Irreversibility of Financial Time Series

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June 9, 2010

1 Introduction

The flow of time from the past to the future is something that most people take for granted, but what is it that makes it so intuitively obvious? Some processes have no apparent direction of time. For example, take a video of a ball flying through a vacuum. played forwards or backwards, the sequence of images looks 'right'. In fact if the original direction in which the video should be played was lost, it would be impossible to say which way it should be played.

The reason it looks 'right' forwards or backwards is that the physical equations that describe the motion of the ball are symmetric with respect to time reversal. The trajectory of the ball is described by Newton's law $F = ma$. The path is found by integrating this equation *twice* with respect to time. The double integration is what ensures that the solution, the trajectory, is symmetric with respect to time reversal.

However, there are many clear examples of non-reversible processes. The classic example is a mixing process. Take a glass of water, add some dye and video the dye as it spreads out into the water until it is uniformly spread throughout the glass. Now, this video tape played backwards will look 'wrong'; glasses of dyed water do not spontaneously unmix themselves by having the dye accumulate at one point. Why is this? In fact the dynamics of the molecules in the dye and water are reversible. They can be played backwards and still make sense. A microscopic examination of the unmixing process would force us to conclude that the process is 'right'. Yet we know it is 'wrong'. This wrongness must be a result of the statistical nature of the process.

Even though the unmixing process is microscopically 'right' it is exceedingly unlikely. The number of configurations of the molecules that will at some future point lead to an unmixed glass of water is far, far less than the number of configurations that do not. In fact they are so fewer that even if the glass of dyed water was left for many lifetimes of the universe it still would not unmix itself. This is how time-irreversibility creeps into the universe. Statistical processes evolve from unique or special states to typical or non-special states, because they have a very high probability of doing so. They do not go from typical to special states because the probability of doing so is very, very low. This, essentially, is the second law of thermodynamics.

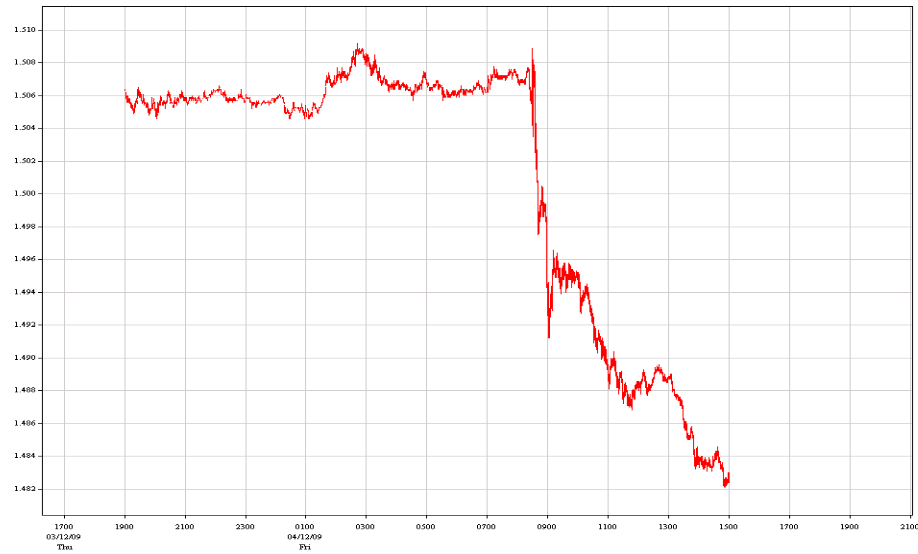

Figure 1: Euro/USD futures contract on 4 December 2009

2 Are Financial Processes Reversible?

Our own expertise suggests the answer to this question is a resounding no! Consider the trading in the Euro/USD FX rate futures on 4 December 2009 (fig. 1).

At 8:30 am, an announcement is made concerning the US jobs market and suddenly market volatility spikes. The market volatility continues for some time before slowly fading. Consider these events if time is reversed. Volatility crescendos into the 8:30 announcement before suddenly dissappearing. It is our opinion that events like this provide the direction of time in the financial markets. It can be compared to mixing dye into a glass of water. News is added to financial system, putting the system in a state akin to the glass of water just after the dye is added. The news disseminates throughout the financial system before a typical state is reached again. Just as the dyed water does not unmix, the financial system can not ”undisseminate” the news; the reverse process of dissemination does not happen.

The work of Chen *et al* [2], based on 10 years of Stock Market data shows that the data is irreversible. What we would like to understand in more detail is whether this irreversibilty is local to events such as the one discussed above or whether it is always apparent in the data. We would also like to know more about the data coarseness and sample sizes on which irreversibility is detectable and how, if at all, time-irreversibility at one data coarseness, say 1 minutely data, can affect time-irreversibility on another data coarseness, say daily data.

We'd also like to know whether the type of irreversibility observed is the same in all types of markets.

3 Definition of a Reversible Time-Series?

A process Y_t is said to be time-reversible if the finite dimensional distribution functions of $Y_{t_1}, Y_{t_2}, \ldots, Y_{t+n}$ and $Y_{t_n}, Y_{t_{n-1}}, \ldots, Y_{t_1}$ are the same for any n. This seems a reasonable definition.

Now there are two branches to take from this point. Can we tell if a stochastic process is time-reversible? And how can we test whether a time-series was generated by a time-reversible process?

The first is an exercise in proving that the definition of the stochastic process implies the definition of time-reversibility. It is easy to see that any iid process is time-reversible. It has also be proven that all stationary Gaussian processes are time-reversible[1]. This implies, of course, that geometric brownian motion, often used to model log return series of stock prices is a reversible process. Evidence that financial time-series are not reversible would be another flaw in this widely used model.

For the latter there are already a number of statistical tests that exist. Chen *et al* [2] look at the test statistic $E[\sin(\omega X_{t,k})]$ where $X_{t,k} = Y_t - Y_{t-k}$, this value is zero if the process is time-reversible. Lim *et al*[3] look at the imaginary part of the double Fourier transform of the third cumulant of the series difference. Both these tests rely on the fact that the Fourier transform of a symmetric function is real for all frequencies (i.e. it requires only cosine waves to describe it). See also [4].

4 Problems to be addressed

Empirical Problems We will provide a set of intra-day data for several different futures contracts, each broken into a training set in which the correct time orientation is given and a test set in which the correct orientation is to be determined.

1. Can the group develop a test to determine the correct time-orientation of each of the futures contracts?
2. Can the group develop and describe a reliable "universal" test which works for all the contracts? In other words, one test appropriate for all of the different contracts?

Theoretical Problems We'd like the group to investigate the reversibility of popular processes used in the literature to model financial data.

1. We'd be interested in examining the GARCH model. The literature and our intution hints that GARCH returns are irreversible. Can this be shown?

2. Repeat Problem 1 for other common (and less common!) models for stock returns, including the multi-fractal model [5], Levy process models, and their special case of jump diffusion models.
3. What about common interest rate models like Vasicek, CIR, Hull-White, etc?

References

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