- Least cost: internal abatement or acquisition of allowances
- Trading between: Regulated emitters, Non-regulated emitters, Non-emitters

General Introduction

EU ETS Market

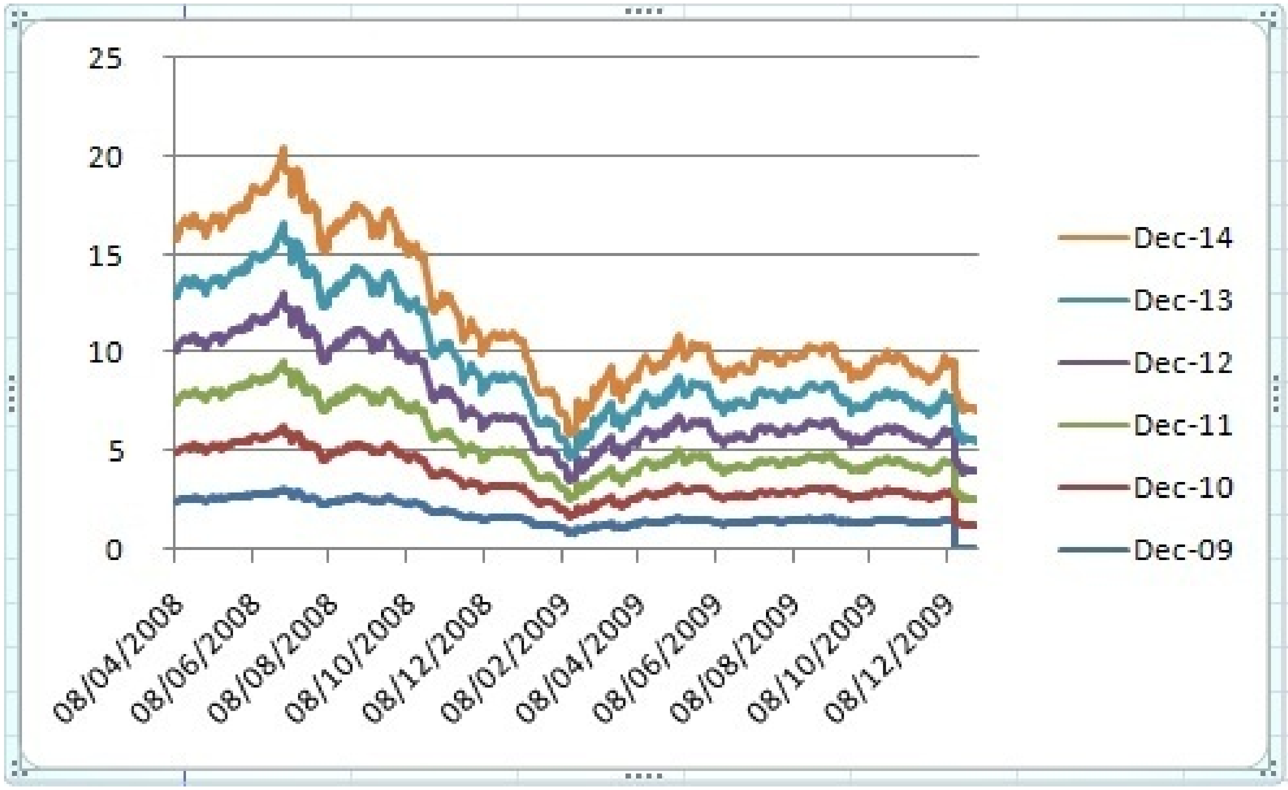


Figure 1: Futures prices for Dec 2009-14 from April 2008 to December 2009 (Source: Bloomberg)

General Introduction

EU ETS Market

- Auctioning up to **10%** of total emissions in Phase II (Article 10 of the EU Directives)
- Point Carbon 2010 survey: **51%** sold some surplus EUAs
- Short: power/heat sector
- Long: pulp/paper and cement/lime/glass sectors

Questions

- Is the cap-and-trade system the most cost effective policy instrument?
- Does it force emissions reductions?
- For which market design do price signals best describe the true cost of emitting a tonne of carbon?
- How can we protect regulated companies and consumers in the transition to a clean energy economy?
- Can we avoid carbon leakage?

