Inflation Linked Bonds: An incentive for lower inflation?

Johannes Geißler

University of St Andrews

June 23, 2010

Table of contents

The Basic Economic Model

Log-real output y_t is given by

$$y_t = y_t^N + \tilde{a}(\pi_t - \pi_t^e),$$

where y_t^n denotes the so called natural rate of output.

The Basic Economic Model

Log-real output y_t is given by

$$y_t = y_t^N + \tilde{a}(\pi_t - \pi_t^e),$$

where y_t^n denotes the so called natural rate of output.
π_t and π_t^e are actual and expected rates of inflation respectively.

The Basic Economic Model

Log-real output y_t is given by

$$y_t = y_t^N + \tilde{a}(\pi_t - \pi_t^e),$$

where y_t^n denotes the so called natural rate of output.
π_t and π_t^e are actual and expected rates of inflation respectively.
$\tilde{a}$ is the slope of the Phillips curve.

The Basic Economic Model

Log-real output y_t is given by

$$y_t = y_t^N + \tilde{a}(\pi_t - \pi_t^e),$$

where y_t^n denotes the so called natural rate of output.
π_t and π_t^e are actual and expected rates of inflation respectively.
$\tilde{a}$ is the slope of the Phillips curve.

In a first order approximation the central bank's accumulated gains (in absolute real terms) over the time interval $[t; T]$ following the policy π_t is given by

$$Y_t = Y_t^n \int_t^T \tilde{a}(\pi_s - \pi_s^e) ds.$$

The Basic Economic Model

Social cost arising from inflation is assumed to be quadratic in π_t. This is weight up against the output benefit via the parameter $\tilde{\lambda}$.

The Basic Economic Model

Social cost arising from inflation is assumed to be quadratic in π_t. This is weight up against the output benefit via the parameter $\tilde{\lambda}$.Hence the bank's instantaneous benefit function reads

$$a(\pi_t - \pi_t^e) - \frac{\tilde{\lambda}}{2}\pi_t^2 = a\left(\pi_t - \pi_t^e - \frac{\lambda}{2}\pi_t^2\right),$$

where $a := \tilde{a}Y_t^N$ and $\lambda := \frac{\tilde{\lambda}}{a}$.

The Basic Economic Model

Social cost arising from inflation is assumed to be quadratic in π_t. This is weight up against the output benefit via the parameter $\tilde{\lambda}$.Hence the bank's instantaneous benefit function reads

$$a(\pi_t - \pi_t^e) - \frac{\tilde{\lambda}}{2}\pi_t^2 = a\left(\pi_t - \pi_t^e - \frac{\lambda}{2}\pi_t^2\right),$$

where $a := \tilde{a}Y_t^N$ and $\lambda := \frac{\tilde{\lambda}}{a}$.
Further we assume

$$d\pi_t^e = \gamma(\pi_t - \pi_t^e)dt \quad \text{and} \quad \pi_t = u_t + \sigma\dot{W}_t.$$

The Basic Economic Model

Social cost arising from inflation is assumed to be quadratic in π_t. This is weight up against the output benefit via the parameter $\tilde{\lambda}$.Hence the bank's instantaneous benefit function reads

$$a(\pi_t - \pi_t^e) - \frac{\tilde{\lambda}}{2}\pi_t^2 = a\left(\pi_t - \pi_t^e - \frac{\lambda}{2}\pi_t^2\right),$$

where $a := \tilde{a}Y_t^N$ and $\lambda := \frac{\tilde{\lambda}}{a}$.
Further we assume

$$d\pi_t^e = \gamma(\pi_t - \pi_t^e)dt \quad \text{and} \quad \pi_t = u_t + \sigma \dot{W}_t.$$

And hence one gets

$$d\pi_t^e = \gamma(u_t - \pi_t^e)dt + \gamma\sigma dW_t.$$

and

$$dP_t = P_t\pi_t dt = P_t(u_t dt + \sigma dW_t)$$

Inflation Linked Bond and the Structuring

Definition

An inflation linked bond (ILB) with maturity time T *issued at time* $s \in [0, T]$ *is a financial contract that pays off* $1 \cdot \frac{P_T}{P_s}$ *Dollar at time* T.

