BID ASK DYNAMIC PRICING IN FINANCIAL MARKETS WITH TRANSACTION COSTS AND LIQUIDITY RISK

Jocelyne Bion-Nadal
CNRS-CMAP Ecole Polytechnique

June 24, 2010

Introduction

Goal: Define an axiomatization for dynamic prices in context of liquidity risk and transaction costs

- Usual setting: linear pricing procedure.
  A no arbitrage dynamic price process is a martingale under a probability measure equivalent to the reference probability measure.

- Real Financial Markets:
  Limit order book associated to a financial product X= Bid and Ask prices associated with nX.
  $C_{ask}(nX) > nC_{ask}(X)$ for n large enough.

  The observation of limit order books $\rightarrow$ ask price is convex.

INTRODUCTION

DYNAMIC PRICING PROCEDURE

Introduce an axiomatic approach of dynamic pricing procedure for the ask price

-indexed by two stopping times

-convex

- consistent in time $\rightarrow$ price consistently options with different maturities.

- No Arbitrage

RELATED LITTERATURE

Axiomatization in the setting of a Brownian filtration: Peng (2004): g-expectations

Close definitions:

Monetary utility function for processes in a discrete time setting: Cheridito, Delbaen, Kupper (2006)

Monetary Concave Utility Functional: Klöppel and Schweizer (2007). One deterministic instant of time.

INTRODUCTION

APPLICATION TO CALIBRATION ON BOTH LIQUID AND ILLIQUID ASSETS

Reference family composed of
- liquid assets represented by their stochastic process S^k
-options on these assets of various maturity dates for which a limit order book is observable today.

EXTENSION TO THE CASE OF MODEL UNCERTAINTY

OUTLINE

PROPERTIES OF LIMIT ORDER BOOKS

Y^l traded financial asset.

Bid	
quantity	limit
M_1	C^1_{bid}
M_2	C^2_{bid}
...	...
M_p	C^p_{bid}

Ask	
limit	quantity
C^1_{ask}	N_1
C^2_{ask}	N_2
...	...
C^q_{ask}	N_q

$$C^p_{bid} < \ldots < C^1_{bid} < C^1_{ask} < \ldots < C^q_{ask} \tag{1}$$

C^0 price of transaction, $N_0 = M_0$ number of shares exchanged at time t_0

$$C^1_{bid} \leq C^0 \leq C^1_{ask} \tag{2}$$

Let $n \leq \sum_{0 \leq i \leq q} N_i = N^l$ Let $j \leq q$ such that $\sum_{0 \leq i \leq j-1} N_i \leq n < \sum_{0 \leq i \leq j} N_i$.

$$C_{ask}(nY^l) = \sum_{0 \leq i \leq j-1} N_i C^i_{ask} + (n - \sum_{0 \leq i \leq j-1} N_i) C^j_{ask}$$

$n \rightarrow C_{ask}(nY^l)$ is convex.

TCPP

Framework

Filtered probability space $(\Omega, \mathcal{F}_\infty, (\mathcal{F}_t)_{t\in R^+}, P)$.
$(\mathcal{F}_t)$ right continuous filtration.
$\mathcal{F}_0$: σ-algebra generated by the P null sets of $\mathcal{F}_\infty$.
One positive asset is taken as numéraire. A financial position at a stopping time τ means an element of $L^\infty(\mathcal{F}_\tau)$.

Modelize the dynamic ask price of X: $(\Pi_{\sigma,\tau}(X))_{\sigma\le\tau}$.
Selling X: same as buying $-X$. So dynamic bid price of X: $-\Pi_{\sigma,\tau}(-X)$.
Dynamic limit order book associated to X at time $\sigma \le \tau$:

$$(-\Pi_{\sigma,\tau}(-nX), \Pi_{\sigma,\tau}(nX))_{n\in N}$$

OUTLINE

Dynamic Pricing Procedure

Definition

A dynamic pricing procedure $(\Pi_{\sigma,\tau})_{0\leq\sigma\leq\tau}$ ($\sigma \leq \tau$: stopping times) is a family of maps $(\Pi_{\sigma,\tau})_{0\leq\sigma\leq\tau} : L^\infty(\mathcal{F}_\tau) \rightarrow L^\infty(\mathcal{F}_\sigma)$ satisfying:

1. monotonicity:
$$\forall\, (X,Y) \in (L^\infty(\mathcal{F}_\tau))^2, \quad \text{if } X \leq Y \text{ then } \Pi_{\sigma,\tau}(X) \leq \Pi_{\sigma,\tau}(Y)$$
2. translation invariance:
$$\forall\, Z \in L^\infty(\mathcal{F}_\sigma)\,,\ \forall\, X \in L^\infty(\mathcal{F}_\tau)\ \Pi_{\sigma,\tau}(X+Z) = \Pi_{\sigma,\tau}(X) + Z$$
3. convexity: $\forall\, (X,Y) \in (L^\infty(\mathcal{F}_\tau))^2\ \forall\, \lambda \in [0,1]$
$$\Pi_{\sigma,\tau}(\lambda X + (1-\lambda)Y) \leq \lambda\Pi_{\sigma,\tau}(X) + (1-\lambda)\Pi_{\sigma,\tau}(Y)$$
4. normalization: $\Pi_{\sigma,\tau}(0) = 0$

DUAL REPRESENTATION

DUAL REPRESENTATION OF A PRICING PROCEDURE CONTINUOUS FROM BELOW

$$\forall X \in L^{\infty}(\mathcal{F}_\tau) \;\; \Pi_{\sigma,\tau}(X) = \text{esssup}_{\mathrm{R} \in \mathcal{M}^1_{\sigma,\tau}}(\mathrm{E_R}(\mathrm{X}|\mathcal{F}_\sigma) - \alpha^{\mathrm{m}}_{\sigma,\tau}(\mathrm{R})) \;\; \mathrm{P\ a.s.} \quad (3)$$

$$\textit{where } \mathcal{M}^1_{\sigma,\tau} = \{R \text{ on } (\Omega, \mathcal{F}_\tau)\,,\; R \ll P,\; R_{|\mathcal{F}_\sigma} = P \text{ and } \; E_R(\alpha^m_{\sigma,\tau}(R)) < \; \infty\} \quad (4)$$

$$\forall R \ll P \;\; \alpha^m_{\sigma,\tau}(R) = R - \text{esssup}_{X \in L^\infty(\Omega, \mathcal{F}_\tau, P)}(E_R(X|\mathcal{F}_\sigma) - \Pi_{\sigma,\tau}(X)) \quad (5)$$

static case: Föllmer and Schied; also Frittelli and Rosazza Gianin
conditional case: Detlefsen and Scandolo and Bion-Nadal

TCPP

Time Consistency

Definition

A dynamic pricing procedure $(\Pi_{\sigma,\tau})_{0\leq\sigma\leq\tau}$ is time-consistent if

$$\forall\, 0 \leq \nu \leq \sigma \leq \tau \;\; \forall\, X \in L^{\infty}(\mathcal{F}_\tau) \;\; \Pi_{\nu,\sigma}(\Pi_{\sigma,\tau}(X)) = \Pi_{\nu,\tau}(X).$$

Time consistency + normalization $\rightarrow$ $\Pi_{\sigma,\tau}$ is the restriction of $\Pi_{\sigma,\infty}$.

Notation

a TCPP is a time-consistent dynamic pricing procedure continuous from below.

Time consistency and cocycle condition

Theorem

Let $(\Pi_{\sigma,\tau})_{0\leq\sigma\leq\tau}$ be a dynamic pricing procedure continuous from below. It is time-consistent if and only if for every probability measure $Q \ll P$, the minimal penalty function satisfies the following cocycle condition for all stopping times $\nu \leq \sigma \leq \tau$:

$$\alpha^m_{\nu,\tau}(Q) = \alpha^m_{\nu,\sigma}(Q) + E_Q(\alpha^m_{\sigma,\tau}(Q)|\mathcal{F}_\nu) \quad Q \; a.s \tag{6}$$

Another characterization of time consistency was given in Cheridito, Delbaen, Kupper (2006) in terms of acceptance sets and also in terms of a concatenation condition.

No Free Lunch TCPP

Definition

Atteignable claims at zero cost via self financing simple strategies:

$$\mathcal{K}_0 = \{X = X_0 + \sum_{1 \leq i \leq n} (Z_i - Y_i),\ (X_0, Z_i, Y_i) \in L^\infty(\mathcal{F}_\infty) \ \mid$$
$$\Pi_{0,\infty}(X_0) \leq 0;\ \Pi_{\tau_i,\infty}(Z_i) \leq -\Pi_{\tau_i,\infty}(-Y_i)\ \forall 1 \leq i \leq n\}$$

where $0 \leq \tau_1 \leq ... \leq \tau_n < \infty$ are stopping times.

