Real Investments and their Financing: a real options approach

Engelbert Dockner, Jøril Mæland, and Kristian R. Miltersen

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Copenhagen Business School
HANDELSHØJSKOLEN

Our Research Question

- Does capital structure decisions (i.e., the financing of the firm) interrelate with (real) investments decisions?
  - Capital structure decisions before the investment
  - The timing of the investment decision itself
  - The financing of the investment
  - Why does capital structure influence investment decisions (Compare Myers, 1977)
  - Are there situations where it does not influence the real decisions? (Compare Modigliani and Miller, 1958 and 1963)
- How does/should capital structure and investments vary across different industries and competitive settings?

What are we doing?

- Providing a micro foundation to the Dynamic Capital Structure Models
- Making a role for investments
- Analyzing different competitive structures and different industries
  - monopoly, imperfect competition, perfect competition
  - mature versus growth industry
- Discuss covenants and introduce the concept of idealized (optimal?) covenants

Example: a firm with an investment option (real option)

- Current Earnings: $X = 1$
- Current Value: $A = 15$
- Current Capital Structure
  - Equity value: $E = 10$
  - Debt value: $D = 5$
  - Coupon: $c = 0.5$
- The investment option
  - Invest $I = 25$ (i.e. improve production facility)
  - The earnings at each instant in time will be doubled
  - The endogenously determined trigger for when to exercise the option is $X_I = 2$

Example: a firm with an investment option (real option)

- The (optimal) situation just after the investment option is exercised:
  - Earnings: $2X_I = 4$
  - Value: $A_+ = 45$
  - Capital Structure
    - Equity value: $E_+ = 23$
    - Debt value: $D_+ = 22$
    - Coupon: $c_+ = 2.0$
- The situation just before the investment option is exercised:
  - Earnings: $X_I = 2$
  - Value: $A = A_+ - I = 20$
  - Capital Structure
    - Coupon: $c_1 = 0.5$
    - Debt value: $D_1 = 7$
    - Equity value: $E_1 = A - D_1 = 13$

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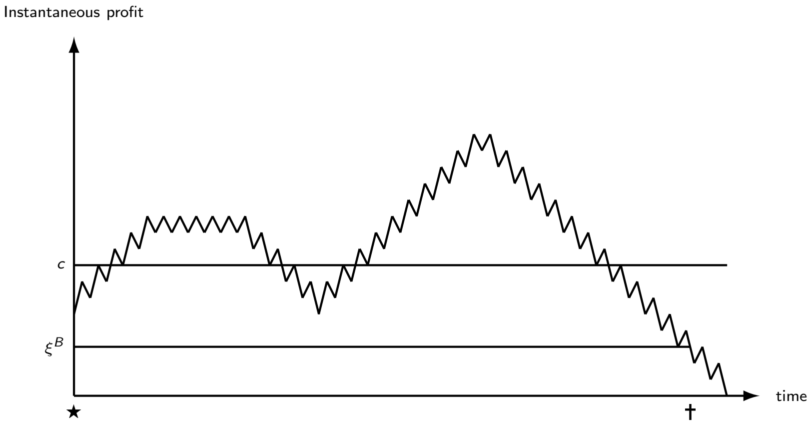
Example: a firm with an investment option (real option)

- How to finance the new investment (at the trigger point, $X_I = 2$)?
  - New debt (assume we approach the same creditors):
    - Coupon of junior debt: $c_2 = c_+ - c_1 = 1.5$
    - Value of junior debt: $D_2 = D_+ - D_1 = 15$
  - New equity: $E_2 = I - D_2 = 10$
- Hence, the situation just after the investment option is exercised:
  - Earnings: $X = 4$
  - Capital Structure
    - Equity value: $E_1 + E_2 = 23$
    - Debt value: $D_1 + D_2 = 22$
    - Coupon: $c_1 + c_2 = 2.0$
  - Value: $A = E_1 + E_2 + D_1 + D_2 = 45$

The Firms Capital Structure Decision

- (Instantaneous) cash flow from the production unit, ξ_t
- Firm is financed by
  - Debt with fixed instantaneous coupon rate, c, and infinite maturity
  - Equity
- Cash flow to
  - Debt: $(1 - \tau_i)c$
  - Equity: $(1 - \tau_e)(\xi_t - c)$
  - An Investor who have invested in both debt and equity: $(1 - \tau_e)\xi_t + (\tau_e - \tau_i)c$
- The curse of having debt: Bankruptcy
  - The equity holders have a real option to stop paying the coupons. I.e., if ξ_t becomes too low relative to c the equity holders will exercise this option. Hence, there is a trigger value, X^B. (I.e., in terms of the state variable X.)