Hybrid system

- Removes the deficiencies of both the cap-and-trade and the carbon tax markets.
- Protects the economy by fixing a safety valve price.
- Special case: Cap-and-trade market.
- Baumol and Oates (1988), Weitzman (1974), Montero (2002), Roberts and Spence (1976), Prizer (2002), McKibbin and Wilcoxen (2002), Jacoby and Ellerman (2004)

Market Design

100% auctioning

- Eliminate windfall profits (Woerdman, Couwenberg and Nentjes, 2009).
- Incentives to innovate (Cramton and Kerr, 1998)
- Stable long-term price signal (Hepburn et al., 2006)
- Distribute % income to
  - final consumer as tax reduction
  - avoid carbon leakage
  - invest in green projects

Market Design

Principle market players

Regulator

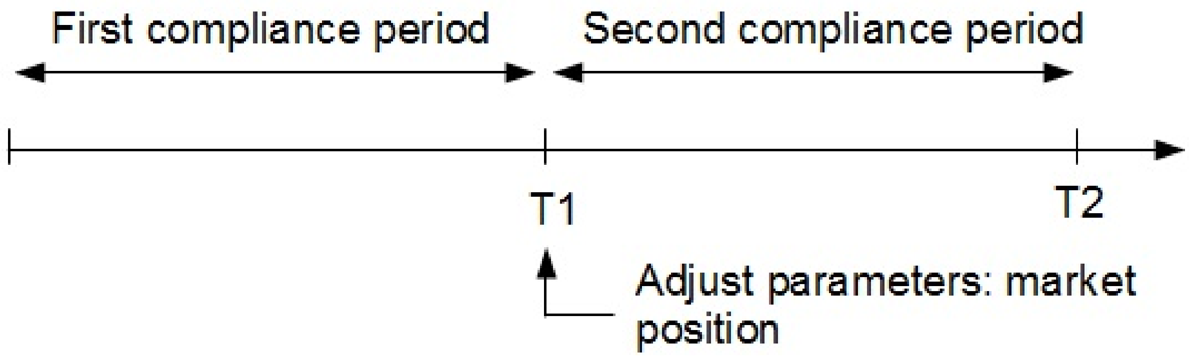
- Incomplete information: Abatement cost and emission quantity
- Sets:
  - ▷ Auctioning price P_0
  - ▷ Initial endowment N_0
  - ▷ Length of compliance periods
  - ▷ Penalty π

Emitter

- Abatement cost available
- Accurate emission prediction
- Avoid paying P_0
- Buy allowances $\leq N_0$

Two period Market Model

- Compliance dates T_1 and T_2, $T_1 < T_2$
- Banking allowed: Do not affect market position at T_1
- Borrowing Forbidden
- Insufficient certificates at T_1: Later delivery + Penalty π to pay at T_1
- Safety valve price P_{max}
- Adjust market parameters at time T_1^+



Two period Market Model

Define

- The discounted price process of the future contracts:
  - X_t^1 matures at T_1
  - X_t^2 matures at T_2
- No-arbitrage condition: $X_t^1 \leq \pi e^{-r(T_1 - t)} + X_t^2$
- Stopping time τ

$$\tau(\omega) = \min\{t / X_t^1(\omega) = P_{max}\}$$

- T_1-contingent claim: $H = f(X) \in L^2(P)$

Two period Market Model

- $\forall t \geq \tau(\omega), X_t^1 = P_{max}$
- $\tau(\omega) < T_1$
  - ▷ $X_{T_1}^2 = P_{max} - \pi$
  - ▷ $H(\omega) = f(P_{max})$
- $\tau(\omega) \geq T_1$
  - ▷ Short: $X_{T_1}^2 < X_{T_1}^1$

    $\Rightarrow H(\omega) = f(\min(X_{T_1}^1(\omega), \pi + X_{T_1}^2(\omega), P_{max}))$
  - ▷ Not short: $X_{T_1}^2 \geq X_{T_1}^1$

    $\Rightarrow H(\omega) = f(X_{T_1}^1(\omega))$

Two period Market Model

Effective payoff

$$
\begin{aligned}
H \;=\; & \mathbb{I}_{\tau \geq T_1} \\
& [\mathbb{I}_{X^2_{T_1} < X^1_{T_1}} f(\min(X^1_{T_1}, \pi + X^2_{T_1}, P_{max})) \\
& + \mathbb{I}_{X^2_{T_1} \geq X^1_{T_1}} \; f(X^1_{T_1})] \\
& + \mathbb{I}_{\tau < T_1} \; f(P_{max})
\end{aligned}
$$

Example: H $:= T_1$-Call option at strike K written on X^1_t

- $f(X) = (X - K)^+$
- Depends on $X^2_{T_1}$

Pricing and Hedging Solution

Mean reversion Jump diffusion dynamic
(Daskalakis et al., 2009)