No arbitrage

$\mathcal{K}_0 \cap L^\infty_+(\Omega, \mathcal{F}_\infty, P) = \{0\} \iff \mathcal{K} \cap L^\infty_+(\Omega, \mathcal{F}_\infty, P) = \{0\}$ where $\mathcal{K}$ is the cone generated by $\mathcal{K}_0$

Definition

The TCPP has No Free Lunch if $\overline{\mathcal{K}} \cap L^\infty_+(\Omega, \mathcal{F}_\infty, P) = \{0\}$ where $\overline{\mathcal{K}}$ is the weak* closure of $\mathcal{K}$

No Free Lunch TCPP

Definition

Atteignable claims at zero cost via self financing simple strategies:

$$\mathcal{K}_0 = \{X = X_0 + \sum_{1 \leq i \leq n} (Z_i - Y_i),\ (X_0, Z_i, Y_i) \in L^\infty(\mathcal{F}_\infty) \ |$$
$$\Pi_{0,\infty}(X_0) \leq 0;\ \Pi_{\tau_i,\infty}(Z_i) \leq -\Pi_{\tau_i,\infty}(-Y_i)\ \forall 1 \leq i \leq n\}$$

where $0 \leq \tau_1 \leq ... \leq \tau_n < \infty$ are stopping times.

No arbitrage

$\mathcal{K}_0 \cap L^\infty_+(\Omega, \mathcal{F}_\infty, P) = \{0\} \iff \mathcal{K} \cap L^\infty_+(\Omega, \mathcal{F}_\infty, P) = \{0\}$ where $\mathcal{K}$ is the cone generated by $\mathcal{K}_0$

Definition

The TCPP has No Free Lunch if $\overline{\mathcal{K}} \cap L^\infty_+(\Omega, \mathcal{F}_\infty, P) = \{0\}$ where $\overline{\mathcal{K}}$ is the weak* closure of $\mathcal{K}$

Fundamental Theorem

Theorem

Let $(\Pi_{\sigma,\tau})_{\sigma\leq\tau}$ be a TCPP. The following conditions are equivalent:
i) The TCPP has No Free Lunch
iii) There is a probability measure R equivalent to P with zero minimal penalty: $\alpha^m_{0,\infty}(R) = 0$
iv) There is a probability measure R equivalent to P such that for every stopping time σ,

$$\forall X \in L^\infty(\Omega, \mathcal{F}_\infty, P) \quad -\Pi_{\sigma,\infty}(-X) \leq E_R(X|\mathcal{F}_\sigma) \leq \Pi_{\sigma,\infty}(X) \tag{7}$$

REGULARITY OF PATHS

THEOREM

Let $(\Pi_{\sigma,\tau})_{\sigma\leq\tau}$ be a No Free Lunch TCPP. For every probability measure R equivalent to P with zero penalty, for every $X \in L^\infty(\Omega, \mathcal{F}_\infty, P)$, $(\Pi_{t,\infty}(X))_t$ is a R-supermartingale (resp. $-(\Pi_{t,\infty}(-X))_t$ is a R-submartingale). It admits a cadlag version.

$$-\Pi_{\sigma,\infty}(-X)) \leq E_R(X|\mathcal{F}_\sigma) \leq \Pi_{\sigma,\infty}(X) \tag{8}$$

$x \in \boldsymbol{R}^+*,\ X(t,x,\omega)$ ask price at time t per share for an order of size x:

$$X(t,x,\omega) = \frac{\Pi_{t,\infty}(xX)(\omega)}{x}$$

Properties of the supply curve

Proposition

1. *For every x,* $(t,\omega) \rightarrow X(t,x,\omega)$ *is a càdlàg stochastic process.*
2. $\forall t \in \mathbf{R}^+$, *P a.s.,* $x \rightarrow X(t,x,\omega)$ *is non decreasing, continuous, admits a right and a left derivative at any point and is derivable almost surely.*
3. *limit in zero:* $x \rightarrow X(t,x,.)$ *has a right limit in* 0

$$X^+(t,0,.) = \mathrm{esssup}_{Q \in \mathcal{M}^0} E_Q(X|\mathcal{F}_t)$$

where $\mathcal{M}^0 = \{Q \sim P \mid \alpha^m_{0,\infty}(Q) = 0\}$

4. *Asymptotic limit:* $X(t,x,.)$ *has a limit as* $x \rightarrow +\infty$*:*

$$X^\infty(t,\omega) = \mathrm{esssup}_{Q \in \mathcal{M}^{1,e}(P)} (E_Q(X|\mathcal{F}_t)$$

with $\mathcal{M}^{1,e}(P) = \{Q \sim P \mid E_Q(\alpha^m_{t,\infty}(Q)) < \infty\}$