A Firms Capital Structure



A Micro Foundation

- The price of the product at a given quantity demanded, q

$$p(q) = aX_t^{\gamma} q^{-\theta}$$

where $a, \gamma, \theta > 0$ and

$$dX_t = X_t \mu dt + X_t \sigma dW_t, X_0 = 1$$

- Costs of producing a given quantity, q

$$C(q) = kq^{\kappa}$$

Convex costs of producing, i.e. $\kappa > 1$. I.e, decreasing returns of scale

- Profit from producing q units

$$qp(q) - C(q) = aX_t^{\gamma} q^{1-\theta} - kq^{\kappa}$$

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A Micro Foundation

Solutions

- Monopoly: Use market power. I.e., take price impact into account when optimizing over q: $q_M^*(X_t)$
- Duopoly: Both competitors take price impact into account
    - Cournot competition: $q_C^*(X_t)$
    - Bertrand competition: $q_B^*(X_t)$
- Perfect Competition: Each producer takes the price, aX_t^κ, as given, i.e, $\theta = 0$. Hence, profit from producing becomes

$$aX_t^\gamma q - kq^\kappa$$

  Therefore, $q_P^*(X_t)$

In all cases will we get instantaneous profit from producing on the form $\xi_t = \omega X_t^\epsilon c^\eta$, $\epsilon > 0$, $\eta < 0$, and $\omega > 0$

A Micro Foundation

- The instantaneous profit in the monopoly and perfect competition cases

$$\xi_t = \omega X_t^{\epsilon} k^{\eta}$$

- Parameters

$$\epsilon = \frac{\gamma\kappa}{\kappa + \theta - 1} > 0$$

$$\eta = -\frac{1-\theta}{\kappa + \theta - 1} < 0$$

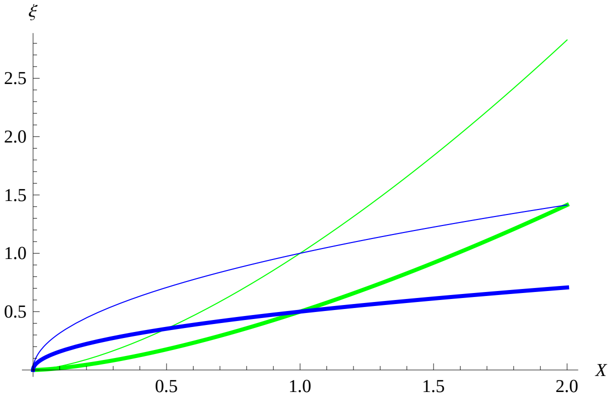
$$\omega = (1-\theta)^{\frac{1-\theta}{\kappa+\theta-1}} \left(\frac{a}{\kappa}\right)^{\frac{\kappa}{\kappa+\theta-1}} (\kappa + \theta - 1) > 0$$

Investments and Bankruptcy

- An investment can reduce variable production costs, k
  - New approach (as far as we know)
  - Others have looked at capacity constraints: Numerically very complicated
- After a bankruptcy the variable production costs, k, may have increased

Different Competitive Settings and Different Industries

- Different types of industries
  - Competitive (low θ) versus non competitive (high θ)
  - Mature (low γ) versus growth (high γ)
  - High versus low κ



Combining Investments and Capital Structure

- We have an option to improve the production
  - Invest I at a given date τ
  - After the investment the parameter in the optimal instantaneous cash flow

    $$\xi_t = \omega X_t^{\epsilon} k^{\eta}$$

    changes from unity (1) to k_f
- The firm has already some debt in its capital structure with instantaneous coupon, c
- (Part of) the capital needed for the new investment, I, will be raised by issuing more debt in the firm (with instantaneous coupon, c_J)

Combining Investments and Capital Structure

- We have to be careful with covenants of debt and with how to split the firm value in case of bankruptcy between the two classes of debt
- Typically, a firm has to default on all its debt at the same point in time.
- The decision to make the investment (and how to finance it) will be taken by the equity holders, i.e., maximizing their future cash flow

Combining Investments and Capital Structure

- What is happening?
  - Cash flows before the investment
    - Debt: $(1-\tau_i)c$
    - Equity: $(1-\tau_e)(\omega X_t^{\epsilon} - c)$
  - Cash flows after the investment
    - Debt: $(1-\tau_i)(c + c_J)$
    - Equity: $(1-\tau_e)(\omega X_t^{\epsilon} k_f^{\eta} - c - c_J)$
  - The capital raised by issuing the new debt helps the equity holders finance the new investment, I

Combining Investments and Capital Structure

- A couple of (interesting) questions
  - Does it delay or accelerate the decision to make the investment that
    - there is already some debt in the firms original capital structure (delay)
    - that (part of) the investment capital, I, can be raised by issuing new debt? (accelerate)
  - Can we separate the effect of the two issues?
  - Does it change the *original* decision to issue debt in the firm that the firm has a (valuable) real option investment opportunity? (reduce initial debt for two reasons—(i) bankruptcy kills the investment option (ii) we get a second chance to increase debt)

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The Initial Conditions

- Initial conditions when debt is issued

$$D(1) = P$$
$$E(1) = A - P$$

- P is the principal of the debt (issue at par)
- A is the value of the firm (including its real option to invest)
- $A = E(1) + D(1)$

The Boundary Conditions at Bankruptcy

- The boundary conditions at the bankruptcy trigger level, B

$$E(B) = 0$$
$$E'(B) = 0$$
$$D(B) = (1-\alpha)A_0 B^{\epsilon} k_b^{\eta}$$

- α reflects direct bankruptcy costs
- k_b^{η} reflects indirect bankruptcy costs
- A_0 is the value of a similar firm optimally financed but *without* the investment option

