BOOTSTRAP-BASED TESTS FOR TRENDS IN HYDROLOGICAL TIME SERIES, WITH APPLICATION TO ICE PHENOLOGY DATA

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MOTIVATION

It has been documented that the global average surface temperature has increased by about 0.6^oC over last 100 years due to atmospheric concentrations of trace gases such as carbon dioxide (IPCC, 2001).

What implications do green-house gases have on:

- precipitation?
- streamflow?
- lake ice phenology (ice cover duration, freeze-up and break-up dates)?

APPROACH: check for trends in hydro-meteorological data

AVAILABLE TESTS FOR TRENDS AND DEPENDENCE EFFECT

Consider the case of a possible linear trend

$$y_t = a + bt + e_t.$$

We are interested in $H_0 : b = 0$ vs. $H_1 : b \neq 0$.

MOST POPULAR TREND TESTING PROCEDURES:

classical t-test;

rank-based Mann-Kendall test;

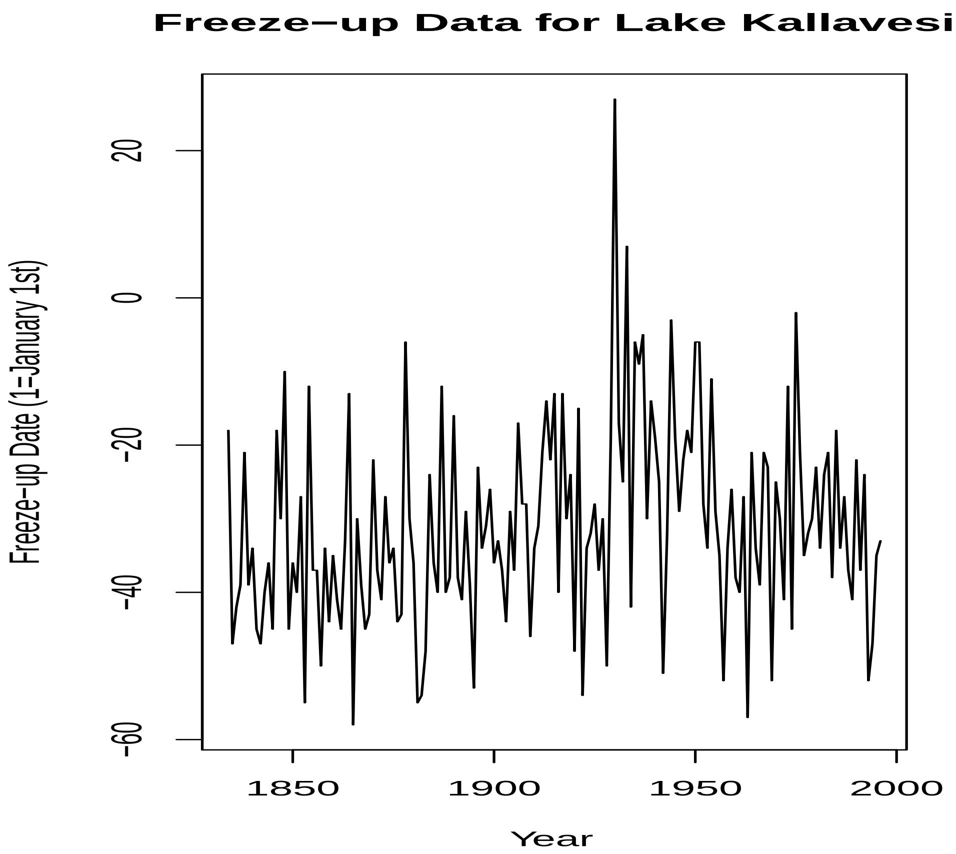
rank-based Sen's slope.

