

**Humboldt Universität zu Berlin**

Department of Mathematics - **Applied Financial Mathematics**



# Hidden Liquidity and the Optimal Display of Iceberg Orders

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(Joint work with Ulrich Horst)

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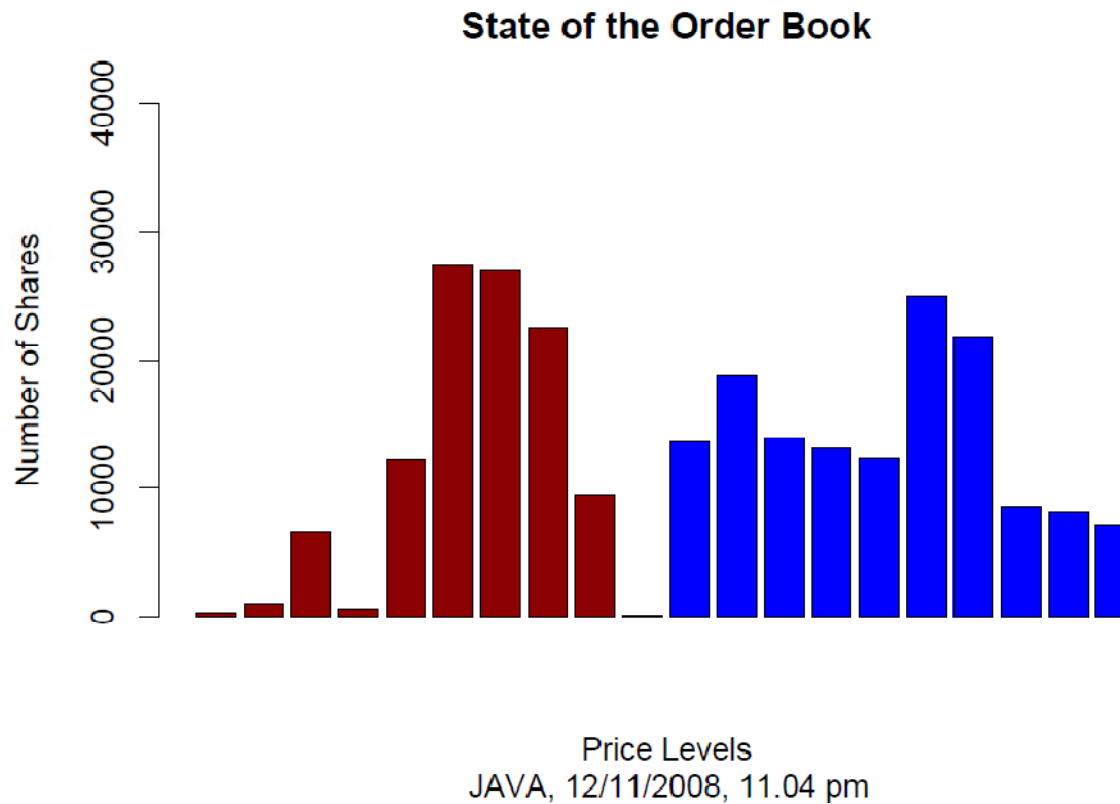
# Outline

- Hidden Liquidity and Icebergs in Electronic Exchanges
- How much should the trader hide?
  - The Model
  - Model Input: Hidden Liquidity Statistics
  - Calibration