Indifference price with exponential utility

Pricing by indifference with respect to utility function: Hodges and Neuberger (1989)
Exponential utility $u(x) = -\frac{1}{\alpha}e^{-\alpha x}$: Rouge and El Karoui (2000), Cheridito and Kupper (2006), Klöppel and Schweizer (2007)...
Indifference price of $X \in L^\infty(\mathcal{F}_\tau)$:
$\Pi_{\sigma,\tau}(X) = \frac{1}{\alpha} ln(E(e^{\alpha X}|\mathcal{F}_\sigma))$. Dual representation:

$$\Pi_{\sigma,\tau}(X) = \text{esssup}_{\mathrm{R}\in\mathcal{M}^e(\mathrm{P})}(\mathrm{E_R}(\mathrm{X}|\mathcal{F}_\sigma) - \frac{1}{\alpha}\mathrm{H}_\sigma(\mathrm{R}|\mathrm{P}))$$

$\mathcal{M}^e(P) = \{Q \sim P\}$, $H_\sigma(R|P) = E_P(\ln(\frac{dR}{dP})\frac{dR}{dP}|\mathcal{F}_\sigma)$
$\Pi_{\sigma,\tau}$ is a No Free Lunch TCPP.
Dynamic pricing using BSDE give also exemples of TCPP.

TCPP from portfolio constraints

Pricing and hedging under constraints on the set of admissible porfolios.

Föllmer and Kramkov (1997) and Klöppel and Schweizer (2007) have computed the value process associated with the minimal $\mathcal{H}$-constrained hedging portfolio (under some conditions on $\mathcal{H}$) is

$$\Pi_{\tau,T}(X) = \text{esssup}_{\mathrm{Q}\in\mathcal{P}(\mathcal{H})}(\mathrm{E}_{\mathrm{Q}}(\mathrm{X}|\mathcal{F}_\tau) - \mathrm{E}_{\mathrm{Q}}[\mathcal{A}^{\mathcal{H}}(\mathrm{Q})_{\mathrm{T}} - \mathcal{A}^{\mathcal{H}}(\mathrm{Q})_\tau|\mathcal{F}_\tau]) \quad (9)$$

where $\mathcal{A}^{\mathcal{H}}(Q)$ is the smallest increasing predictable process A such that $Y - A$ is a local Q supermartingale for any $Y \in \{H.S \mid H \in \mathcal{H}\}$.

It defines a No Free Lunch TCPP.

OUTLINE

Economic Model

Reference Family

- d+1 liquid assets described by their locally bounded stochastic process

$$(S^k)_{0 \le k \le d}$$

- p non liquid assets: options known only at one stopping time τ_l (maturity date). Modelized by an essentially bounded $\mathcal{F}_{\tau_l}$-measurable map Y^l.
  At one instant of time (time 0), a limit order book $(C_{bid}(mY^l))_{\le M^l}, (C_{ask}(nY^l))_{n \le N^l})$ is observed for each Y^l.

Calibration on option prices

Definition

A TCPP $(\Pi_{\sigma,\tau})_{0\leq\sigma\leq\tau}$ is calibrated on the reference family $((S^k)_{0\leq k\leq d}, (Y^l)_{1\leq l\leq p})$ and the observed limit order books $(C_{bid}(mY^l)_{m\leq M^l}, C_{ask}(nY^l)_{n\leq N^l})$ if

- it extends the dynamics of the process $(S^k)_{0\leq k\leq d}$:$\forall n \in \mathbf{Z}$

$$\text{if } \; S^k_\tau \in L^\infty(\mathcal{F}_\tau) \; \text{ then } \; \Pi_{\sigma,\tau}(nS^k_\tau) = nS^k_\sigma$$

