Improved Modeling of Double Default Effects in Basel II - An Endogenous Asset Drop Model without Additional Correlation

Sebastian Ebert & Eva Lütkebohmert

Bonn Graduate School of Economics and University of Freiburg

6^{th} Bachelier World Congress, Toronto, June 2010

Agenda

Credit Risk in Basel II

Internal Ratings Based (IRB) approach under Pillar 1 of Basel II:

Benchmark model to quantify minimal capital requirements for portfolio credit risk

- since 2007 binding for all banks in the European Union
- minimal capital requirement is the 99.9% value-at-risk of the credit portfolio loss distribution
- based on a conditional independence framework: Asymptotic Single Risk Factor (ASRF) model by Gordy (2003) where all idiosyncratic risk is assumed to be diversified away
- Merton model of default

Credit Risk in Basel II

Internal Ratings Based (IRB) approach under Pillar 1 of Basel II:

Benchmark model to quantify minimal capital requirements for portfolio credit risk

- since 2007 binding for all banks in the European Union
- minimal capital requirement is the 99.9% value-at-risk of the credit portfolio loss distribution
- based on a conditional independence framework: Asymptotic Single Risk Factor (ASRF) model by Gordy (2003) where all idiosyncratic risk is assumed to be diversified away
- Merton model of default

Credit Risk in Basel II

Internal Ratings Based (IRB) approach under Pillar 1 of Basel II:

Benchmark model to quantify minimal capital requirements for portfolio credit risk

- since 2007 binding for all banks in the European Union
- minimal capital requirement is the 99.9% value-at-risk of the credit portfolio loss distribution
- based on a conditional independence framework: Asymptotic Single Risk Factor (ASRF) model by Gordy (2003) where all idiosyncratic risk is assumed to be diversified away
- Merton model of default

Credit Risk in Basel II

Internal Ratings Based (IRB) approach under Pillar 1 of Basel II:

Benchmark model to quantify minimal capital requirements for portfolio credit risk

- since 2007 binding for all banks in the European Union
- minimal capital requirement is the 99.9% value-at-risk of the credit portfolio loss distribution
- based on a conditional independence framework: Asymptotic Single Risk Factor (ASRF) model by Gordy (2003) where all idiosyncratic risk is assumed to be diversified away
- Merton model of default

Credit Risk in Basel II

Internal Ratings Based (IRB) approach under Pillar 1 of Basel II:

Benchmark model to quantify minimal capital requirements for portfolio credit risk

- since 2007 binding for all banks in the European Union
- minimal capital requirement is the 99.9% value-at-risk of the credit portfolio loss distribution
- based on a conditional independence framework: Asymptotic Single Risk Factor (ASRF) model by Gordy (2003) where all idiosyncratic risk is assumed to be diversified away
- Merton model of default

Double Default

Hedged exposures are lost if

1. the obligor defaults AND
2. the guarantor defaults. Thus: **"double default"**

Hedging Instruments: Credit Derivatives such as CDS, collateral securitization, guarantees...

Treatment of Double Default Effects in Basel II:

- Original New Basel Accord (2003): Substitution approach
- Amendment to Basel II (2005), "IRB-Treatment of Double Default Effects" (additional correlation approach)
- December (2009): Basel Committee again focuses on counterparty risk in Basel II outside the IRB approach

Double Default

Hedged exposures are lost if

1. the obligor defaults AND
2. the guarantor defaults. Thus: **“double default”**

Hedging Instruments: Credit Derivatives such as CDS, collateral securitization, guarantees...

Treatment of Double Default Effects in Basel II:

- Original New Basel Accord (2003): Substitution approach
- Amendment to Basel II (2005), “IRB-Treatment of Double Default Effects” (additional correlation approach)
- December (2009): Basel Committee again focuses on counterparty risk in Basel II outside the IRB approach

Double Default

Hedged exposures are lost if

1. the obligor defaults AND
2. the guarantor defaults. Thus: **“double default”**

Hedging Instruments: Credit Derivatives such as CDS, collateral securitization, guarantees...

Treatment of Double Default Effects in Basel II:

- Original New Basel Accord (2003): Substitution approach
- Amendment to Basel II (2005), “IRB-Treatment of Double Default Effects” (additional correlation approach)
- December (2009): Basel Committee again focuses on counterparty risk in Basel II outside the IRB approach

Double Default

Hedged exposures are lost if

1. the obligor defaults AND
2. the guarantor defaults. Thus: **"double default"**

Hedging Instruments: Credit Derivatives such as CDS, collateral securitization, guarantees...