Inflation Linked Bond and the Structuring

Definition

An inflation linked bond (ILB) with maturity time T issued at time $s \in [0, T]$ is a financial contract that pays off $1 \cdot \frac{P_T}{P_s}$ Dollar at time T.

- When selling N ILB's the bank enters a liability at maturity of $N \frac{P_T}{P_s}$.

Inflation Linked Bond and the Structuring

Definition

An inflation linked bond (ILB) with maturity time T issued at time $s \in [0, T]$ is a financial contract that pays off $1 \cdot \frac{P_T}{P_s}$ Dollar at time T.

- When selling N ILB's the bank enters a liability at maturity of $N\frac{P_T}{P_s}$.
- Each ILB can be structured into a zero coupon bond B_s with face value 1 and an inflation compensation component $\frac{P_T - P_s}{P_s}$.

Inflation Linked Bond and the Structuring

Definition

An inflation linked bond (ILB) with maturity time T issued at time $s \in [0, T]$ is a financial contract that pays off $1 \cdot \frac{P_T}{P_s}$ Dollar at time T.

- When selling N ILB's the bank enters a liability at maturity of $N\frac{P_T}{P_s}$.
- Each ILB can be structured into a zero coupon bond B_s with face value 1 and an inflation compensation component $\frac{P_T - P_s}{P_s}$.
- At time s the bank buys B_s from a private bank, where the price is $e^{-r_i(T-s)}$.

Inflation Linked Bond and the Structuring

Definition

An inflation linked bond (ILB) with maturity time T issued at time $s \in [0, T]$ is a financial contract that pays off $1 \cdot \frac{P_T}{P_s}$ Dollar at time T.

- When selling N ILB's the bank enters a liability at maturity of $N\frac{P_T}{P_s}$.
- Each ILB can be structured into a zero coupon bond B_s with face value 1 and an inflation compensation component $\frac{P_T - P_s}{P_s}$.
- At time s the bank buys B_s from a private bank, where the price is $e^{-r_i(T-s)}$.
- The bank is left with the inflation compensation component, which it values in real terms.

Inflation Linked Bond and the Structuring

Definition

An inflation linked bond (ILB) with maturity time T issued at time $s \in [0, T]$ is a financial contract that pays off $1 \cdot \frac{P_T}{P_s}$ Dollar at time T.

- When selling N ILB's the bank enters a liability at maturity of $N\frac{P_T}{P_s}$.
- Each ILB can be structured into a zero coupon bond B_s with face value 1 and an inflation compensation component $\frac{P_T - P_s}{P_s}$.
- At time s the bank buys B_s from a private bank, where the price is $e^{-r_i(T-s)}$.
- The bank is left with the inflation compensation component, which it values in real terms.

Hence the liability for the bank at maturity is given by

$$-N\left(\frac{P_T - P_s}{P_s}\right) \cdot \left(\frac{P_T}{P_s}\right)^{-1} = -N\frac{P_T - P_s}{P_T} = -N\left(1 - \frac{P_s}{P_T}\right).$$

The HJB-problem

Using the instantaneous benefit and the approximation $1 - x \approx \log(x^{-1})$ for the terminal obligation we see the central bank needs to optimize

$$V(t, \pi^e, P, N) := \max_{u_v} \mathbb{E}\Bigg(a \int_t^T e^{-r(v-t)} \left(u_v - \pi_v^e - \frac{\lambda}{2} u_v^2 \right) dv \\ - e^{-r(T-t)} N \log\left(\frac{P_T}{P_s}\right) \Bigg| \pi_t^e = \pi^e, P_t = P \Bigg)$$