- it is compatible with the observed limit order books for $(Y^l)_{1\leq l\leq p}$

$$\forall 1 \leq l \leq p \quad C_{bid}(nY^l) \leq -\Pi_{0,\tau_l}(-nY^l) \leq \; \Pi_{0,\tau_l}(nY^l) \leq C_{ask}(nY^l)$$

CHARACTERIZATION OF CALIBRATION

THEOREM

A TCPP is calibrated on the reference family if and only if

- *Every probability measure $R \ll P$ such that $\alpha^m_{0,\infty}(R) < \infty$ is a local martingale measure with respect to every process S^k.*
- *For every probability measure $R \ll P$, for every stopping time τ,*

$$\alpha^m_{0,\tau}(R) \geq \sup_{\{\tau_l \leq \tau\}} \left(\sup_{m \leq M^l} (C_{bid}(mY^l) - E_R(mY^l), \sup_{n \leq N^l} (E_R(nY^l) - C_{ask}(nY^l)) \right) \quad (10)$$

HEDGING FOR TCPP CALIBRATED ON LIQUID OPTIONS

$\Pi_{\sigma,\tau}$ be a No Free Lunch TCPP calibrated on the reference family. Assume that for any $n \in N$, $C_{ask}(nY^l) = C_{bid}(nY^l) = nC^l$.

$$\Pi_{\sigma,\tau}(X) = \mathrm{esssup}_{\mathrm{Q}\in\mathcal{Q}}(\mathrm{E_Q}(\mathrm{X}|\mathcal{F}_\sigma) - \alpha^{\mathrm{m}}_{\sigma,\tau}(\mathrm{Q}))$$

PROPOSITION

$Z^l_t = \Pi_{t,\infty}(Y^l)$ is a martingale for every Q in $\mathcal{Q}$. The options $(Y^l)_{0\leq l\leq p}$ can be used to hedge dynamically (using $\Pi_{t,\infty}(Y^l)$) as well as the assets $(S^k)_{1\leq k\leq d}$.

OUTLINE

PRICING UNDER MODEL UNCERTAINTY

FRAMEWORK No reference probability measure is given but a weakly relatively compact set $\mathcal{Q}$ of probability measures not all absolutely continuous with respect to some probability measure.
Framework first introduced by Denis Martini (2006)
Example: The laws of X_t^σ, where

$$dX_t^\sigma = b_t dt + \sigma_t dW_t \quad \sigma_t \in [\underline{\sigma}, \overline{\sigma}]$$

Let Ω be a Polish space. For example $\Omega = \mathcal{C}_0(\boldsymbol{R}^+, \boldsymbol{R}^d)$. Let

$$c(f) = \sup_{Q \in \mathcal{Q}} E_Q(f) \quad \forall f \in \mathcal{C}_b(\Omega)$$

$L^1(c)$ denotes the Banach space obtained by completion and separation of $\mathcal{C}_b(\Omega)$ for the semi-norm c. $L^1(c)$: introduced by Feyel de la Predelle (1989).

CANONICAL CLASS OF PROBABILITY MEASURE ASSOCIATED TO $L^1(c)$

USUAL EQUIVALENCE CLASS OF MEASURES

let μ_0 be a non negative finite measure on $(\Omega, \mathcal{B}(\Omega))$. A non negative measure μ on $(\Omega, \mathcal{B}(\Omega)$ belongs to the (usual) equivalence class of the probability measure μ_0 if and only if

$$\forall A \in \mathcal{B}(\Omega), \quad \mu(A) = 0 \iff \mu_0(A) = 0$$

Or equivalently

$$\mu \sim \mu_0 \iff [\forall X \in L^\infty(\Omega, \mathcal{B}(\Omega), \mu_0)_+, \quad X = 0 \iff \int X d\mu = 0]$$

Canonical class of probability measure associated to $L^1(c)$

When $\mathcal{Q}$ is not finite, characteristic functions of Borelian sets are not all in $L^1(c)$ (recall: $c(f) = \sup_{Q \in \mathcal{Q}} E_Q(f)\ \ f \in \mathcal{C}_b(\Omega)$).
Issues:

- Can one associate a probability measure to $L^1(c)$?
- If yes, can one define a natural equivalence relation so that one gets a unique class characterizing the null elements in the cone $L^1(c)_+$?