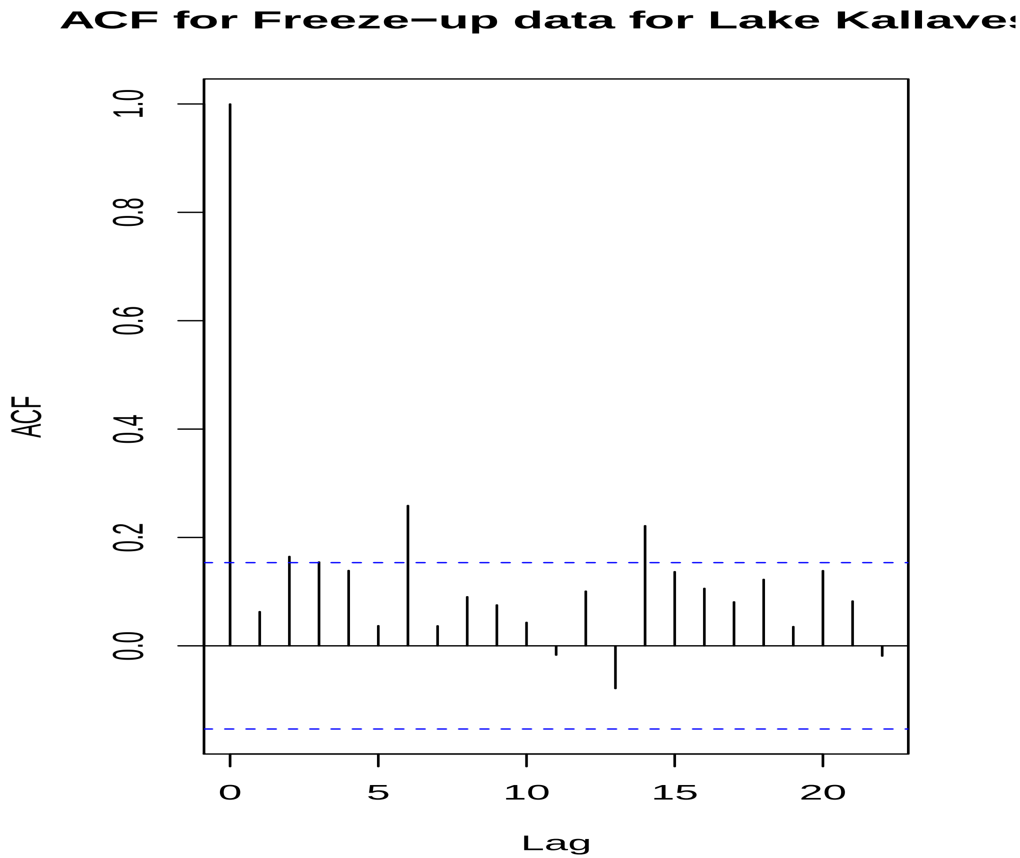
AVAILABLE TESTS FOR TRENDS AND DEPENDENCE EFFECT: CONTD

All of these tests assume that the data are independent and identically (normally) distributed. Impact of violations is:

- minor for the distributional assumption;
- potentially **disastrous** for the independence assumption $\Rightarrow$ inflated Type I Error and over-rejection of H_0.

But hydrological data typically exhibit a strong serial correlation!





Time series and ACF plot of observed freeze-up dates for Lake Kallavesi, Finland, 1834–1996.

AVAILABLE TESTS FOR TRENDS AND DEPENDENCE EFFECT: CONTD

Nominal level $\alpha = 0.05$		
Distribution	Mann-Kendall	Student's t
Normal	0.373	0.361
Exponential	0.363	0.406
Lognormal	0.361	0.357
t_5	0.359	0.365

Observed Type I error for the simulated data from the autoregressive model. Sample size is 128 measurements (equals to the number of the break-up observations for Lake Baikal). Number of MC simulations is 10000.

OVER-REJECTION IS UP TO 8 TIMES!