- $dX_t^i = \theta_i dt +$
  $X_{t-}^i(\mu_i dt + \sigma_{i1} dw_{1t} + \sigma_{i2} dw_{2t} + \varphi_{i1} dN_{1t} + \varphi_{i2} dN_{2t}), \quad X_0^i > 0, \quad \varphi_{i1} > -1$
- $(N_{1t}, N_{2t})'$: Poisson process with intensity $\nu = (\nu_1, \nu_2)'$
- $(w_{1t}, w_{2t})'$: independent Brownian motions

Probability space $(\Omega, \mathcal{F}, P)$

- Complete
- $\mathcal{F}_0$ is trivial and contains all nut sets of $\mathcal{F}$
- $\mathcal{F}_t$: P-augmented right continuous filtration generated by w_t and N_t, $\forall t \leq T_1$

Pricing and Hedging Solution

Market incomplete

- Existence of intrinsic risk
- Equivalent martingale measure $\mathcal{M}^e(X) \neq \{\}$

Doob-Meyer Decomposition

- $X_t^i = X_0^{i*} + M_t^i + A_t^i, \quad i = 1..2$
- M_t^i: Local P-martingale
- A_t^i: predictable process with finite variation
- X_0^{i*}: $\mathcal{F}_0$-measurable

Pricing and Hedging Solution

Define

- Space of square integrable martingales: $\mathcal{M}^2(P)$
- Dynamic strategies: $\phi = (\xi_t, \eta_t)_{0 \leq t \leq T}$
- Portfolio value process: $V_t = \xi' \cdot X_t + \eta_t$
- Cumulative gains: $G_t(\xi) = \int_0^t \xi_s dX_s$
- Cumulative cost: $C_t = V_t - G_t(\xi)$
- Risk: $R_t(\phi) = E((C_T(\phi) - C_t(\phi))^2)$
- Optimality Equation: ϕ^* s.t. $R_t(\phi^*) \leq R_t(\phi)$ for all admissible ϕ

Pricing and Hedging Solution

Föllmer-Schweizer Decomposition

$$C(\phi) \in \mathcal{M}^2(P), \quad C(\phi) \perp M^i \text{ under } P$$

$$\Updownarrow$$

$$H = H_0 + \int_0^T \xi^H \cdot dX_t + L_T^H, P \;\; a.s$$

where

- $H_0 \in \mathbb{R}$,
- $\xi^H \in \Theta$,
- $L^H \in \mathcal{M}^2(P)$ and $L^H \perp M^i$,
- $\Theta = \{(\xi)_t/\mathbb{R}^2 -$ predictable process, $(E[\int_0^{T_1} \xi_t' d < M >_t \xi_t])^{1/2} < \infty,$ and $E[(\int_0^{T_1} |\xi_t' dA_t|)^2] < \infty\}$.

Pricing and Hedging Solution

Let

$$\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}, \quad \Phi = \begin{pmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \end{pmatrix},$$

$$\Lambda = \begin{pmatrix} \sigma_{11}^2 + \sigma_{12}^2 & \sigma_{11}\sigma_{21} + \sigma_{12}\sigma_{22} \\ \sigma_{11}\sigma_{21} + \sigma_{22}\sigma_{12} & \sigma_{21}^2 + \sigma_{22}^2 \end{pmatrix},$$

$$\Xi = \begin{pmatrix} \varphi_{11}^2\nu_1 + \varphi_{12}^2\nu_2 & \varphi_{11}\varphi_{21}\nu_1 + \varphi_{12}\varphi_{22}\nu_2 \\ \varphi_{21}\varphi_{11}\nu_1 + \varphi_{22}\varphi_{12}\nu_2 & \varphi_{21}^2\nu_1 + \varphi_{22}^2\nu_2 \end{pmatrix},$$

$$\alpha = (\Lambda + \Xi)^{-1}(\mu + \Phi\nu).$$

Pricing and Hedging Solution

Mean-variance tradeoff process

Define

$$\widehat{\lambda}_t^i := \frac{1}{X_{t^-}^i}\alpha_i, \quad \text{for} \quad i = 1, 2$$

$$\widehat{K}_t = \int_0^t \widehat{\lambda}_s' d < M >_s \widehat{\lambda}_s$$

$\widehat{K}_t$ Properties

- Deterministic
- Uniformly bounded in (t, ω)

$\Rightarrow \exists!$ F-S Decomposition (Monat and Stricker, 1995)