Theorem

There is a probability measure P on $(\Omega, \mathcal{B}(\Omega))$ characterizing the null elements of $L^1(c)_+$.

$$\forall X \in L^1(c)_+, \quad X = 0 \Longleftrightarrow E_P(X) = 0$$

Canonical class of probability measure associated to $L^1(c)$

When $\mathcal{Q}$ is not finite, characteristic functions of Borelian sets are not all in $L^1(c)$ (recall: $c(f) = \sup_{Q \in \mathcal{Q}} E_Q(f) \;\; f \in \mathcal{C}_b(\Omega)$).
Issues:

- Can one associate a probability measure to $L^1(c)$?
- If yes, can one define a natural equivalence relation so that one gets a unique class characterizing the null elements in the cone $L^1(c)_+$?

Theorem

There is a probability measure P on $(\Omega, \mathcal{B}(\Omega))$ characterizing the null elements of $L^1(c)_+$.

$$\forall X \in L^1(c)_+, \quad X = 0 \Longleftrightarrow E_P(X) = 0$$

Definition

$\mathcal{M}^+(c)$ is the set of non negative finite measures on $(\Omega, \mathcal{B}(\Omega))$ defining an element of $L^1(c)^*$.

The equivalence relation $\mathcal{R}_c$ is defined on $\mathcal{M}^+(c)$ by

$$\mu \mathcal{R}_c \nu \iff \tag{11}$$

$$\{X \in L^1(c), X \geq 0 \mid \int X d\mu = 0\} = \{X \in L^1(c), X \geq 0 \mid \int X d\mu = 0\}$$

Definition

The class of P for the equivalence relation $\mathcal{R}_c$ is called the canonical c-class. It is the set of non negative measures belonging to $L^1(c)^*$ characterizing the null elements of the cone $L^1(c)_+$.

Pricing under model uncertainty

Definition

$\Pi : L^1(c) \to \boldsymbol{R}$ is a pricing function if it satisfies: monotonicity, convexity, translation invariance and normalization.

General result: DUAL REPRESENTATION THEOREM.

Specific result for Π sublinear:

Theorem

Let Ω be a Polish space. Let c as above. Every regular sublinear pricing function Π on $L^1(c)$ admits a dual representation

$$\forall X \in L^1(c), \quad \Pi(X) = \sup_{n \in N} E_{Q_n}[X]$$

$Q_n \ll P$, P belongs to the canonical c-class.

Pricing under model uncertainty

Definition

$\Pi : L^1(c) \rightarrow \mathbf{R}$ is a pricing function if it satisfies: monotonicity, convexity, translation invariance and normalization.

General result: DUAL REPRESENTATION THEOREM.

Specific result for Π sublinear:

Theorem

Let Ω be a Polish space. Let c as above. Every regular sublinear pricing function Π on $L^1(c)$ admits a dual representation

$$\forall X \in L^1(c), \quad \Pi(X) = \sup_{n \in \mathbf{N}} E_{Q_n}[X]$$

$Q_n \ll P$, P belongs to the canonical c-class.

Conclusion

- For every No Free Lunch Dynamic Pricing Procedure, the ask price process $(\Pi_{\sigma,\tau}(X))_\sigma$ has regular paths. For every R equivalent to P with zero penalty:

$$-\Pi_{\sigma,\tau}(-X)) \leq E_R(X|\mathcal{F}_\sigma) \leq \Pi_{\sigma,\tau}(X)$$

- Every dynamic pricing procedure calibrated on a reference family composed of liquid assets and options admits a dual representation in terms of equivalent local martingale measures for the liquid assets.
- Under model uncertainty ($\rightarrow$ set of probability measures):
  For the appropriate semi-norm c, $c(f) = \sup_{Q\in\mathcal{Q}} E_Q(f)\ \forall f \in \mathcal{C}_b(\Omega)$, there is a probability measure P characterizing the null elements in the cone $L^1(c)_+$.
  Its c-class is unique.

Papers

- “Bid-Ask Dynamic Pricing in Financial Markets with Transaction Costs and Liquidity Risk”, J. B.N. Journal of Mathemitical Economics, 45, 2009, p 738-750.
- “Dynamic pricing models calibrated on both liquid and illiquid assets”, J.B.N. preprint.
- “Risk measuring under model uncertainty” J.B.N. and M. Kervarec, preprint.

http://www.cmap.polytechnique.fr/~bionnada

jocelyne.bion-nadal@cmap.polytechnique.fr