Interest rates and credit risk **Assignment #2** **February 10, 2010**

This assignment is due in class on Wednesday February 24, 2010.

Exercise 1. The *leverage ratio* or *debt/equity ratio* of a firm is defined to be the total debt of the firm divided by the equity $L_t = D_t/E_t$. A highly leveraged firm has a lot of debt which might indicate a danger of default, and certainly a susceptibility to rising interest rates.

1. Consider the Merton model with parameters $\mu = r = 0.05$ and $\sigma = 0.20$. Use MATLAB or similar to plot the value of the debt D_0 as a function of debt maturity T for the following leverage levels: $10, 3, 1, 1/3, 1/10$. (Hint: W.L.O.G. you may set $K = 1$. Then you will need to use a root finding method (e.g. Newton-Raphson) to find the value of A_0 for a given level of leverage.)

2. Thinking of $D_0(T)$ as the price of a zero coupon defaultable bond, compute the *credit spread* (defaultable bond yield minus default free bond yield) as a function over T, again for the same leverage levels.

Exercise 2. For any $s < T$, compute the conditional probability density function $p_\tau(t|\mathcal{F}_s), t > s$ for the time to default τ in the Black-Cox model. What is the behaviour of p_τ as $t \to s+$? Does this model have a default intensity?

Exercise 3. Course Notes Exercise 24.

Exercise 4. Course Notes Exercise 25.

Exercise 5. Course Notes Exercise 28.

Exercise 6. Let the spot interest rate r_t be $\mathcal{G}_t$-adapted. Use Proposition 5.3.1 directly to prove *Lando's formula* for the price of a zero-coupon, zero-recovery bond:

$$H_t^c \bar{P}_t(T) := E^Q[H_T^c e^{-\int_t^T r_s ds}|\mathcal{F}_t] = H_t^c E^Q[e^{-\int_t^T [r_s + \lambda_s] ds}|\mathcal{G}_t] \tag{1}$$