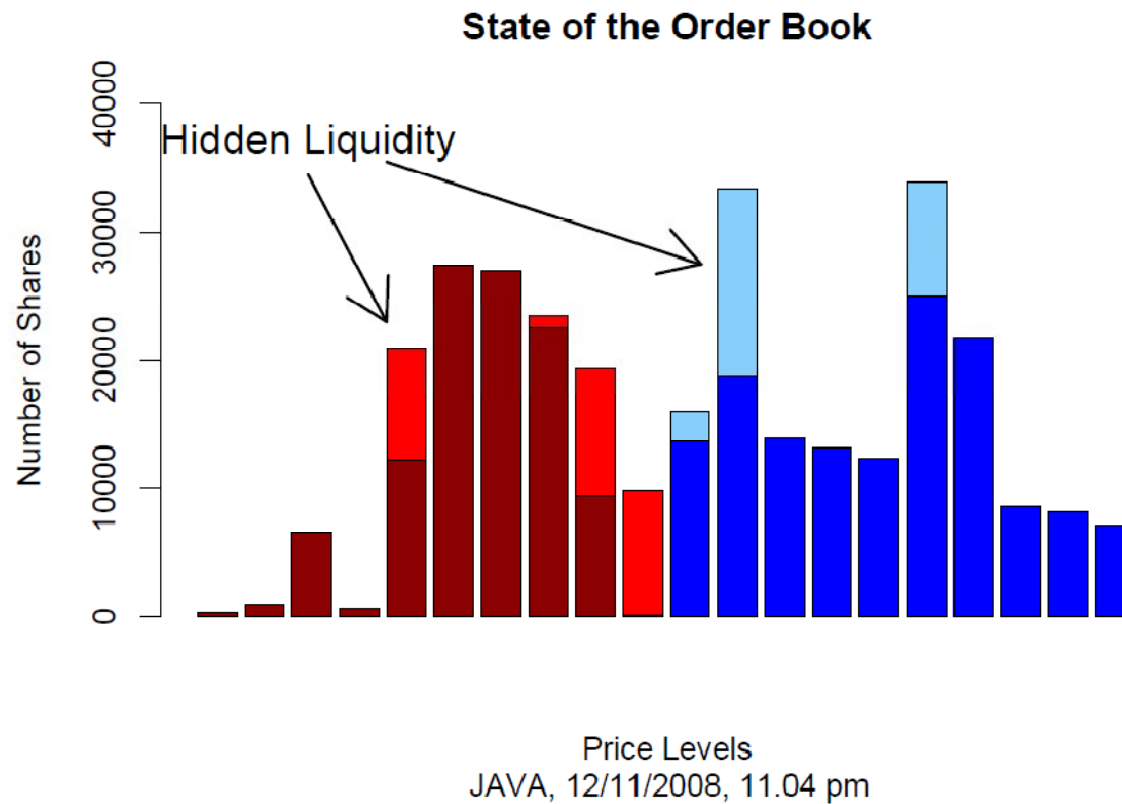
# Limit Order Books

- Almost all electronic exchanges are based on Limit Order Books (LOBs)
  - **Market Orders:** immediate execution
  - **Limit Orders:** stored in the LOB
- Orders are executed according to a set of **Priority Rules:**
  - Price Priority
  - Display Priority
  - Time Priority
- Large orders (limit or market) **move the market**
- Orders may be shielded from public view (Hidden Liquidity)

# The Displayed Limit Order Book



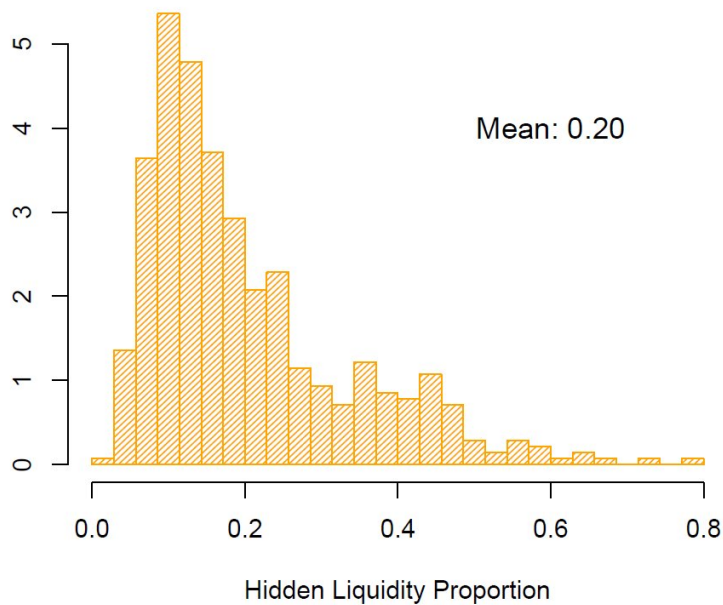
# The „True“ Order Book



# Significance of Hidden Liquidity

How much is hidden ?