The Boundary Conditions at Investment

- The boundary conditions at the investment trigger level, F

$$E(F) = E_0(F) + \left(P_0 F^{\epsilon} k_f^{\eta} - D(F)\right) - I = A_0 F^{\epsilon} k_f^{\eta} - D(F) - I$$
$$E'(F) = \epsilon A_0 F^{\epsilon-1} k_f^{\eta} - D'(F)$$

- The debt has no value matching condition
- $P_0 F^{\epsilon} k_f^{\eta} - D(F)$ is the proceeds from issuing a junior loan under idealized (optimal) covenants
- Coupon rate to junior loan $c_0 F^{\epsilon} k_f^{\eta} - c$
- Equity holders choice of investment trigger using idealized covenants is identical to a central planner/manager who optimizes total firm value

$$E(F) + D(F) = A_0 F^{\epsilon} k_f^{\eta} - I$$
$$E'(F) + D'(F) = \epsilon A_0 F^{\epsilon-1} k_f^{\eta}$$

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Alternative Boundary Conditions at Investment

- The boundary conditions at investment with no new debt financing (Myers)
  - The boundary conditions at the investment trigger level, F

$$E(F) = E_0^c(F) - I$$
$$E'(F) = E_0^{c\prime}(F)$$
$$D(F) = D_0^c(F)$$

  - E_0^c and D_0^c denotes values with the same c as chosen initially.
- The boundary condition at refinancing
  - The boundary conditions at the trigger level, F, chosen exogenously

$$E(F) = E_0(F) + \left(P_0 F^{\epsilon} k_f^{\eta} - D(F)\right) = AF^{\epsilon} k_f^{\eta} - D(F)$$

  - We pick the same F as for the investment case with idealized covenants

Some Numbers

- Short term (after-tax) interest rate $r = 0.05$
- Volatility on the X process $\sigma = 0.3$
- Drift of the X process $\mu = 0.02$
- Price elasticity of demand $\theta = 0.4$
- Income elasticity of demand $\gamma = 0.5$
- Convexity of cost function $\kappa = 1.2$
- Direct bankruptcy cost $\alpha = 0.2$
- Indirect bankruptcy costs $k_b = 1.2$
- The effective tax rate on dividends $\tau_e = 0.42$
- The tax rate on coupon payments $\tau_i = 0.34$
- Improvement from investment $k_f = 0.6$
- Investment costs $I = 5$

Some numbers

$\theta = 0.4$ $\epsilon = 1.0$	Pure E No Inv.	E&D No Inv., No RF	E&D No Inv., RF	Pure E Inv.	E&D Inv., No RF	E&D Inv. and RF
Firm Value	4.83	4.99	5.04	6.06	6.23	6.33
Bankruptcy Trigger		.23	.21		.20	.18
Investment Trigger			(5.61)	5.77	5.81	5.61
Leverage		.45	.41		.37	.33

$\theta = 0.5$ $\epsilon = .85$	Pure E No Inv.	E&D No Inv., No RF	E&D No Inv., RF	Pure E Inv.	E&D Inv., No RF	E&D Inv. and RF
Firm Value	4.72	4.89	4.93	5.15	5.32	5.39
Bankruptcy Trigger		.21	.20		.20	.19
Investment Trigger			(8.22)	8.80	8.85	8.22
Leverage		.49	.46		.45	.43

$\theta = 0.3$ $\epsilon = 1.2$	Pure E No Inv.	E&D No Inv., No RF	E&D No Inv., RF	Pure E Inv.	E&D Inv., No RF	E&D Inv. and RF
Firm Value	7.48	7.67	7.77	12.91	13.12	13.37
Bankruptcy Trigger		.25	.22		.18	.15
Investment Trigger			(3.99)	3.96	4.00	3.99
Leverage		.40	.34		.27	.21

Some Empirical Implications

- (Initial) leverage ratios depend on
  - Industry: More growth, less leverage
  - Competitiveness: More competitive, less leverage
  - Convexity of costs: ambiguous
  - Moneyness of real investment option(s): more in-the-money, less leverage
- Investment triggers
- Bankruptcy triggers
- In order to implement first best decisions of investments a rich menu of debt covenants to pick from is essential in designing debt contracts

Why are we doing this?

- Investment (and bankruptcy) behavior and the competitive environment
- How does capital structure influence investment decisions (Compare Myers, 1977)
- How taxes influence investments (and bankruptcy) across different industries
- Analyzing bankruptcy treatment
- Return requirements for different types of (optimal) financing of investments
- A rigorous treatment of Weighted Average Cost of Capital (WACC)
- Separation between direct and indirect bankruptcy costs
- Only one investment option per firm (Will be lost in case of bankruptcy before investment option is exercised)

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With Competitive Interactions

- So far we have to force firms to have 100% equity financing after bankruptcy
- A leader (who invests first) and a follower
- Preemption for some parameter values
- A new role for debt: To reduce preemption

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