The HJB-problem

Using the instantaneous benefit and the approximation $1 - x \approx \log(x^{-1})$ for the terminal obligation we see the central bank needs to optimize

$$V(t, \pi^e, P, N) := \max_{u_v} \mathbb{E}\left(a \int_t^T e^{-r(v-t)} \left(u_v - \pi_v^e - \frac{\lambda}{2} u_v^2 \right) dv \right.$$
$$\left. - e^{-r(T-t)} N \log\left(\frac{P_T}{P_s}\right) \Big| \pi_t^e = \pi^e, P_t = P \right)$$

subject to

$$d\pi_t^e = \gamma(\pi_t - \pi_t^e)dt \quad ; \quad \pi_t = u_t + \sigma \dot{W}_t.$$

and

$$V(T, \pi^e, P, N) = -N \log\left(\frac{P}{P_s}\right)$$

The HJB-problem

This can be solved as

$$V(t, \pi^e, P, N) = -N \log\left(\frac{P}{P_s}\right) e^{-r(T-t)} + A_t \pi^e + C_t + D_t(N)$$

The HJB-problem

This can be solved as

$$V(t,\pi^e,P,N) = -N\log\left(\frac{P}{P_s}\right)e^{-r(T-t)} + A_t\pi^e + C_t + D_t(N)$$

with

$$\begin{aligned}
A_t &= \frac{a}{\gamma+r}\left(e^{-(\gamma+r)(T-t)}-1\right)\\
C_t &= \frac{a}{\lambda(\gamma+r)^2}\left[\frac{r}{2}\left(1+e^{-r(T-t)}\right) - re^{-(\gamma+r)(T-t)}\right.\\
&+ \left.\frac{\gamma^2}{2(2\gamma+r)}\left(e^{-r(T-t)}-e^{-2(\gamma+r)(T-t)}\right)\right]\\
D_t(N) &= e^{-r(T-t)}\left[\frac{N^2}{2a\lambda r}\left(1-e^{-r(T-t)}\right)+\frac{N\sigma^2}{2}(T-t)\right.\\
&- \left.\frac{N}{\lambda(\gamma+r)^2}\left(r(\gamma+r)(T-t)+\gamma\left(1-e^{-(\gamma+r)(T-t)}\right)\right)\right].
\end{aligned}$$

Implications for Monetary Policy

So we get

$$
\begin{aligned}
\pi_t^*(N) &= \frac{1}{\lambda(\gamma+r)} + \left(\gamma e^{-(\gamma+r)(T-t)} + r\right) - \frac{Ne^{-r(T-t)}}{a\lambda} + \sigma \dot{W}_t, \\
\pi_t^{e^*}(N) &= \pi_s^e e^{-\gamma(t-s)} + \frac{\gamma^2 e^{-(\gamma+r)T} e^{-\gamma(t-s)}}{\lambda(\gamma+r)(2\gamma+r)} \left(e^{(2\gamma+r)t} - e^{(2\gamma+r)s}\right) \\
&+ \frac{r}{\lambda(\gamma+r)} \left(e^{\gamma t} - e^{\gamma s}\right) \\
&- \frac{N\gamma e^{-rT} e^{\gamma(t-s)}}{a\lambda(r-\gamma)} \left(e^{(r-\gamma)t} - e^{(r-\gamma)s}\right) + \gamma\sigma e^{-\gamma(t-s)} \int_s^t e^{\gamma\nu} dW_\nu.
\end{aligned}
$$

Implications for Monetary Policy

So we get

$$\pi_t^*(N) = \frac{1}{\lambda(\gamma + r)} + \left(\gamma e^{-(\gamma+r)(T-t)} + r\right) - \frac{Ne^{-r(T-t)}}{a\lambda} + \sigma \dot{W}_t,$$

$$\begin{aligned} \pi_t^{e^*}(N) &= \pi_s^e e^{-\gamma(t-s)} + \frac{\gamma^2 e^{-(\gamma+r)T} e^{-\gamma(t-s)}}{\lambda(\gamma + r)(2\gamma + r)} \left(e^{(2\gamma+r)t} - e^{(2\gamma+r)s}\right) \\ &+ \frac{r}{\lambda(\gamma + r)} \left(e^{\gamma t} - e^{\gamma s}\right) \\ &- \frac{N\gamma e^{-rT} e^{\gamma(t-s)}}{a\lambda(r - \gamma)} \left(e^{(r-\gamma)t} - e^{(r-\gamma)s}\right) + \gamma\sigma e^{-\gamma(t-s)} \int_s^t e^{\gamma\nu} dW_\nu. \end{aligned}$$