SIEVE BOOTSTRAPPED TREND TESTS

1. Approximate $\{y_i\}_{i=1}^{T}$ by an autoregressive filter, AR$(p(T))$
2. Estimate the AR parameters $\widehat{\phi} = (\widehat{\phi}_1, \ldots, \widehat{\phi}_p)'$, using YW or LS
3. Get the residuals $\widehat{v}_t = \sum_0^p \widehat{\phi}_j (y_{t-j} - \bar{y})$; $\widehat{\phi}_0 = 1$
4. Draw a resample of v_t^* from $\tilde{v}_t = \widehat{v}_t - \bar{v}$
5. Define y_t^* by the recursion $\sum_0^p \widehat{\phi}_j (y_{t-j}^* - \bar{y}) = v_t^*$
6. Compute trend test statistic Tr^* on y_t^*
7. Repeat B times

Then, $\mathrm{SB\ \ p-value} = \frac{\#(|\mathrm{Tr}_1^*|,\ldots,|\mathrm{Tr}_B^*|)\geq|\hat{\mathrm{T}}\mathrm{r}|}{\mathrm{B}}$

SIMULATIONS FOR SIEVE BOOTSTRAPPED TREND TESTS: SIZE

Distribution	ARMA	Sen's	MK	t-test
Exp(0.1)	AR(1)	0.044	0.045	0.046
$\sqrt{\beta_1} = 2$	AR(2)	0.059	0.054	0.055
$\beta_2 = 9$	AR(6)	0.058	0.044	0.062
	ARMA(1, 1)	0.056	0.049	0.058
Lognormal(3.642, 0.25)	AR(1)	0.057	0.051	0.052
$\sqrt{\beta_1} = 0.778$	AR(2)	0.061	0.054	0.051
$\beta_2 = 4.096$	AR(6)	0.065	0.059	0.056
	ARMA (1, 1)	0.062	0.056	0.061
t_5	AR(1)	0.060	0.057	0.055
$\sqrt{\beta_1} = 0$	AR(2)	0.053	0.048	0.052
$\beta_2 = 9$	AR(6)	0.062	0.048	0.055
	ARMA (1, 1)	0.059	0.044	0.054
$N(0, 1)$	AR(1)	0.059	0.050	0.053
$\sqrt{\beta_1} = 0$	AR(2)	0.063	0.053	0.050
$\beta_2 = 3$	AR(6)	0.064	0.055	0.057
	ARMA (1, 1)	0.060	0.049	0.047

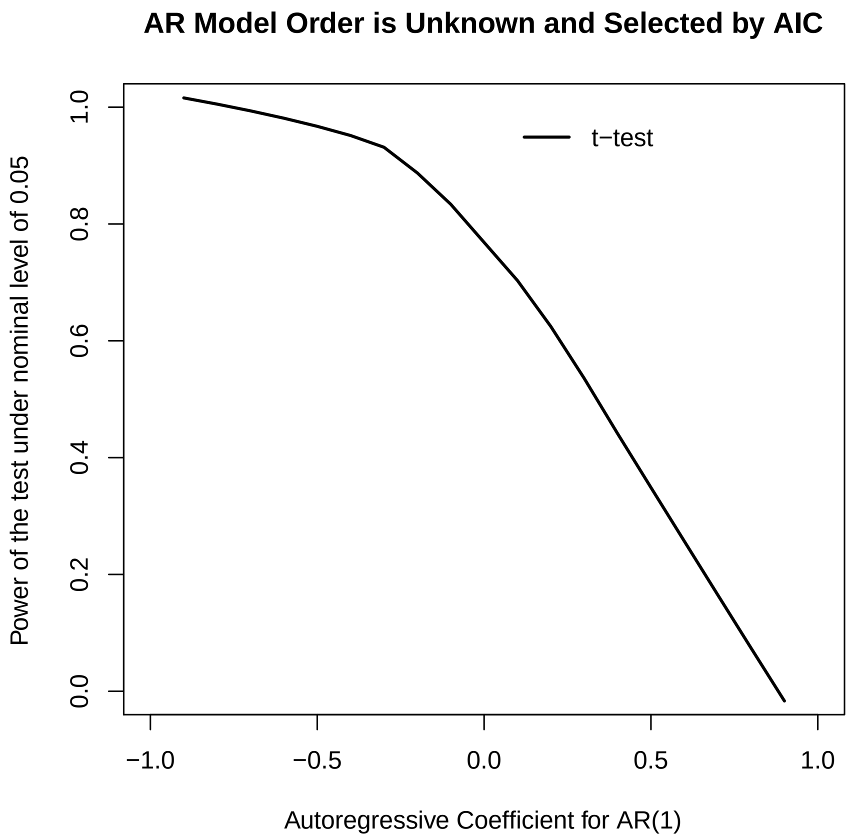
B is 1000; MC is 1000; Sample Size is 200; α is 0.05.