Treatment of Double Default Effects in Basel II:

- Original New Basel Accord (2003): Substitution approach
- Amendment to Basel II (2005), "IRB-Treatment of Double Default Effects" (additional correlation approach)
- December (2009): Basel Committee again focuses on counterparty risk in Basel II outside the IRB approach

Contribution of this paper

We

1. reveal structural weaknesses of the IRB treatment of double default effects and any additional correlation approach,
2. propose a new asset drop model that addresses all mentioned weaknesses and which is
3. just as easily applicable as it does not pose extensive data requirements and economic capital can still be computed analytically.

Contribution of this paper

We

1. reveal structural weaknesses of the IRB treatment of double default effects and any additional correlation approach,
2. propose a new asset drop model that addresses all mentioned weaknesses and which is
3. just as easily applicable as it does not pose extensive data requirements and economic capital can still be computed analytically.

Contribution of this paper

We

1. reveal structural weaknesses of the IRB treatment of double default effects and any additional correlation approach,
2. propose a new asset drop model that addresses all mentioned weaknesses and which is
3. just as easily applicable as it does not pose extensive data requirements and economic capital can still be computed analytically.

Additional Correlation Approach under Basel II

The normally distributed asset returns r_n and r_{g_n} of obligor n and its guarantor are no more conditionally independent on the systematic risk factor X but

$$r_n = \sqrt{\rho_n}X + \sqrt{1-\rho_n}\left(\sqrt{\psi_{n,g_n}}Z_{n,g_n} + \sqrt{1-\psi_{n,g_n}}\epsilon_n\right)$$

ρ_n : asset correlation of obligor n
ψ_{n,g_n} : sensitivity of both n and g_n to stochastic factor Z_{n,g_n}
ϵ_n : idiosyncratic risk factor of obligor n.

This implies the double default probability

$$\begin{aligned}\mathbb{P}(\mathrm{DD}) &:= \mathbb{P}\left(\{\text{default of obligor n}\} \cap \{\text{default of guarantor } g_n\}\right) \\ &= \Phi_2\left(\Phi^{-1}(\mathrm{PD}_n), \Phi^{-1}(\mathrm{PD}_{g_n}); \rho_{n,g_n}\right).\end{aligned}$$

ρ_{n,g_n} : additional correlation parameter

Additional Correlation Approach under Basel II

The normally distributed asset returns r_n and r_{g_n} of obligor n and its guarantor are no more conditionally independent on the systematic risk factor X but

$$r_n = \sqrt{\rho_n} X + \sqrt{1-\rho_n} \left(\sqrt{\psi_{n,g_n}} Z_{n,g_n} + \sqrt{1-\psi_{n,g_n}} \epsilon_n \right)$$

ρ_n : asset correlation of obligor n
ψ_{n,g_n} : sensitivity of both n and g_n to stochastic factor Z_{n,g_n}
ϵ_n : idiosyncratic risk factor of obligor n.

This implies the double default probability

$$\begin{aligned} \mathbb{P}(\mathrm{DD}) &:= \mathbb{P}\left(\{\text{default of obligor n}\} \cap \{\text{default of guarantor}\, g_n\}\right) \\ &= \Phi_2\left(\Phi^{-1}(\mathrm{PD}_n), \Phi^{-1}(\mathrm{PD}_{g_n}); \rho_{n,g_n}\right). \end{aligned}$$

ρ_{n,g_n} : additional correlation parameter

Criticism of the additional correlation approach

1. Correlation, a symmetric measure of dependency, is used to describe an *asymmetric relationship*
2. What is an appropriate value for ρ_{n,g_n}?
   $\longrightarrow$ In Basel II set $\rho_{n,g_n} \equiv 0.5$ for all n and g_n.
   $\longrightarrow$ Grundke (2008) empirically evaluates this assumption
3. *Additional* correlation directly violates the conditional independence assumption of the ASRF model
4. Assumes that all guarantors are a) *distinct* and b) *external* to the portfolio
   $\longrightarrow$ no reflection of overly excessive contracting of the same guarantor
5. Not robust towards application under Pillar 2