Distribution of Hidden Liquidity Proportion  
in US Markets (S&P 500)



Europe :

proportion of posted hidden  
liquidity can take up to 40-50 %

## Some Statistical Properties of Hidden Liquidity

- Correlation of **hidden liquidity ratio** in the Spread with:
  - Average Spread: 0.859
  - Average Price: 0.755
  - Average Daily Trading Volume (ADV): -0.212
  - Average Trade Size: -0.322
- HL ratio well explained by average spread ( $R^2 > 0.7$ ):

$$H_{\text{Ratio}} = -0.04 + 0.09 \text{ Spread}$$

## Typical Example: The Iceberg Order

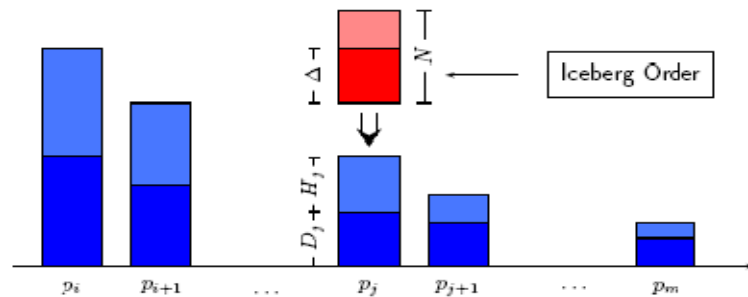
- **Only a fraction** of the order is **openly displayed** in the LOB
- The hidden part **loses time priority** over the displayed part
  
- **How much should we display?**



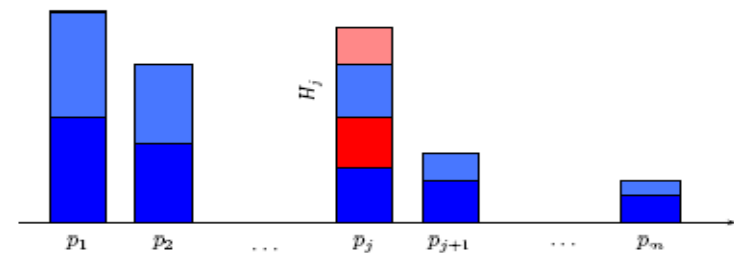
# The Model:

Visible Liquidity has Priority over Hidden Liquidity

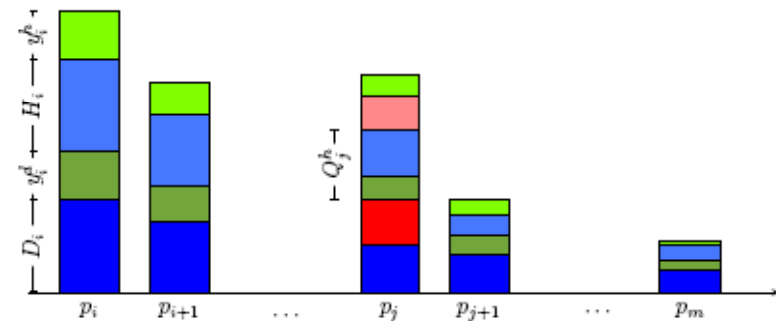
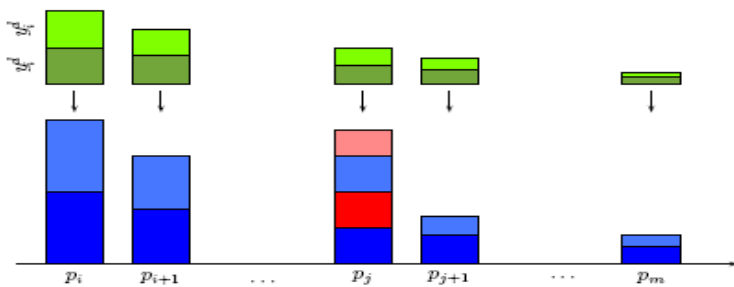
Iceberg Submission



After Iceberg Submission



Arrival of Competing Sell Orders



# The Model

- What is the **optimal display size** of an iceberg order?
- **Model assumptions:**
  - Order placed at a single price level (top of book or in spread)
  - Select display size to maximize expected **execution volume**
- **Model Input:**
  - Initial LOB Liquidity
  - Order (market and limit) arrival volumes
  - **Hidden Liquidity !**

