# Assignment 3
# Foundations of Mathematical Finance - Winter 2010
# Fields Institute

17/03/2010

**1.** Let $\tau$ be a stopping time and $t \in \mathbb{R}_+$.

(i) Show that, in general, $\tau - t$ is a *not* a stopping time.

(ii) Show that $\tau + t$ is a predictable stopping time.

(iii) Show that if $\tau \equiv t$ a.s., then $\mathcal{F}_\tau = \mathcal{F}_t$ and $\mathcal{F}_{\tau^-} = \bigvee_{s<t} \mathcal{F}_s$.

**2.** Let $\tau$ be an exponentially distributed random variable with parameter $\lambda = 1$ and let $S_t = \mathbf{1}_{\{t \geq \tau\}}$ with its natural filtration. It is clear that $S_t$ is a submartingale and can be decomposed as $S = M + A$ where $A$ is the compensator

$$A_t = \int_0^{s \wedge \tau} \lambda ds = s \wedge \tau.$$

Now let

$$H_t = \begin{cases} \frac{1}{1-t}, & \text{for } 0 \leq t < 1 \\ 0, & \text{for } t \geq 1 \end{cases} .$$

Show by an explicit calculation that the stochastic integral $(H \circ S)_t$ exists, whereas $(H \circ A)_t$ doesn't.

**3.** Let $\tau$ be an exponentially distributed random variable with parameter $\lambda = 2$ and let $B$ be a Bernoulli random variable with $P[B = 1] = P[B = -1] = \frac{1}{2}$ assumed to be independent from $\tau$.

(i) Define

$$M_t = \begin{cases} 0, & \text{for } t < \tau \\ B, & \text{for } t \geq \tau \end{cases}$$

and show that $M$ is a martingale with respect to its natural filtration.

(ii) Define $H_t = 1/t$ for $t > 0$ and show that this (deterministic) process is $M$–integrable and show that its stochastic integral $X = H \circ M$ in the sense of semi-martingales is given by

$$X_t = \begin{cases} 0, & \text{for } t < \tau \\ \frac{B}{\tau}, & \text{for } t \geq \tau \end{cases}$$

(iii) Show that $E[|X_t|] = +\infty$, which implies that $X$ is not a martingale.