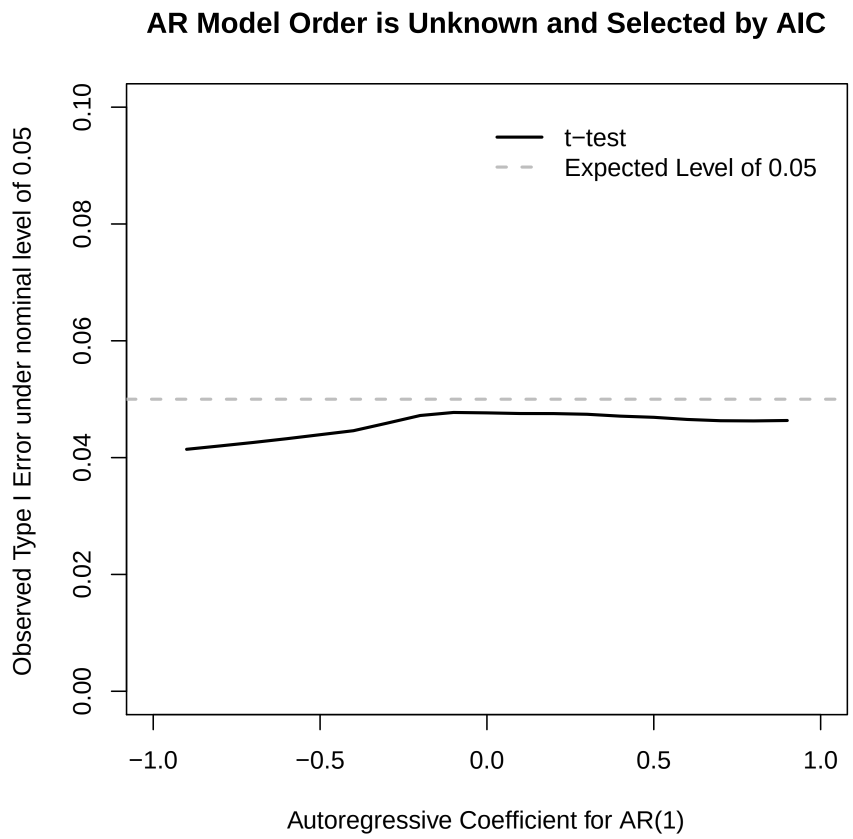
SIMULATIONS FOR SB TREND TESTS: POWER

Distribution	ARMA	Sen's	MK	t-test
Exp(0.1)	AR(1)	0.920	0.933	0.702
$\sqrt{\beta_1} = 2$	AR(2)	0.916	0.940	0.718
$\beta_2 = 9$	AR(6)	0.401	0.453	0.285
	ARMA(1, 1)	0.866	0.890	0.674
Lognormal(3.642, 0.25)	AR(1)	0.730	0.718	0.661
$\sqrt{\beta_1} = 0.778$	AR(2)	0.736	0.720	0.670
$\beta_2 = 4.096$	AR(6)	0.348	0.333	0.277
	ARMA (1, 1)	0.685	0.683	0.627
t_5	AR(1)	0.779	0.783	0.663
$\sqrt{\beta_1} = 0$	AR(2)	0.802	0.798	0.668
$\beta_2 = 9$	AR(6)	0.377	0.394	0.285
	ARMA (1, 1)	0.725	0.714	0.617
$N(0, 10^2)$	AR(1)	0.679	0.662	0.635
$\sqrt{\beta_1} = 0$	AR(2)	0.703	0.681	0.670
$\beta_2 = 3$	AR(6)	0.296	0.261	0.237
	ARMA (1, 1)	0.678	0.646	0.630

B is 1000; MC is 1000; Sample Size is 200; α is 0.05; b is 0.04.