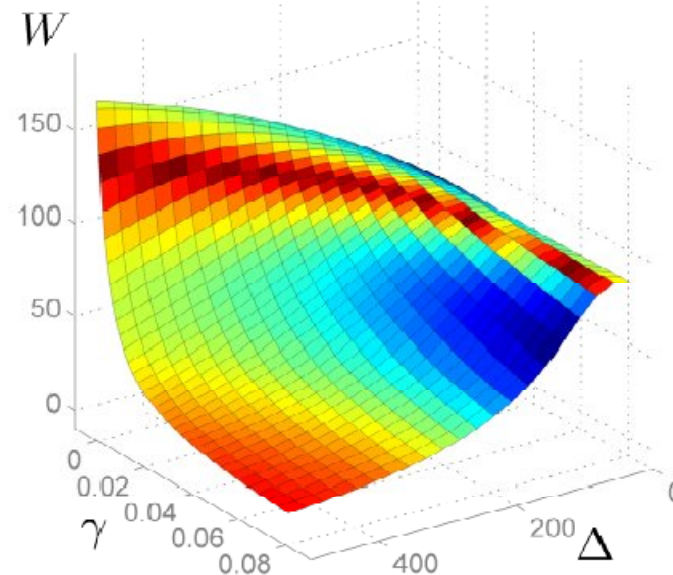
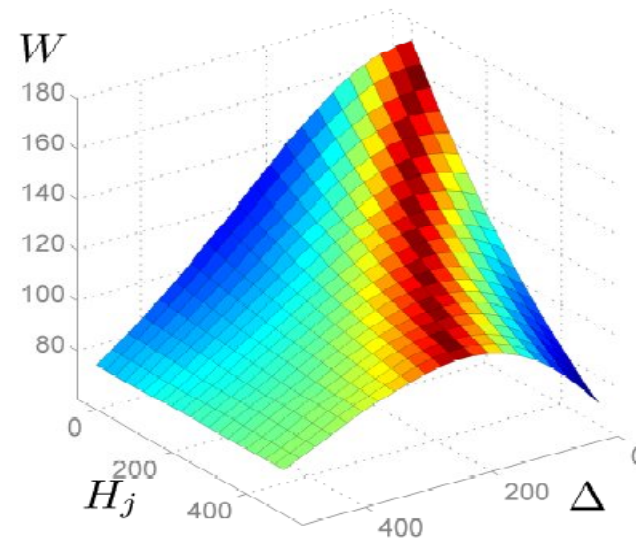
# The Model:

## Optimal Display Curves

- Market Impact Model

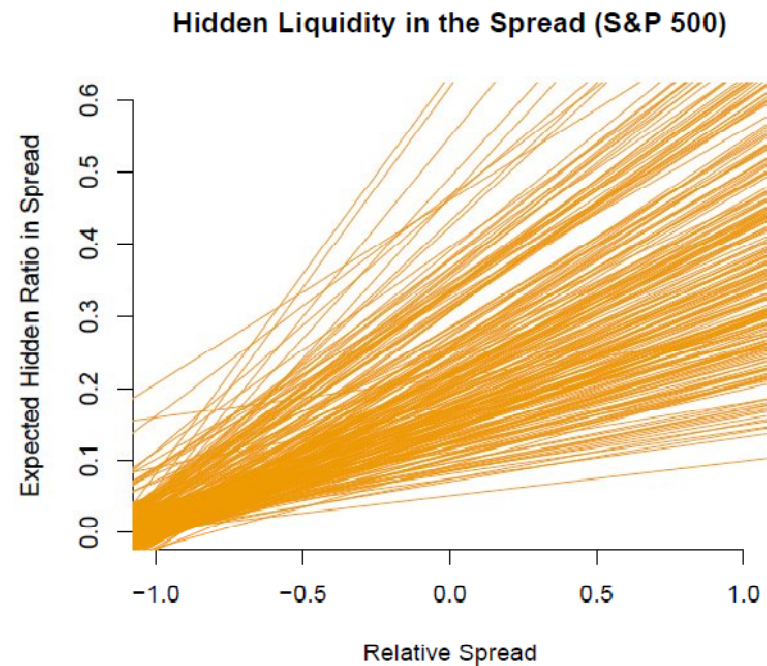
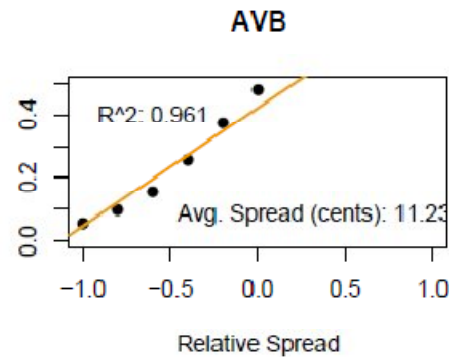
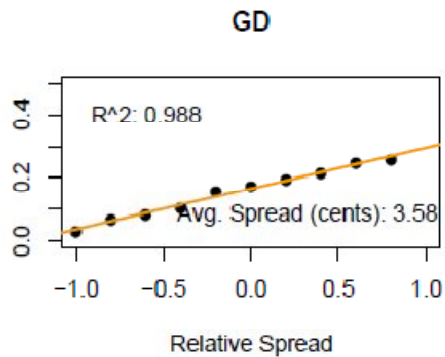
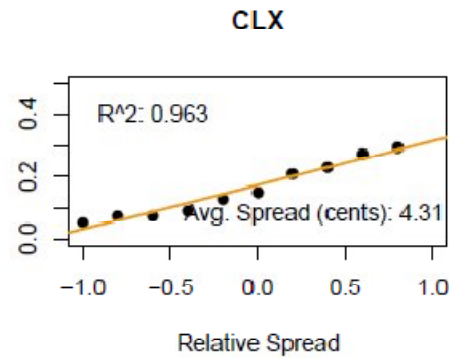
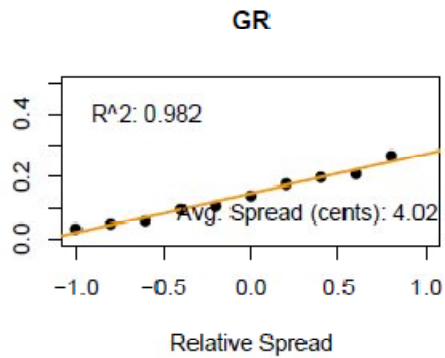
$$\beta(\Delta) = \beta_0 + \gamma \cdot \Delta^2$$

- Presence of Hidden Liquidity  $H_j$   
„encourages“ display
- Market Sensitivity  $\gamma$   
„discourages“ display



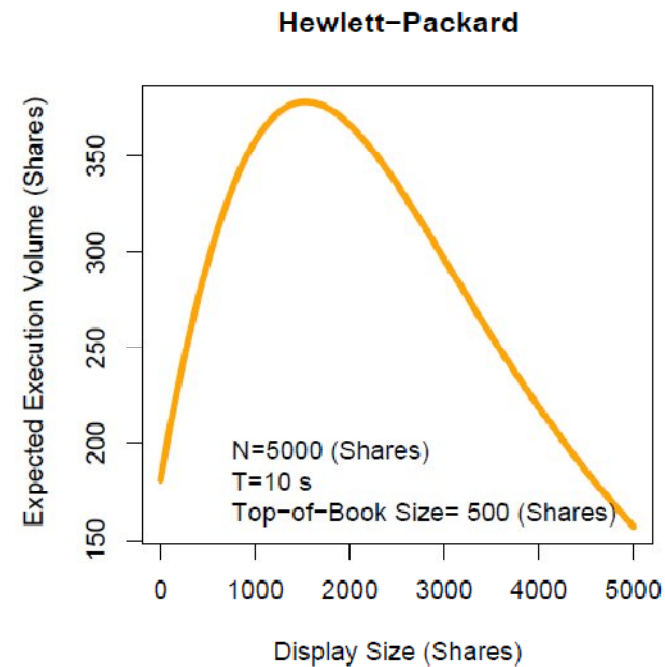
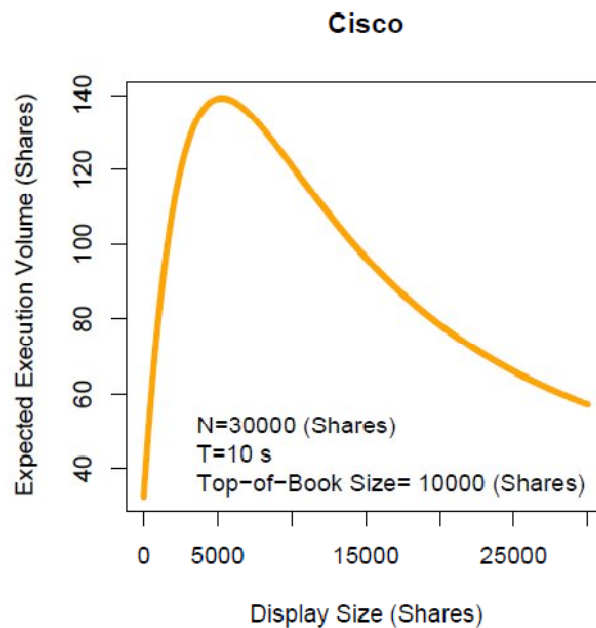
# Model Calibration:

## Forecasting Hidden Liquidity



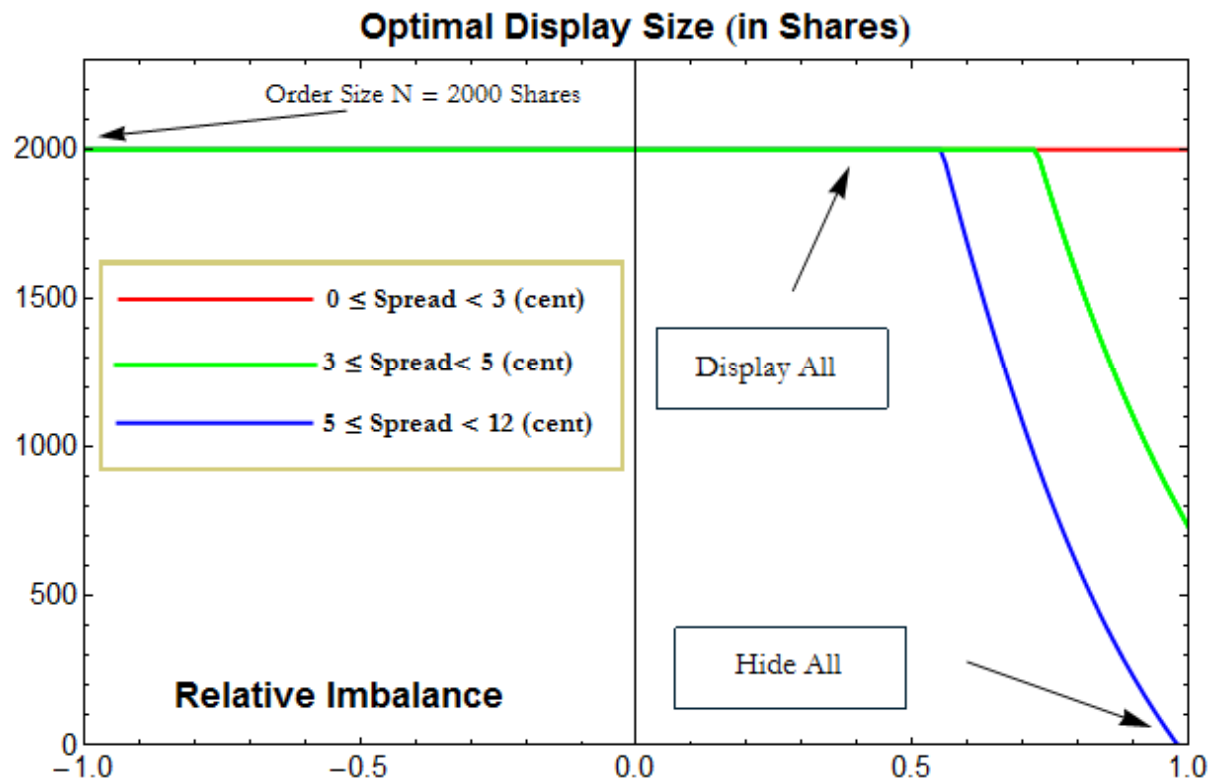
# Model Calibration:

## Obtaining Optimal Display Sizes



# Optimal Display Size:

The role of Imbalance and Spread



# Conclusion

- Hidden Liquidity is important feature in LOB-marjkets
- Statistical properties of HL and forecasting
- Model for Optimal Iceberg Implementation
- Optimal Display Strategies