Criticism of the additional correlation approach

1. Correlation, a symmetric measure of dependency, is used to describe an *asymmetric relationship*
2. What is an appropriate value for ρ_{n,g_n}?
   $\longrightarrow$ In Basel II set $\rho_{n,g_n} \equiv 0.5$ for all n and g_n.
   $\longrightarrow$ Grundke (2008) empirically evaluates this assumption
3. *Additional* correlation directly violates the conditional independence assumption of the ASRF model
4. Assumes that all guarantors are a) *distinct* and b) *external* to the portfolio
   $\longrightarrow$ no reflection of overly excessive contracting of the same guarantor
5. Not robust towards application under Pillar 2

Criticism of the additional correlation approach

1. Correlation, a symmetric measure of dependency, is used to describe an *asymmetric relationship*
2. What is an appropriate value for ρ_{n,g_n}?
   $\longrightarrow$ In Basel II set $\rho_{n,g_n} \equiv 0.5$ for all n and g_n.
   $\longrightarrow$ Grundke (2008) empirically evaluates this assumption
3. *Additional* correlation directly violates the conditional independence assumption of the ASRF model
4. Assumes that all guarantors are a) *distinct* and b) *external* to the portfolio
   $\longrightarrow$ no reflection of overly excessive contracting of the same guarantor
5. Not robust towards application under Pillar 2

Criticism of the additional correlation approach

1. Correlation, a symmetric measure of dependency, is used to describe an *asymmetric relationship*
2. What is an appropriate value for ρ_{n,g_n}?
   $\longrightarrow$ In Basel II set $\rho_{n,g_n} \equiv 0.5$ for all n and g_n.
   $\longrightarrow$ Grundke (2008) empirically evaluates this assumption
3. *Additional* correlation directly violates the conditional independence assumption of the ASRF model
4. Assumes that all guarantors are a) *distinct* and b) *external* to the portfolio
   $\longrightarrow$ no reflection of overly excessive contracting of the same guarantor
5. Not robust towards application under Pillar 2

Criticism of the additional correlation approach

1. Correlation, a symmetric measure of dependency, is used to describe an *asymmetric relationship*
2. What is an appropriate value for ρ_{n,g_n}?
   $\longrightarrow$ In Basel II set $\rho_{n,g_n} \equiv 0.5$ for all n and g_n.
   $\longrightarrow$ Grundke (2008) empirically evaluates this assumption
3. *Additional* correlation directly violates the conditional independence assumption of the ASRF model
4. Assumes that all guarantors are a) *distinct* and b) *external* to the portfolio
   $\longrightarrow$ no reflection of overly excessive contracting of the same guarantor
5. Not robust towards application under Pillar 2

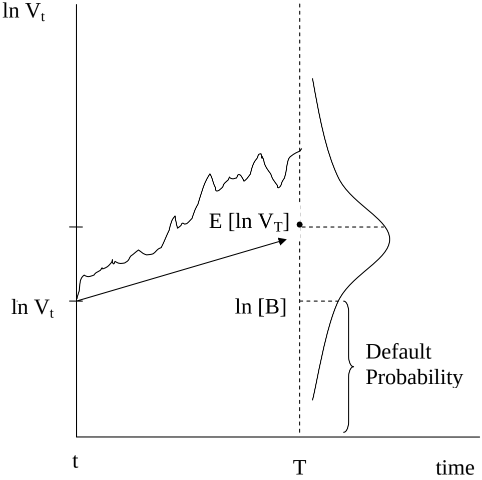
Criticism of the additional correlation approach

1. Correlation, a symmetric measure of dependency, is used to describe an *asymmetric relationship*
2. What is an appropriate value for ρ_{n,g_n}?
   $\longrightarrow$ In Basel II set $\rho_{n,g_n} \equiv 0.5$ for all n and g_n.
   $\longrightarrow$ Grundke (2008) empirically evaluates this assumption
3. *Additional* correlation directly violates the conditional independence assumption of the ASRF model
4. Assumes that all guarantors are a) *distinct* and b) *external* to the portfolio
   $\longrightarrow$ no reflection of overly excessive contracting of the same guarantor
5. Not robust towards application under Pillar 2

Criticism of the additional correlation approach

1. Correlation, a symmetric measure of dependency, is used to describe an *asymmetric relationship*
2. What is an appropriate value for ρ_{n,g_n}?
   $\longrightarrow$ In Basel II set $\rho_{n,g_n} \equiv 0.5$ for all n and g_n.
   $\longrightarrow$ Grundke (2008) empirically evaluates this assumption
3. *Additional* correlation directly violates the conditional independence assumption of the ASRF model
4. Assumes that all guarantors are a) *distinct* and b) *external* to the portfolio
   $\longrightarrow$ no reflection of overly excessive contracting of the same guarantor
5. Not robust towards application under Pillar 2

Motivation for Asset Drop Model: Merton Model



Asset Drop Model

Idea: Adjust PD_{g_n} appropriately to *effective default probability* PD'_{g_n}.