SIMULATIONS FOR SIEVE BOOTSTRAPPED TREND TESTS





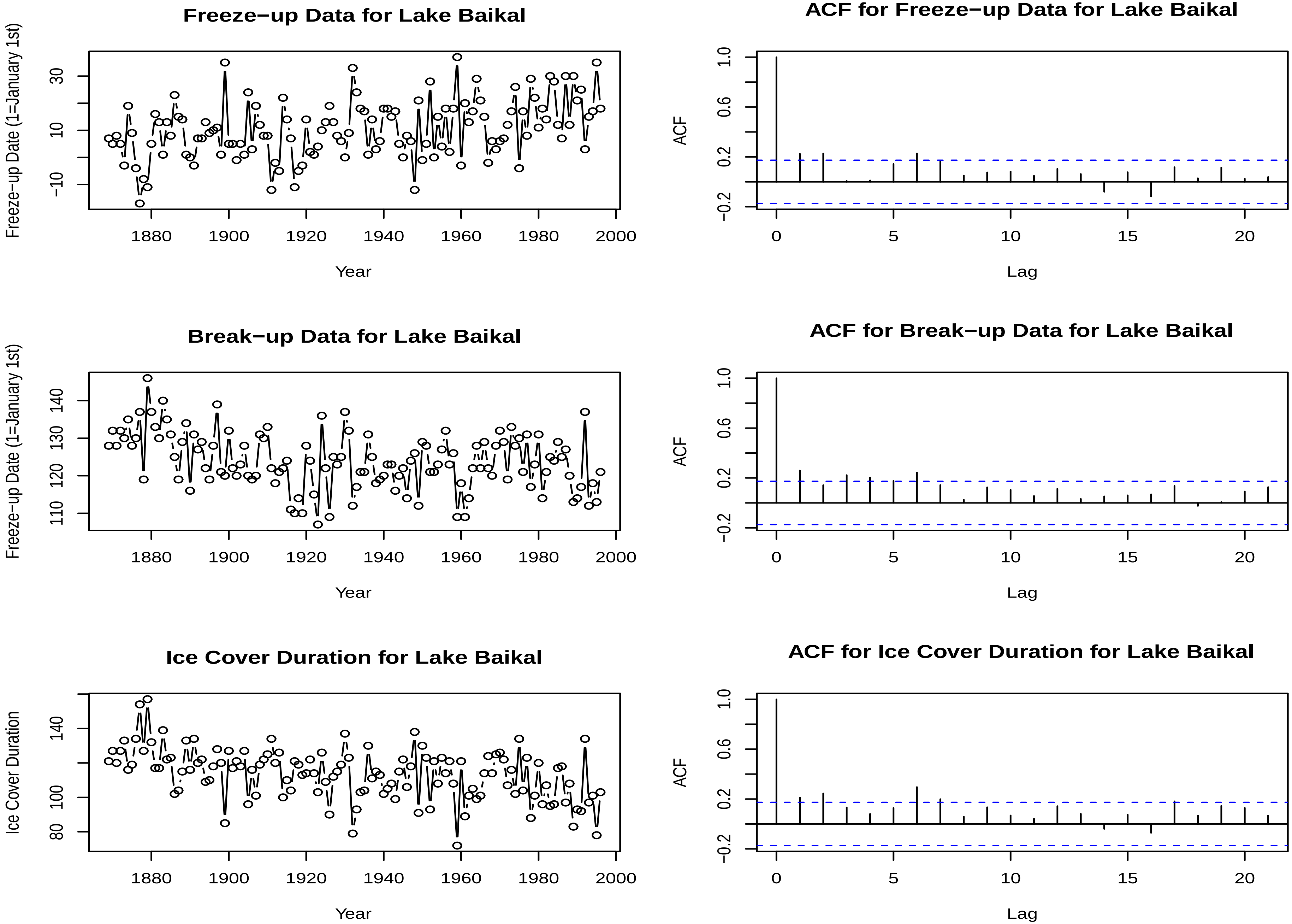
Power (left) and size (right) of the t-test in respect to the magnitude of the autocorrelation parameter, under $N(0,1)$.