Expected inflation turns negative if N is greater than

$$\frac{ar}{(1 - e^{-r(T-s)})(\gamma + r)^2} \left(r(\gamma + r)(T - s) + \gamma(1 - e^{-(\gamma+r)(T-s)})\right).$$

Pricing

By utility indifference pricing we get

$$p_s(N) = \underbrace{e^{-r_i(T-s)}}_{\text{normal bond}} - \underbrace{P_s\left(\frac{D_s(N)}{N}\right)}_{\text{inflation compensation}}$$

for the price set by the central bank.

Pricing

By utility indifference pricing we get

$$p_s(N) = \underbrace{e^{-r_i(T-s)}}_{\text{normal bond}} - \underbrace{P_s\left(\frac{D_s(N)}{N}\right)}_{\text{inflation compensation}}$$

for the price set by the central bank.On the demand side we assume a Black-Scholes type financial market:

- There is a nominal bond and interest rate r_i.

Pricing

By utility indifference pricing we get

$$p_s(N) = \underbrace{e^{-r_i(T-s)}}_{\text{normal bond}} - \underbrace{P_s\left(\frac{D_s(N)}{N}\right)}_{\text{inflation compensation}}$$

for the price set by the central bank.On the demand side we assume a Black-Scholes type financial market:

- There is a nominal bond and interest rate r_i.
- There is a Stock with volatility $\tilde{\sigma}$ and drift μ.

Pricing

By utility indifference pricing we get

$$p_s(N) = \underbrace{e^{-r_i(T-s)}}_{\text{normal bond}} - \underbrace{P_s\left(\frac{D_s(N)}{N}\right)}_{\text{inflation compensation}}$$

for the price set by the central bank.On the demand side we assume a Black-Scholes type financial market:

- There is a nominal bond and interest rate r_i.
- There is a Stock with volatility $\tilde{\sigma}$ and drift μ.
- The price level is not tradeable.

Pricing

By utility indifference pricing we get

$$p_s(N) = \underbrace{e^{-r_i(T-s)}}_{\text{normal bond}} - \underbrace{P_s\left(\frac{D_s(N)}{N}\right)}_{\text{inflation compensation}}$$

for the price set by the central bank.On the demand side we assume a Black-Scholes type financial market:

- There is a nominal bond and interest rate r_i.
- There is a Stock with volatility $\tilde{\sigma}$ and drift μ.
- The price level is not tradeable.

Therefore the market price of risk is $\rho = \frac{mu - r_i}{\tilde{\sigma}}$ and under the risk free measure we have $\tilde{E}_s(P_T) = e^{-\rho(T-s)}E(P_T)$.

Pricing

By utility indifference pricing we get

$$p_s(N) = \underbrace{e^{-r_i(T-s)}}_{\text{normal bond}} - \underbrace{P_s\left(\frac{D_s(N)}{N}\right)}_{\text{inflation compensation}}$$

for the price set by the central bank.On the demand side we assume a Black-Scholes type financial market:

- There is a nominal bond and interest rate r_i.
- There is a Stock with volatility $\tilde{\sigma}$ and drift μ.
- The price level is not tradeable.