Within a structural model of default:

$$\mathrm{PD}_{g_n} = \mathbb{P}(V_{g_n}(T) < B_{g_n}),$$

$V_{g_n}(t)$: total asset value of g_n in period t, B_{g_n} : default threshold.

Denote with $\hat{E}_{n,g_n}$ the nominal g_n guarantees for n. Then

$$\mathrm{PD}'_{g_n} = \mathbb{P}(V_{g_n}(T) - \hat{E}_{n,g_n} < B_{g_n}) = \mathbb{P}(V_{g_n}(T) < B_{g_n} + \hat{E}_{n,g_n}) \quad (1)$$

$\longrightarrow$ Within Merton's model:

$$\mathrm{PD}'_{g_n} = 1 - \Phi\left(\frac{\ln\left(\frac{V_{g_n}(0)}{B_{g_n} + \hat{E}_{n,g_n}}\right) + (r - \frac{n}{2}\sigma^2_{g_n})T}{\sigma_{g_n}\sqrt{T}}\right). \quad (2)$$

Asset Drop Model

Idea: Adjust PD_{g_n} appropriately to *effective default probability* PD'_{g_n}.
Within a structural model of default:

$$\mathrm{PD}_{g_n} = \mathbb{P}(V_{g_n}(T) < B_{g_n}),$$

$V_{g_n}(t)$: total asset value of g_n in period t, B_{g_n} : default threshold.

Denote with $\hat{E}_{n,g_n}$ the nominal g_n guarantees for n. Then

$$\mathrm{PD}'_{g_n} = \mathbb{P}(V_{g_n}(T) - \hat{E}_{n,g_n} < B_{g_n}) = \mathbb{P}(V_{g_n}(T) < B_{g_n} + \hat{E}_{n,g_n}) \quad (1)$$

$\longrightarrow$ Within Merton's model:

$$\mathrm{PD}'_{g_n} = 1 - \Phi\left(\frac{\ln\left(\frac{V_{g_n}(0)}{B_{g_n} + \hat{E}_{n,g_n}}\right) + (r - \frac{n}{2}\sigma^2_{g_n})T}{\sigma_{g_n}\sqrt{T}}\right). \quad (2)$$

Asset Drop Model

Idea: Adjust PD_{g_n} appropriately to *effective default probability* PD'_{g_n}.

Within a structural model of default:

$$\mathrm{PD}_{g_n} = \mathbb{P}(V_{g_n}(T) < B_{g_n}),$$

$V_{g_n}(t)$: total asset value of g_n in period t, B_{g_n} : default threshold.

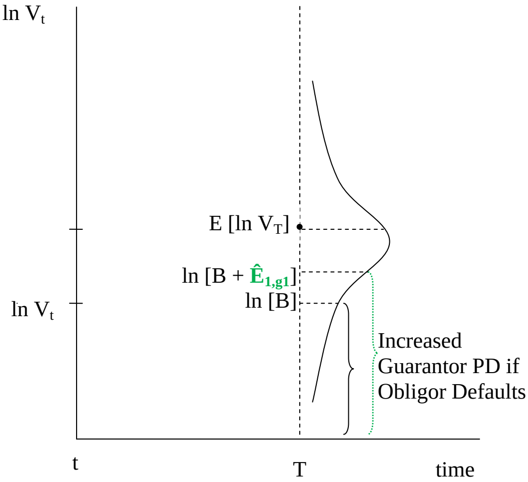
Denote with $\hat{E}_{n,g_n}$ the nominal g_n guarantees for n. Then

$$\mathrm{PD}'_{g_n} = \mathbb{P}(V_{g_n}(T) - \hat{E}_{n,g_n} < B_{g_n}) = \mathbb{P}(V_{g_n}(T) < B_{g_n} + \hat{E}_{n,g_n}) \quad (1)$$

$\longrightarrow$ Within Merton's model:

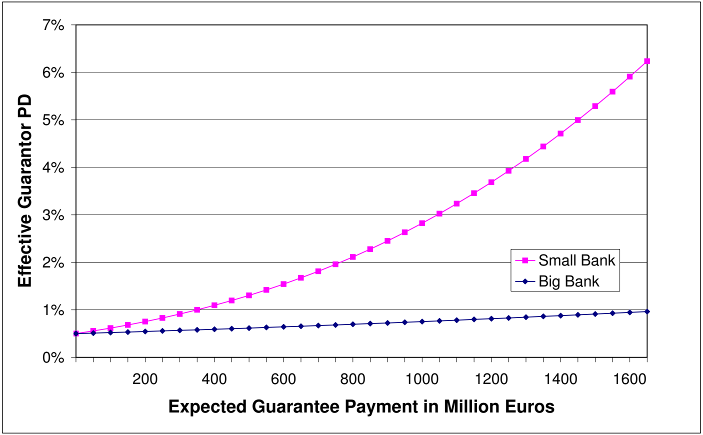
$$\mathrm{PD}'_{g_n} = 1 - \Phi\left(\frac{\ln\left(\frac{V_{g_n}(0)}{B_{g_n}+\hat{E}_{n,g_n}}\right) + (r - \frac{n}{2}\sigma_{g_n}^2)T}{\sigma_{g_n}\sqrt{T}}\right). \quad (2)$$

Asset Drop Model



Example 1: PD increase

Consider two guarantors g_1 ("big bank") and g_2 ("small bank").



Here: $V_{g_1}(0) = 50$ and $V_{g_2}(0) = 10$ billion Euros, respectively, $\sigma^2_{g_1} = \sigma^2_{g_2} = 30\%$, $T = 1$, $r = 0.02\%$ and

$\mathrm{PD}_{g_1} = \mathrm{PD}_{g_2} = 0.5\%$ (implies $B_{g_1} = 22.5$ and $B_{g_2} = 4.5$ billion Euros.)

Treatment of Different Hedging Constellations

$\longrightarrow$ Convexity punishes overly excessive contracting of the same guarantor

$\longrightarrow$ Treatment of guarantor within the portfolio: Joint loss distribution L_{1,g_1} of obligor 1 and its guarantor g_1:

$$\mathbb{P}(L_{1,g_1} = l) = \begin{cases} \mathrm{PD}'_{g_1}\,\mathrm{PD}_1 & \text{for } l = s_1\,\mathrm{ELGD}_1\,\mathrm{ELGD}_{g_1} \\ & \quad + s_{g_1}\,\mathrm{ELGD}_{g_1} \\ \mathrm{PD}_{g_1}(1-\mathrm{PD}_1) & \text{for } l = s_{g_1}\,\mathrm{ELGD}_{g_1} \\ (1-\mathrm{PD}'_{g_1})\,\mathrm{PD}_1 + & \\ (1-\mathrm{PD}_{g_1})(1-\mathrm{PD}_1) & \text{for } l = 0. \end{cases}$$

implies

$$\mathbb{E}[L_{1,g_1}] = s_{g_1}\,\mathrm{ELGD}_{g_1}\overbrace{\mathrm{PD}_{g_1}(1+\mathrm{PD}_1\,\lambda_{1,g_1})}^{\text{adjusted PD}_{g_1}} + s_1\,\mathrm{ELGD}_1\,\mathrm{ELGD}_{g_1}\overbrace{\mathrm{PD}_1\,\mathrm{PD}'_{g_1}}^{\mathbb{P}(\mathrm{DD})}$$

Treatment of Different Hedging Constellations

⟶ Convexity punishes overly excessive contracting of the same guarantor

⟶ Treatment of guarantor within the portfolio: Joint loss distribution L_{1,g_1} of obligor 1 and its guarantor g_1:

$$\mathbb{P}(L_{1,g_1} = l) = \begin{cases} \mathrm{PD}'_{g_1}\,\mathrm{PD}_1 & \text{for } l = s_1\,\mathrm{ELGD}_1\,\mathrm{ELGD}_{g_1} \\ & \quad + s_{g_1}\,\mathrm{ELGD}_{g_1} \\ \mathrm{PD}_{g_1}(1-\mathrm{PD}_1) & \text{for } l = s_{g_1}\,\mathrm{ELGD}_{g_1} \\ (1-\mathrm{PD}'_{g_1})\,\mathrm{PD}_1 + \\ (1-\mathrm{PD}_{g_1})(1-\mathrm{PD}_1) & \text{for } l = 0. \end{cases}$$

implies

$$\mathbb{E}[L_{1,g_1}] = s_{g_1}\,\mathrm{ELGD}_{g_1}\overbrace{\mathrm{PD}_{g_1}(1+\mathrm{PD}_1\,\lambda_{1,g_1})}^{\text{adjusted }\mathrm{PD}_{g_1}} + s_1\,\mathrm{ELGD}_1\,\mathrm{ELGD}_{g_1}\overbrace{\mathrm{PD}_1\,\mathrm{PD}'_{g_1}}^{\mathbb{P}(\mathrm{DD})}$$