CASE STUDY: LAKE KALLAVESI, FINLAND

Trend Test	Critical Values	Order Selection	Freeze-up Dates
p-values for tests for independent data			
Student's t	t_{n-2}	-	9.91×10^{-3}
Mann-Kendall	$N(0,1)$	-	3.14×10^{-3}
p-values for tests for dependent data			
Student's t with AR(6)	Sieve	AR(6)	3.05×10^{-1}
Mann-Kendal with AR(6)	Sieve	AR(6)	1.16×10^{-1}
Sen's slope with AR(6)	Sieve	AR(6)	1.14×10^{-1}
Likelihood Ratio with AR(6)	Sieve	AR(6)	3.47×10^{-1}
Likelihood Ratio with AR(6)	χ^2_1	AR(6)	2.50×10^{-1}

Number of bootstrap replications is 10000. Sample size is 163.

RESULTS CHANGED FROM HIGHLY STATISTICALLY SIGNIFICANT TO INSIGNIFICANT!



Summary plot of for Lake Baikal, Russia, 1869–1996.

CASE STUDY: LAKE BAIKAL, RUSSIA

Dataset / Method	ACF Plot	AIC
Freeze-up Dates	7	6
Break-up Dates	6	6
Ice Cover Duration	7	6

Summary of the selected orders for the approximating AR(p) models, identified using the ACF plots and AIC.

CASE STUDY: LAKE BAIKAL, RUSSIA

Trend Test	Critical Values	Order Selection	Freeze-up Dates	Break-up Dates	Ice Cover Duration
p-values for tests for independent data					
Student's t	t_{n-2}	-	6.18×10^{-6}	2.85×10^{-4}	1.39×10^{-7}
Mann-Kendall	$N(0,1)$	-	2.61×10^{-5}	9.67×10^{-4}	4.74×10^{-7}
p-values for tests for dependent data					
Student's t with AR(p)	Sieve	ACF	9.90×10^{-3}	2.77×10^{-1}	1.81×10^{-2}
		AIC	3.93×10^{-2}		8.60×10^{-3}
Mann-Kendall with AR(p)	Sieve	ACF	3.15×10^{-2}	1.04×10^{-1}	2.25×10^{-2}
		AIC	2.65×10^{-2}		6.20×10^{-3}
Sen's slope with AR(p)	Sieve	ACF	3.05×10^{-2}	8.45×10^{-1}	2.80×10^{-2}
		AIC	2.16×10^{-2}		5.00×10^{-3}
Likelihood Ratio with AR(p)	Sieve	ACF	8.90×10^{-3}	3.18×10^{-1}	6.20×10^{-3}
		AIC	6.82×10^{-2}		2.90×10^{-3}
Likelihood Ratio with AR(p)	χ^2_1	ACF	1.14×10^{-2}	1.87×10^{-1}	2.47×10^{-2}
		AIC	1.21×10^{-2}		1.70×10^{-2}

Number of bootstrap replications is 10000. Sample size is 128.

RESULTS FOR BREAK-UP DATA CHANGED FROM HIGHLY STATISTICALLY SIGNIFICANT TO INSIGNIFICANT!

p-**VALUES FOR FREEZE-UP AND ICE COVER DURATION ALSO INCREASED SUBSTANTIALLY**

DISCUSSION

1. Overall impact of serial correlation on trend tests can be disastrous and can lead to unreliable or even false conclusions, especially for highly correlated hydro-meteorological data

2. New sieve bootstrap-modified trend test are robust across correlation models and distributions

3. Remarkably, the recent analysis of Kouraev et al. (2007) notes the lack of a clear trend in the Baikal break-up dates after the 1920s, which is confirmed by our SB tests.