Therefore the market price of risk is $\rho = \frac{mu - r_i}{\tilde{\sigma}}$ and under the risk free measure we have $\tilde{E}_s(P_T) = e^{-\rho(T-s)}E(P_T)$.Hence the arbitrage free price is given by

$$\tilde{p}_s(N) = e^{-(r_i+\sigma\rho)(T-s)}E_s(P_T^*(N)) = e^{-r_i(T-s)}P_s e^{\int_s^T (u_\nu^*(N)-\frac{1}{2}\sigma^2)d\nu + \int_s^T \sigma d\tilde{W}_\nu}$$

Some simulation

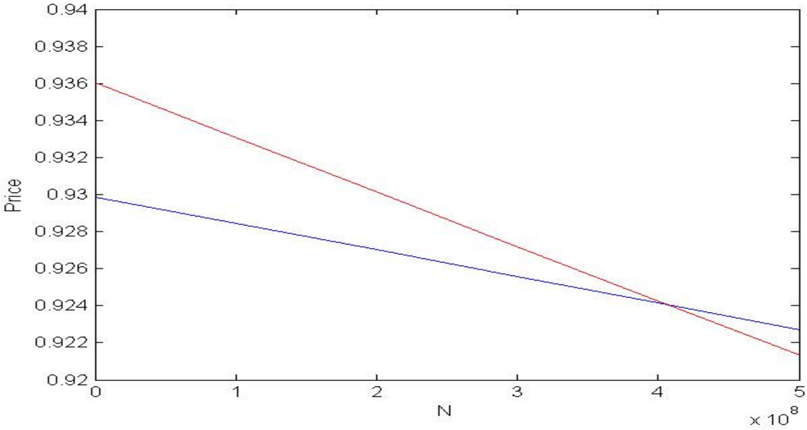

Figure: There is excess demand for ILB's whenever the Bank chooses $N \leq 4.067 * 10^8$. Supply meets demand when $N = 4.067 * 10^8$.

Some simulation

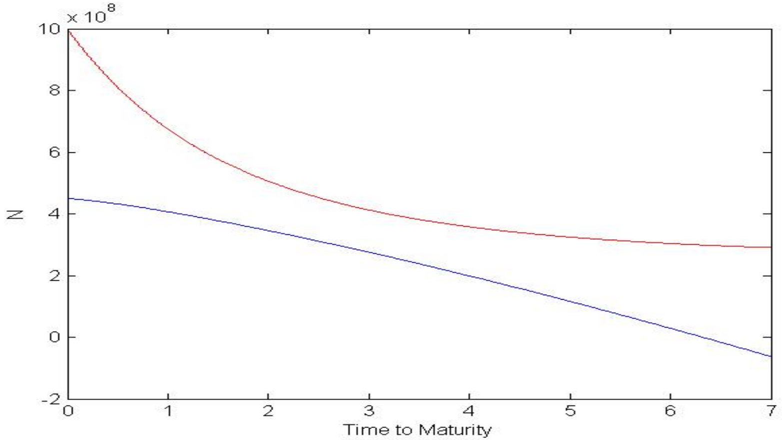

Figure: The number of ILB's the bank can issue changes in time to maturity and first becomes positive for approx. 6.5. However the equilibrium N will never lead to an expected constant price level (red line is alway above the blue line).

Some simulation

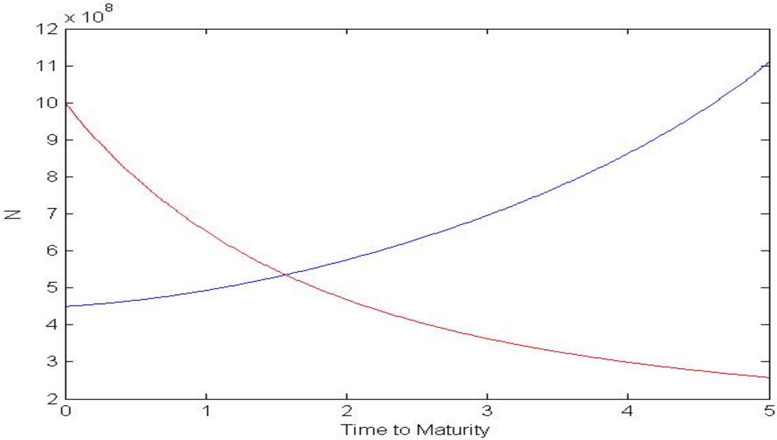

Figure: with slightly other parameters the situation changes dramatically. When issuing the equilibrium N ILB's with time to maturity of about 3 we observe decreasing expected price level.