Treatment of Different Hedging Constellations

$\longrightarrow$ Convexity punishes overly excessive contracting of the same guarantor

$\longrightarrow$ Treatment of guarantor within the portfolio: Joint loss distribution L_{1,g_1} of obligor 1 and its guarantor g_1:

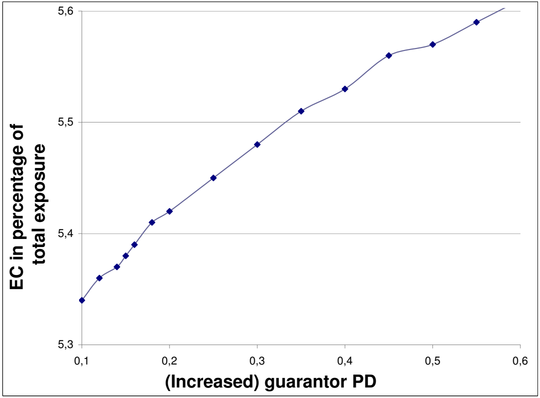
$$\mathbb{P}(L_{1,g_1} = l) = \begin{cases} \mathrm{PD}'_{g_1}\,\mathrm{PD}_1 & \text{for } l = s_1\,\mathrm{ELGD}_1\,\mathrm{ELGD}_{g_1} \\ & \quad +s_{g_1}\,\mathrm{ELGD}_{g_1} \\ \mathrm{PD}_{g_1}(1-\mathrm{PD}_1) & \text{for } l = s_{g_1}\,\mathrm{ELGD}_{g_1} \\ (1-\mathrm{PD}'_{g_1})\,\mathrm{PD}_1 + & \\ (1-\mathrm{PD}_{g_1})(1-\mathrm{PD}_1) & \text{for } l = 0. \end{cases}$$

implies

$$\mathbb{E}[L_{1,g_1}] = s_{g_1}\,\mathrm{ELGD}_{g_1} \overbrace{\mathrm{PD}_{g_1}(1+\mathrm{PD}_1\,\lambda_{1,g_1})}^{\text{adjusted } \mathrm{PD}_{g_1}} + s_1\,\mathrm{ELGD}_1\,\mathrm{ELGD}_{g_1} \overbrace{\mathrm{PD}_1\,\mathrm{PD}'_{g_1}}^{\mathbb{P}(\mathrm{DD})}$$

Example 2: Economic Capital (EC)

With IRB treatment of double default effects: 5.40% of total exposure (99.9% VaR) level. With asset drop technique:



Portfolio with 110 obligors, each has exposure 1, maturity 1 year. The first ten are hedged by the last ten (guarantors are in the portfolio). For obligors PD = 1%, LGD = 45%. For guarantors PD = 0.1%, LGD = 100%.

Summary

We criticize the IRB double default treatment for

1. using correlation to model an *asymmetric relationship*
2. not reflecting important characteristics of obligors and guarantors: $\rho_{n,g_n} \equiv 0.5 \; \forall \; n \; \forall \; g_n$.
3. violating the conditional independence assumption of the ASRF model
4. assuming all guarantors to be a) *distinct* and b) *external* to the portfolio.

We propose a novel asset drop model that

1. addresses these criticisms
2. is equally simple and
3. does not require extensive data or computation time.

Summary

We criticize the IRB double default treatment for

1. using correlation to model an *asymmetric relationship*
2. not reflecting important characteristics of obligors and guarantors: $\rho_{n,g_n} \equiv 0.5 \; \forall \, n \, \forall \, g_n$.
3. violating the conditional independence assumption of the ASRF model
4. assuming all guarantors to be a) *distinct* and b) *external* to the portfolio.

We propose a novel asset drop model that

1. addresses these criticisms
2. is equally simple and
3. does not require extensive data or computation time.