Pricing and Hedging Solution

Minimal Martingale Measure $\widehat{P}$

Density process

$$Z_t = \varepsilon(-\int_0^{\cdot} \widehat{\lambda}_s dM_s)_t, \quad 0 \leq t \leq T_1,$$

where

$$\varepsilon(X) = 1 + \int_0^t \varepsilon(X)_{s^-} dY_s, \quad 0 \leq t \leq T_1.$$

- $Z_t > 0$ if $\exists \delta / (\alpha\Phi)_i < 1 - \delta$ (Arai, 2004)
- $Z_t \in \mathcal{M}^2(P)$ (Choulli et al., 1998)

Pricing and Hedging Solution

Under $\widehat{P}$,

$$V_t = \widehat{E}[H|\mathfrak{F}_t]$$

Pricing Procedure

- Dynamics under $\widehat{P}$

$$\begin{aligned} dX_t^i = \quad & \theta_i dt + X_{t^-}^i((\mu_i - \sigma_{i1}(\alpha_1\sigma_{11} \\ & +\alpha_2\sigma_{21}) - \sigma_{i2}(\sigma_{12}\alpha_1 + \sigma_{22}\alpha_2))dt \\ & +\sigma_{i1}dw_{1t}^{\widehat{P}} + \sigma_{i2}dw_{2t}^{\widehat{P}} + \varphi_{i1}dN_{1t}^{\widehat{P}} + \varphi_{i2}dN_{2t}^{\widehat{P}}), \quad X_0^i > 0 \end{aligned}$$

  - ▷ $(w_{1t}^{\widehat{P}}, w_{2t}^{\widehat{P}})'$: $\widehat{P}$-standard Brownian motions
  - ▷ $N_t^{\widehat{P}} = (N_{1t}^{\widehat{P}}, N_{2t}^{\widehat{P}})'$: Poisson process under $\widehat{P}$ with intensity $\nu^{\widehat{P}} = (\nu_1(1-\alpha_1\varphi_{11})(1-\alpha_2\varphi_{21}), \nu_2(1-\alpha_1\varphi_{12})(1-\alpha_2\varphi_{22}))'$

- $V_0 = \widehat{E}[H]$

Pricing and Hedging Solution

Pricing T_2-contingent claim

- Market efficiency vs Structural adjustment at T_1^+

- Pricing under Larger filtration $\widetilde{\mathcal{F}} \supseteq \mathcal{F}$ such that $\forall t \leq T_1, \widetilde{\mathcal{F}}_t = \mathcal{F}_t$
- Two period setting: $H = f(X^2_{T_2})$
- Given H is attainable under $\widetilde{\mathcal{F}}$:

$$H = H_0 + \int_0^{T_2} \widetilde{\xi}_s dX_s^2$$

where
$\widetilde{\xi}$: $\widetilde{\mathcal{F}}-$measurable admissible strategy
H_0: $\widetilde{\mathcal{F}}_0-$measurable

Pricing and Hedging Solution

Pricing T_2-contingent claim

- For $t < T_1$: $(\mathcal{F})_{t\geq 0}$ available $\Longrightarrow$ intrinsic risk for H

$$H = \underbrace{H_0 + \int_0^{T_1} \widetilde{\xi}_s dX_s^2}_{\text{Full hedge}} + \underbrace{\int_{T_1^+}^{T_2} \widetilde{\xi}_s dX_s^2}_{\text{Partial hedge}}$$

Full hedge Partial hedge

$$\int_{T_1^+}^{T_2} \widetilde{\xi}_s dX_s^2 = \int_{T_1^+}^{T_2} \xi_s dX_s^2 + N_{T_2}$$

where ξ_s : $\mathcal{F}_s$-measurable, $N_{T_2} \in \mathcal{L}^2(\Omega, F_{T_2}, P)$ and $N_t \perp M_t^2$

Pricing and Hedging Solution

Pricing T_2-contingent claim

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Full hedge

Partial hedge

Intrinsic risk

$$\int_{T_1^+}^{T_2} \widetilde{\xi}_s dX_s^2 = \int_{T_1^+}^{T_2} \xi_s dX_s^2 + \boxed{N_{T_2}}$$

where $\xi_s : \mathcal{F}_s$-measurable, $N_{T_2} \in \mathcal{L}^2(\Omega, F_{T_2}, P)$ and $N_t \perp M_t^2$

Concluding remarks

- Quadratic Hedging

$$\min E[(H - V_0 - \int_0^T \xi_s dX_s)^2]$$

- Get around incomplete information by Super-Hedging

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